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ONE-PARAMETER BIFURCATION ANALYSIS OF PREY-PREDATOR MODEL WITH HARVESTING STRATEGIES

(Analisis Dwicabangan Satu-Parameter bagi Model Mangsa-Pemangsa dengan Strategi Penuaian)

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ABSTRACT

This study investigates the effect of changes in the parameter of harvesting effort in both prey and predator species. As we know, prey and predator correlate to each other. Thus, it is important to know the dynamics of their population when the interaction is affected by the harvesting activity. To do this, we consider an ecological model of prey-predator interactions with the presence of harvesting effort. Then, we employ stability analysis, bifurcation analysis and numerical simulations to illustrate the dynamical behaviours of the prey-predator system. This study also analyses the behaviour of prey-predator interactions as the harvesting parameters of prey and predator species are varied. With the help of mathematical software such as XPPAUT and Matlab, a few graphs of bifurcation, phase plane, and time series are plotted. Maple software is used to find the Jacobian matrix and also the critical points. By using bifurcation analysis, there is an occurrence of one transcritical bifurcation point. Our finding demonstrates that as the harvesting parameter exceeds the transcritical bifurcation point, the prey-predator system changes from stable to unstable or vice versa.

Keywords: harvesting activity; prey-predator model; stability analysis; transcritical bifurcation

ABSTRAK

Kajian ini menyelidik kesan perubahan parameter penuaian kepada spesies mangsa dan pemangsa. Seperti yang diketahui, mangsa dan pemangsa saling berhubungan. Oleh itu, adalah penting untuk diketahui dinamik populasi mangsa dan pemangsa apabila interaksi tersebut dipengaruhi oleh aktiviti penuaian. Untuk tujuan tersebut, digunakan model ekologi (perikanan) mangsa-pemangsa dengan melibatkan aktiviti penuaian. Kemudian, digunakan analisis kestabilan, analisis dwicabangan dan simulasi berangka untuk menggambarkan tingkah laku dinamik sistem mangsa-pemangsa. Kajian ini juga menganalisis tingkah laku interaksi mangsa-pemangsa apabila parameter penuaian spesies mangsa dan pemangsa berubah. Beberapa jenis graf telah dibina dengan menggunakan bantuan perisian matematik seperti XPPAUT dan Matlab. Perisian Maple juga digunakan untuk mencari matrik Jacobi dan juga titik kritikal. Dengan menggunakan analisis dwicabangan, didapati wujud satu titik dwicabangan transkritikal. Hasil kajian menunjukkan bahawa apabila parameter penuaian melebihi titik dwicabangan transkritikal, sistem mangsa-pemangsa berubah daripada stabil kepada tidak stabil dan juga sebaliknya.

Kata kunci: aktiviti penuaian; model mangsa-pemangsa; analisis kestabilan; dwicabangan transkritikal

1. Introduction

1.1. *The prey-predator system*

The prey-predator relationship is the interaction between two or more organisms that consequently affect each other towards their population and ecosystem. In this relationship, one is hunting the other one. The one that hunts is the predator; meanwhile, the attacked one is the prey. These two species are related to the ecosystem. The prey-predator relationship develops over time as many generations of the species has been interacting with each other. Scientists studying the population dynamics have noticed that the prey-predator relationship has greatly impacted the population of the species (Chakraborty & Das 2015; Dubey 2007; Yang & Jia 2017). The prey is part of the predator's environment, and within the absence of prey, the predator dies as the predator fully depends on prey as its basic living, such as for growth and reproduction.

Prey-predator interaction occurs in all populations. Ecologists have reported examples of prey-predator interactions in various species, such as fish, plankton, birds, and mammals, including huge herbivorous and carnivorous (Stevens 2012). As for fishery model, prey-predator are often influenced by the size and structure of the fish. Normally, large fish such as the shark (selachimorpha) naturally devour the smaller fish populations. Usually, these predators have their own predatory tactics to attack their prey, such as ambushing and they often target small, weak fish to conserve their energy. Thus, numerous researchers have focused on the issues associated with multispecies fisheries, particularly in prey-predator modelling (Ang *et al.* 2018; Chakraborty *et al.* 2013; Fitzgerald *et al.* 2019; Kar & Matsuda 2007; Lv *et al.* 2013).

Independently Lotka-Volterra has proposed a system of two equations for investigating the prey-predator interaction. Lotka-Volterra method is the first system developed of mathematical ecology and the simplest prey-predator interaction model. These are the premises in this model from Hussein (2010) as follows:

- In the Malthusian way, the prey grows unbounded in the absence of a predator.
- The impact of predation is to reduce the rate of increase of prey in relation to the prey and predator population.
- The death rate of the predator induces exponential decay because there is an absence of prey and predator population.
- The consumption rate term, which relates to the available prey as well as the size of the predator population.

1.2. *Harvesting effort*

Nowadays, harvesting is a common situation where a constant number of populations are removed during each time period. This activity can somehow leave a big impact on the system's dynamic. This happens due to excessive demand for resources, which led to the over exploitation in some biological resources. There is also a need to establish a sustainability policy in the different fields of human activity to preserve the environment (Sahoo *et al.* 2016). Shatnawi (2016) presented a deterministic, continuous Lotka-Volterra model which has been expanded by including the time delay in growth rate for prey; meanwhile for the predator, it has a constant rate of harvesting.

While in Das *et al.* (2009), the authors have discussed the effect of toxicants by some other species in prey-predator system in which both species are harvested. They modified the Lotka-Volterra system by considering the environmental factors to limit the growth of prey population to a finite range without the predator. The local and global stabilities are examined. Alternatively, Ang *et al.* (2018) investigated the prey-predator system in a fisheries model, taking into account the impact of toxicants and harvesting strategies. The harvesting effort in the model is observed by varying the harvesting parameter which affects both prey and predator population. Based on the steady state diagram in their research, low level of harvesting activities leads to coexistence, and inversely, a higher level of harvesting can lead to the extinction of the entire population within a short time.

1.3. *Bifurcation theory*

Bifurcation theory is the mathematical analysis of changes in a given system of qualitative or topological structure. It occurs when a slight smooth change in a system's parameter (bifurcation parameter) induces a sudden qualitative change in its behaviour. Bifurcation exists in both continuous and discrete structures. There are two main bifurcation classes: local bifurcation, which can be studied by modifying the local stability characteristics of equilibria or other qualitative sets; and global bifurcation, frequently occurring whenever the system's greater qualitative sets overlap with one another or with the system's equilibrium.

Bifurcation analysis is done by varying the parameter and known as the bifurcation parameter. Manaf and Mohd (2019) investigated two systems of prey-predator incorporating the herd behaviours. For each bifurcation parameter, the stability and bifurcation analysis result is obtained by using the Matlab and XPPAUT, whereas the bifurcation parameter increase, the behaviour of both models become unstable. Kar (2003) has come out with a model based on Generalized Gauss type prey predator model with harvesting that is stated to be harmless under certain conditions depending on delay. It showed that when there is time delay, there is also switching in stability. Abdul Satar and Naji (2019) studied a food web model consisting of prey predator-scavenger represented by a linear type of harvest and nonlinear type of environment toxicant. The stability of equilibriums and bifurcation analysis is used to understand the effects of varying the system parameters.

This paper will focus on the transcritical bifurcation which is a specific type of local bifurcation. For all values of a variable, a transcritical bifurcation is one where a fixed point remains and is never demolished. However another fixed point interchanges its stability with the other fixed point as the variables vary (Strogatz 2018). In other words, there is an unstable and a stable fixed point both before and after the bifurcation. As they overlap, their stability is exchanged. Therefore, the stable fixed point become unstable and vice versa.

This paper is organised as follows. Section 2 represents the prey-predator system with harvesting effort. The existence of steady states, stability and bifurcation analysis are discussed in Section 3. Section 4 contains the results and discussions, while the conclusions are discussed in Section 5.

2. Prey-predator System with Harvesting Effort

The system used in this study consists of prey-predator interaction in the harvested fishery model. By omitting the toxicant parameter in the equation from Ang *et al.* (2018), which give the system of the differential equation as below:

$$\frac{dX}{dt} = x\left(1 - \frac{X}{K}\right) - \gamma_1 XY - aEX$$
$$\frac{dY}{dt} = r_2 Y + \gamma_2 XY - cEY$$

where X and Y represent the prey and predator populations, respectively. The parameter K denotes the prey population's environmental carrying capacity, implying that the prey population increases logistically in the absence of predator species. These two species are affected by combining harvesting effort E, where a and c represents the catchability coefficients of prey and predator species, respectively. All of these parameters are considered to have positive values. The model is non-dimensionalised to minimise the number of parameters and simplify the model for better understanding (Ang *et al.* 2018). The model is shown as below:

$$\begin{aligned} \frac{dx}{dy} &= x(1-\alpha x) - xy - \beta x \\ \frac{dy}{dt} &= -\delta y + xy - \varepsilon y \end{aligned} \tag{1}$$

where

$$\alpha = \frac{r_1}{k\gamma_2}, \quad \beta = \frac{aE}{r_1}, \quad \delta = \frac{r_2}{r_1}, \quad \varepsilon = \frac{cE}{r_1}$$

and the parameters α, β, δ and ε are all positive constants. Variable x represents the prey population, and variable y represents the predator population. Parameter α denotes the ratio of growth rate to the product of growth rate multiplied by x and environmental carrying capacity. While parameter β is the ratio of catchability coefficient and harvesting effort to the growth rate, parameter δ defines the y to x growth rate ratio. Finally, the parameter ε represents the product of catchability coefficient and harvesting effort to the growth rate.

3. Stability and Bifurcation Analysis

To find the steady states or critical points, the prey-predator system (1) is assumed to be equal to zero. Then the system (1) is solved by using Maple software in order to form the Jacobian matrix as represented by:

$$J_{(x,y)} = \begin{bmatrix} (1-\alpha x) - \alpha x - y - \beta & -x \\ y & -\delta + x - \varepsilon \end{bmatrix}$$

The eigenvalues of each critical point can be determined by substituting the critical points into the Jacobian matrix. From the eigenvalues that have been obtained, we can conclude that either the system is stable or unstable by looking at the signs of the eigenvalue either it is all positive, all negative or opposite signs. The critical point is stable if all eigenvalues are negative signs. Meanwhile, the critical point is unstable if all the eigenvalues have the opposite sign. For bifurcation analysis, we captured the transcritical bifurcation point using numerical software XPPAUT. This transcritical bifurcation point indicates the changes in the stability of the system. The changes in stability between prey and predator can be analysed by varying the harvesting parameter β and ε.

In this paper, the analysis is divided into two cases. For case I, the harvesting parameter of prey, β is varied while for case II, the harvesting parameter of the predator ε is varied. For both cases, the initial population for prey $x(t)$ and predator $y(t)$ are set to 1.2 and 1.0 respectively (Ang *et al.* 2018). All the parameter values for each case are obtained by trial and error, which are summarised in Table 1.

Table 1: Parameter used in the prey-predator system (1)

Parameter	Definition	Parameter values	
		Case I	Case II
α	the ratio of growth rate to the product of growth rate multiplied by x and environmental carrying capacity	0.85	0.85
β	the ratio of catchability coefficient and harvesting effort to the growth rate (harvesting parameter for prey x)	0.7, 1.32 (varied)	0.15
δ	the y to x growth rate ratio	0.85	0.85
ε	the product of catchability coefficient and harvesting effort to the growth rate (harvesting parameter for predator y)	0.85	0.05, 0.2 (varied)

4. Results and Discussions

To analyse the model dynamical behaviour, the parameter variation technique is employed in association with the numerical software XPPAUT. All parameters α, β, δ, ε are set to 0.85, 0.15, 0.85 and 0.85, respectively. These values are chosen since when the XPPAUT is performed, we can locate the transcritical bifurcation point, where the system stability interchanges. As a result, it can assist us in achieving our main goal, which is to investigate the system stability in the presence of harvesting effort.

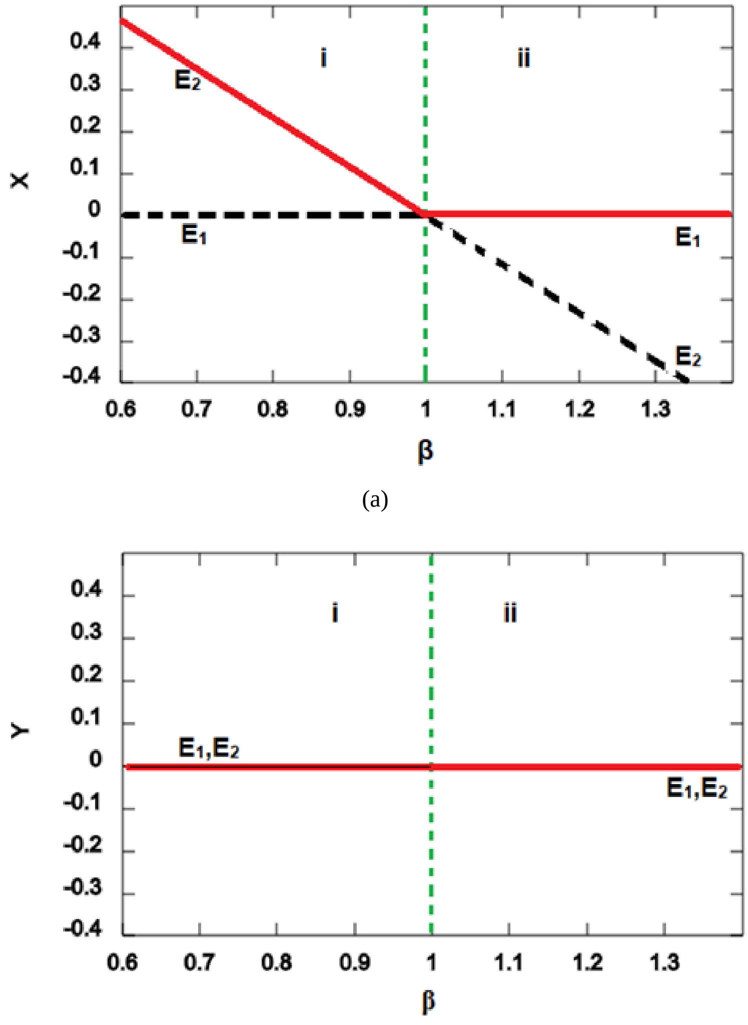
4.1. *Case I: Harvesting activities in prey populations*

This section examines the influence of the harvesting parameter on the prey population in system (1). The steady state diagrams for the parameter β are shown in Figure 1. Region (i) is the region of the behaviour for both populations before reaching the transcritical bifurcation point while region (ii) is the behaviour for both populations after crossing the transcritical bifurcation point. To illustrate, the solid red lines indicate the stable steady states, whereas the black dashed lines represent the unstable steady states. The transcritical bifurcation point at $\beta = 1$ is depicted by the vertical green dashed line.

By using bifurcation analysis, there are different types of behaviour we can see as the harvesting parameter of prey β changes. When $\beta = 0.7$, single-species steady state, $E_2(0.35, 0)$ is stable and the prey species is free from the predator, as the predator has become extinct at this state. However, as the harvesting parameter exceeds the transcritical point which is at $\beta = 1.32$, the steady state E_2 becomes unstable. Meanwhile for steady state $E_1(0,0)$, as we increase the parameter value of β, the system switches from unstable state to stable state. For steady state $E_3(1.7, -1.15)$, there is no change in the stability before and after passing the transcritical bifurcation point thus, we do not label E_3 in the bifurcation diagram in Figure 1. The changes in stability are summarised in Table 2.

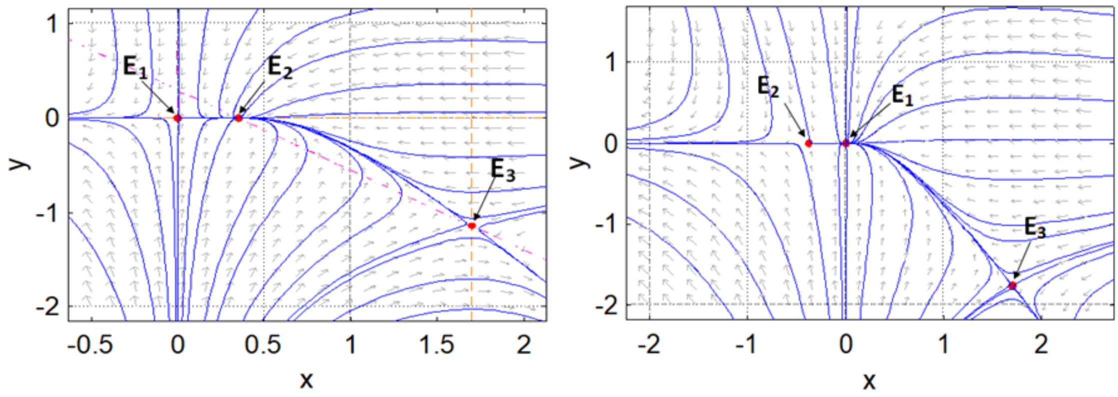
Table 2: Summary of stability analysis with respect to harvesting parameter β

Bifurcation parameters	Steady-states	Eigenvalues	Characteristics	Figure
$\beta = 0.7$	$E_1(0,0)$	$\lambda_1 = -1.7$, $\lambda_2 = 0.3$	Unstable saddle node	2(a)
	$E_2(0.35,0)$	$\lambda_1 = -0.3$, $\lambda_2 = -1.3471$	Asymptotically stable node	
	$E_3(1.7,-1.15)$	$\lambda_1 = -2.2936$, $\lambda_2 = 0.8487$	Unstable saddle node	
$\beta = 1.0$		Transcritical bifurcation point		
$\beta = 1.32$	$E_1(0,0)$	$\lambda_1 = -0.32$, $\lambda_2 = -1.7$	Asymptotically stable node	2(b)
	$E_2(-0.38,0)$	$\lambda_1 = 0.32$, $\lambda_2 = 2.0765$	Unstable node	
	$E_3(1.7,-1.77)$	$\lambda_1 = 1.1543$, $\lambda_2 = 2.5993$	Unstable node	



(b)

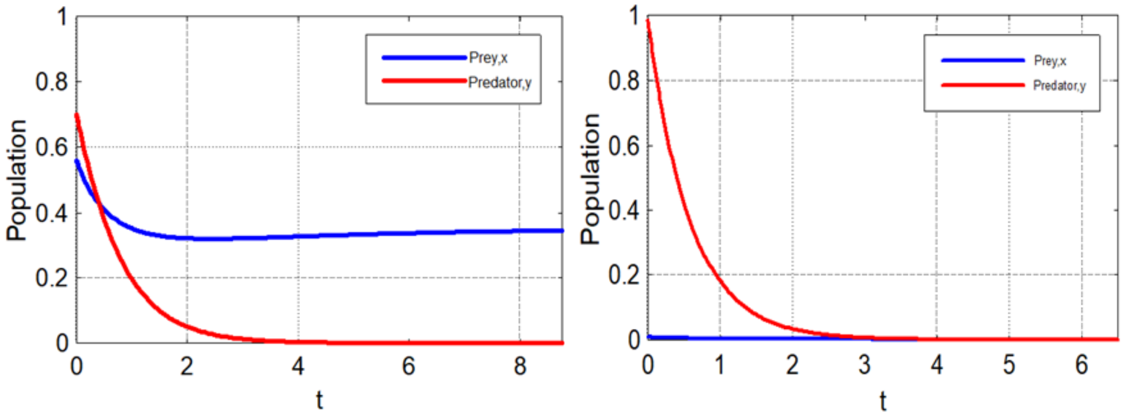
Figure 1: Bifurcation diagram of system (1) with respect to varying harvesting parameter β with $\alpha = 0.85$, $\delta = 0.85$, $\varepsilon = 0.85$ for (a) prey, x and (b) predator, y.



(a) (b)

Figure 2: The phase-plane diagram of prey-predator system (1) with parameter value (a) $\beta = 0.7$ and (b) $\beta = 1.32$

Figure 2(a) represents the phase plane diagram of prey-predator system in region (i) of Figure 1(a). In this region, the steady state E_1 shows a pattern of unstable saddle node, while E_2 shows a pattern of a stable node. Meanwhile, Figure 2(b) represents region (ii) after the harvesting parameter, β passes the transcritical bifurcation point. In this region, we can see the switching in the stability pattern for steady-state E_1 and E_2 where E_1 is switching to asymptotically stable node and E_2 is switching to unstable node. For E_3, there is no change in the stability from region (i) to region (ii), which is just unstable saddle node.



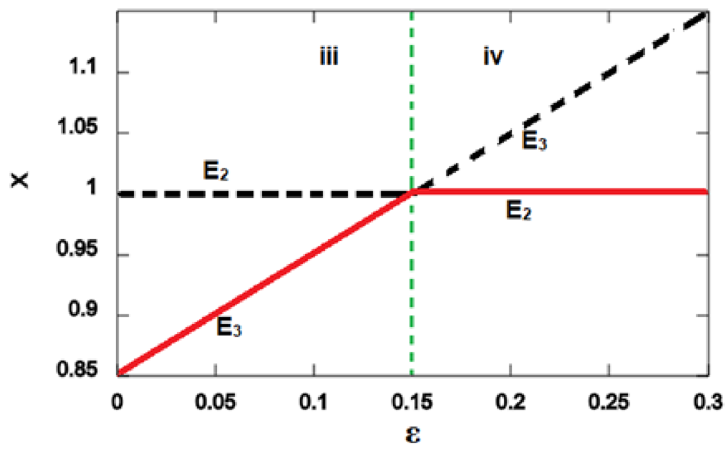
(a) (b)

Figure 3: Time series graph of system (1) with harvesting parameter value (a) $\beta = 0.7$ and (b) $\beta = 1.32$

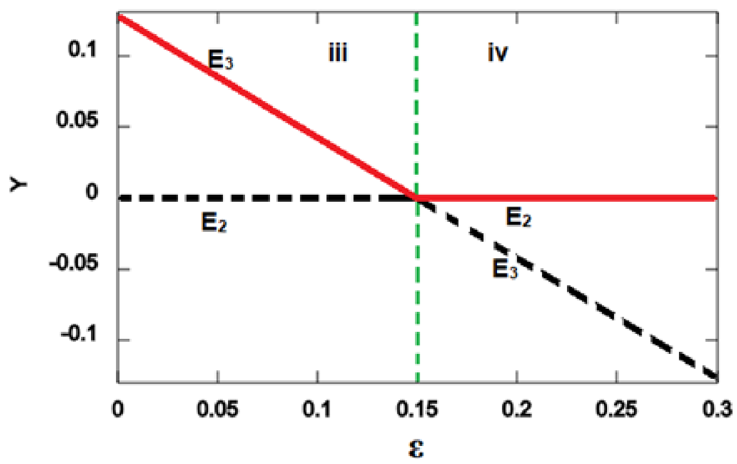
Referring to the time series graph in Figure 3(a), the prey population decreases slightly at the beginning until it reaches stable steady state values. This scenario is the direct consequence of harvesting activities. The predator population is steadily declining until it is extirpated from the system. As illustrated in Figure 3(b), both species are threatened with extinction due to intensive harvesting of prey species. The density of prey species decreases significantly, indicating that harvesting activities significantly affect the population even in the absence of predator predation.

4.2. *Case II: Harvesting Activities in Predator Populations*

In this section, the effect of the harvesting parameter on the predator population is examined. Figure 4(a) and Figure 4(b) display the bifurcation diagrams of the prey-predator system (1) with respect to the harvesting parameter ε. The red solid line represents the stable state, the black dashed line represents unstable steady states, and the green dashed line represents transcritical bifurcation point. Region (iii) is the region of the behaviour for both population before reaching transcritical bifurcation point while region (iv) is the behaviour of both populationsafter passing the transcritical bifurcation.



(a)



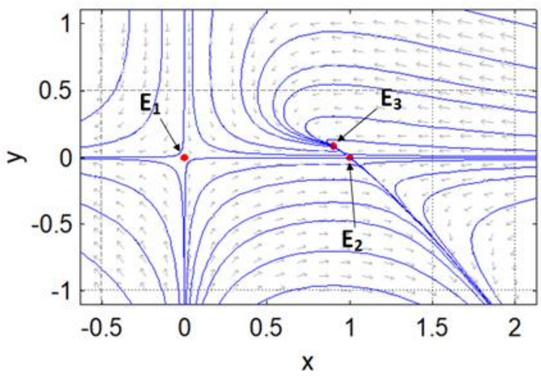
(b)

Figure 4: Bifurcation diagram of system (1) with respect to varying harvesting parameter ε with $\alpha = 0.85$, $\delta = 0.85$, $\beta = 0.15$ for (a) prey, x and (b) predator, y

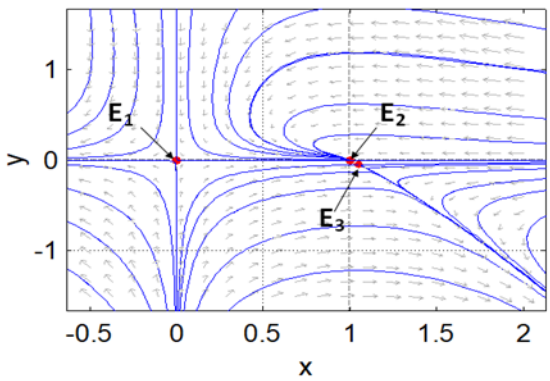
Figures 4(a) and 4(b) demonstrate that as the harvesting parameter increases, the prey population density grows linearly while the predator population declines linearly. This scenario explains the number of predators decreasing as a result of being harvested. Consequently, less prey is being eaten by the predator. As we can see, steady state E_3 is stable when harvesting activity $\varepsilon < 0.15$ (transcritical bifurcation point) where both species coexist. This is indicated with the red solid line in region iii. Despite this, when the harvesting level continues to rise until it crosses the transcritical bifurcation point of $\varepsilon = 0.15$, the steady state branch E_3 exchanges its stability with E_2. This essentially means that the predator population will become extinct, leaving only the prey species to survive. Due to this situation, the prey species is stable for a long time period. While for steady state $E_1(0,0)$, as we increase the harvesting parameter value, there is no change in the stability before and after the transcritical bifurcation. Thus, it is not labelled in the bifurcation diagram in Figure 4. The stability with respect to the harvesting parameter is summarised in Table 3.

Table 3: Summary of stability analysis with respect to harvesting parameter ε

Bifurcation parameters	Steady-states	Eigenvalues	Results	Figure
$\varepsilon = 0.05$	$E_1(0,0)$	$\lambda_1 = 0.85$, $\lambda_2 = -0.9$	Unstable saddle node	5(a)
	$E_2(1,0)$	$\lambda_1 = 0.1$, $\lambda_2 = -0.85$	Unstable saddle node	
	$E_3(0.9, 0.085)$	$\lambda_1 = -0.12$, $\lambda_2 = -0.65$	Asymptotically stable node	
$\varepsilon = 0.15$		Transcritical bifurcation point		
$\varepsilon = 0.2$	$E_1(0,0)$	$\lambda_1 = -1.05$, $\lambda_2 = 0.85$	Unstable saddle node	5(b)
	$E_2(1,0)$	$\lambda_1 = -0.85$, $\lambda_2 = -0.05$	Asymptotically stable node	
	$E_3(1.05, -0.44)$	$\lambda_1 = -0.94$, $\lambda_2 = 0.05$	Unstable saddle node	



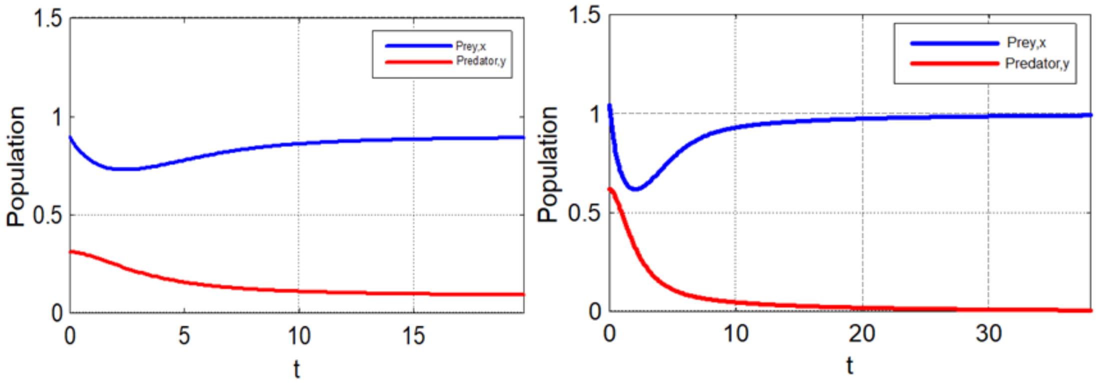
(a)



(b)

Figure 5: The phase-plane diagram of the prey-predator system (1) with parameter values (a) $\varepsilon = 0.05$ and (b) $\varepsilon = 0.2$

Figure 5(a) represents the phase-plane diagram for region (iii) which before the harvesting parameter, ε passes the transcritical bifurcation point. At this region, steady-state E_2 shows a pattern of unstable saddle, while E_3 is showing a pattern of a stable node. Meanwhile, Figure 5(b) represents the phase-plane diagram for region (iv) after the harvesting parameter, ε passing the transcritical bifurcation point. At this region, we can see switching in the stability pattern for steady-state E_2 and E_3 where E_2 is switching to asymptotically stable node and E_3 to unstable saddle node. For E_1, there is no change in stability from region (iii) to region (iv), which is just an unstable saddle node.



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(a) (b)

Figure 6: Time series graph of system (1) with harvesting parameter value (a) $\varepsilon = 0.05$ and (b) $\varepsilon = 0.2$

Figure 6(a) and 6(b) shows a time series graph corresponding to bifurcation diagrams in Figure 4(a) and 4(b). At a low harvesting rate, $\varepsilon = 0.05$, the prey population decreases slightly at first before increasing to a maximum value and reaching stable, steady state values. This situation occurs in response to the predator predation activities on the prey. The predator population decreases continuously until it reaches a stable steady state value. Both species persist where E_3 is stable, as shown in region (iii) of Figure 4. Unnecessarily high harvesting rates, such as $\varepsilon = 0.2$, will eventually lead to the extinction of the predator population. In contrast, the prey population will continue to grow indefinitely until it reaches the logistic limiting factor or carrying capacity.

5. Conclusion

In this research, a prey-predator fisheries model motivated by both species' harvesting activities is investigated. Both species are harvested at different rates. The model steady states are examined, as well as the stability analysis. Our study shows that the harvesting effect can cause a switch in the stability of a system. We can see that the system changed from stable to unstable as the harvesting parameter of prey exceeds the transcritical point. Meanwhile, when the harvesting parameter of the predator exceeds the transcritical point, the system switch from unstable to stable. For case I, only prey survives at a low rate of harvesting activity in prey population and if we increase the harvesting rate further, both prey and predator species will become extinct. For case II, a low rate of harvesting activities in the predator population will result in both species' coexistence. As the harvesting activities in the predator population increase, there is a survival of prey population only and the predator population becomes extinct. Thus, we can conclude that a low rate of harvesting activities in the predator population will resulting in the coexistence of both species. While a high rate of harvesting activities in both prey and predator populations, the extinction of prey or predator species may occur in the system. We reasonably conclude that harvesting activities have a more obvious effect on the dynamical behaviour of the system. In conclusion, acceptable methods to sustainable fishery resource use and ecosystem health, such as sustainable fishing, reviving flawed fisheries, and establishing environmentally protected zones, should be reinforced. Illegal fishing and overfishing in fisheries should be addressed to ensure that subsequent generations may benefit from the marine ecosystems.

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