Journal of Quality Measurement and Analysis *JQMA* 17(1) 2021, 79-91
e-ISSN: 2600-8602 *http://www.ukm.my/jqma*

A MATHEMATICAL MODELLING OF ECONOMIC RESTORATION THROUGH AGRICULTURAL REVITALIZATION IN NIGERIA

(Suatu Pemodelan Matematik bagi Pemulihan Ekonomi Menerusi Pengukuhan Semula Pertanian di Nigeria)

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ABSTRACT

Agricultural sector plays a major role in the economic growth of a nation. Apart from the provision of food which sustains human and animal life, the sector is the highest employer of labour in low income economies. The neglect of the sector due to the discovery of crude oil in Nigeria has led to the decline in the contribution of the sector over the years and an increase in the level of poverty. Based on that, a deterministic compartmental mathematical model was designed to study the least step needed to be taken by the government in the agricultural sector to revive the economy. Following the epidemic modelling approach, an analytic threshold that determined the effect of government effort on agriculture was derived. The model was studied qualitatively using the stability theory of nonlinear deferential equation and then quantitatively using hypothetical values for the model parameters to conduct the simulation. The qualitative result showed that the optimal agricultural output equilibrium of the model was locally asymptotically stable while the quantitative result showed that government effort to revive the agriculture was enough to restore the economy when the threshold quantity $R_I > 1$. The implication of both the qualitative and quantitative results was that a drastic step had to be taken by the government to promote agriculture by allocating enough funds and land to the sector as well as encouraging farmers which would activate economic boom in the long run.

Keywords: agriculture; crude oil; stability

ABSTRAK

Sektor pertanian memainkan peranan utama dalam pertumbuhan ekonomi sesebuah negara. Selain daripada penyediaan makanan yang menampung kehidupan manusia dan haiwan, sektor ini melibatkan pekerja paling ramai daripada kumpulan yang berpendapatan rendah. Berlaku pengabaian sektor ini adalah disebabkan oleh penemuan minyak mentah di Nigeria yang mendorong kepada penurunan sumbangan terhadap sektor ini selama bertahun-tahun dan menyebabkan peningkatan tahap kemiskinan. Berdasarkan isu ini, satu model matematik berpetak berketentuan direka bentuk untuk mengkaji langkah terkecil yang perlu diambil oleh kerajaan dalam sektor pertanian untuk menghidupkan kembali ekonominya. Mengikuti pendekatan pemodelan wabak, satu ambang analisis yang menentukan kesan usaha kerajaan terhadap pertanian telah diterbitkan. Model ini dikaji dengan menggunakan teori kestabilan persamaan pembezaan tak linear secara kualitatif dan kemudian menggunakan nilai hipotesis untuk parameter model secara kuantitatif bagi menjalankan simulasi. Hasil kualitatif menunjukkan bahawa keseimbangan hasil pertanian optimum model adalah stabil secara asimptotik sementara hasil kuantitatif menunjukkan bahawa usaha kerajaan untuk menghidupkan kembali pertanian sudah cukup untuk memulihkan ekonomi apabila kuantiti ambang $R_I > 1$. Implikasi daripada kedua-dua hasil kualitatif dan kuantitatif tersebut adalah bahawa langkah drastik harus diambil oleh kerajaan untuk mempromosikan pertanian dengan memperuntukkan dana dan tanah yang mencukupi untuk sektor ini serta menggalakkan petani-petani yang akan mengaktifkan ledakan ekonomi dalam jangka panjang.

Kata kunci: pertanian; minyak mentah; kestabilan

1. Introduction

Agriculture is the art and science that deals with the cultivation of land and rearing of animals for man's use. It also involves the sales of agricultural produce because production is not complete until the goods produced reach the final consumers (Bulator *et al*. 2018). Agriculture is the oldest profession and supports the existence of life. It is the mainstay of many economies because labour force in middle and low income nations depends on agriculture as main source of income (Olajide & Akinlabi 2012). The largest percentage of the labour force in less developed economies is engaged in various branches of agriculture which include: animal husbandry or animal production, veterinary medicine, soil science, crop production, agricultural economics and farm management, agricultural engineering, agricultural extension and education, forestry and wild life conservation, fishery, apiculture and heliculture (Letey 2015). The farmers with inadequate resources are the brains behind food security in their respective nations. However, in the present situation of soil fertility depletion, climate change, multiplicity of agricultural pests, rise in input prices, reduction in diversity of plant species and precarious agricultural crop prices, farmers in less developed countries find it difficult to make both ends meet (Pramanik & Pappula 2018).

Agriculture is faced with a number of problems in Nigeria. Land availability is limited by land tenure system which makes it difficult for non- indigenes of a community that owns a parcel of land to acquire the land or have access to it. Besides, since most farmers in low income countries are poor, inadequate access to loan or credit facilities hinders the purchase of land, improved seeds, fertilizers, machinery, agricultural tools and other chemicals (Viscarra-Rossel & McBratney 2018). Poor transportation network is another obstacle to agricultural activities in Nigeria. Most farms in rural areas are not linked to roads, and this eventually leads to wastage of food. Wastage of agricultural produce is also enhanced by the presence of bad roads which makes distribution of agricultural produce to markets very difficult. The effect of poor transport system is increase in farmers' cost which eventually reduces their profit margin (Rohrbach *et al.*2011). Apart from that, farm inputs like chemicals, fertilizers, tools and implements are grossly inadequate. The available ones are of inferior quality, outdated and crude. Modern inputs are not produced in Nigeria which makes them expensive to purchase and maintain. On the other hand, inadequate basic amenities like electricity, pipe-borne water and proper health-care make life in the rural areas boring and consequently trigger migration of able-bodied men and youths from rural to urban areas in search of jobs (Olajide & Akinlabi 2012). Lastly, agricultural development in Nigeria is hampered by inconsistent government policies. More often than not, the policies are crashed at the implementation stage because they do defeat the purpose for which they are designed. The policies do fail to recognise the peasant farmers at the grass root level but center on fake city farmers who only collect money and use it for other purposes (Olajide & Akinlabi 2012).

The government of Nigeria like the governments of other West African countries has played significant roles to tackle numerous problems facing agricultural development. The federal government of Nigeria, through its various agencies, has helped in the provision of assistance to farmers in the form of loans, credit and subsidies. Agencies like Nigeria Agricultural and Co-operative Bank (NACB), Agricultural Credit Guarantee Scheme (ACGS) and the National Directorate of Employment (NDE) were established to give loans to farmers (Obasi *et al.* 2017). Besides, successive government in Nigeria has launched one agricultural programme or the other to boost greater production of crops and livestock. The programmes include: Agricultural Loan Scheme, River Basin Development Authority (RBDA), National Agricultural Insurance Scheme (NAIS), Green Revolution, Operation Feed the Nation (OFN), National Agricultural Land Development Agency (NALDA), Agricultural Development Project (ADP), Directorate of Food, Road and Rural Infrastructure (DFFRI), National

Accelerated Industrial Crop Production Program (NAICPS), National Accelerated Food Production Program (NAFPP), Agro-Service Centers, Farm Settlement Scheme and Co-operative Farming (Obasi *et al.* 2017).

Despite launching of various programmes and implementation of numerous policies, agricultural sector is currently under performing in Nigeria given the unemployment trend, inflation of foodstuffs and the present economic predicament in the country (Ayoade *et al.* 2020a; Ayoade *et al.* 2020b). The government of the old Western Region of Nigeria led by late Chief Obafemi Awolowo and later by late Chief S.L. Akintola enjoyed unprecedented economic prosperity before the Nigeria independence in 1950s and after independence in 1960s (Adeniran 2016). The government relied on the proceeds from agriculture especially cocoa to facilitate huge capital projects which remained second to none till today. Some of these projects were the establishment of the first television station in Africa called Western Nigerian Government Broadcasting Corporation (WNTV) in 1959 but now Nigerian Television Authority (NTA) in Ibadan, construction of Liberty Stadium commissioned in 1960 in Ibadan, establishment of University of Ife in 1961 now Obafemi Awolowo University in Ile-Ife, erection of the first skyscraper in tropical Africa: the 26-storey Cocoa House (still the tallest in Ibadan) commissioned in 1965, launching of free primary education for all in 1955 and free health care for children in Western Region (Adeniran 2016). The achievement of Chief Obafemi Awolowo as the Premier of Western Region of Nigeria has become the yard stick to measure the performance of successive government till today.

The bloody coup of January 15, 1966 put the country into serious political disarray and ended the regional government that was put in place at independence on 1st of October 1960 (Ojo & Fagbohun 2014; Obi-Ani & Obi-Ani 2016). The political crisis ushered in neglect of agriculture. Agriculture finally lost its place and began to suffer from total neglect in the 1970s during the oil boom era when attention was shifted from agriculture to crude oil in Nigeria. However, excessive reliance on crude oil has not yielded good results going by the current rising debt profile occasioned by the dwindling oil prices in the world market. Apart from the fact that about six persons continue to fall into absolute poverty within every minute, Nigeria harbours the largest number of poor people in the world despite being the leading producer of crude oil in Africa as about 87 million Nigerians, almost half of the total population believed to be living below poverty line (i.e. on less than World Bank benchmark of \$1.90 per day) (Adebayo 2018). If the poor people in Nigeria were to come together and form a country, the country would be more populous than Germany with the population of the poor Nigerians estimated at 89.5 million in May 2020 (The Vanguard 2020). In order to bring back the glory of agriculture and to restore the economy, we designed a mathematical model to study the minimum effort required by the government to revive the agricultural sector and optimise the agricultural output. Mathematical modelling, which is defined by Benyah (2005) as “the process of evaluating a mathematical representation of some phenomena in order to gain a better understanding of that phenomena” has become an important scientific technique over the years and is becoming more and more powerful tool to solve problems arising from science, engineering, industry, and the society in general. Mathematical models have been used both analytically and numerically to give insight into the dynamics of many real life phenomena (Liu *et al.* 2019; Ayoade *et al.*2019; Ayoade *et al.*2020a; Ayoade *et al.*2020b; Ayoade *et al.*2020c).

2. Model Formulation and Model Basic Properties

In the study, a deterministic compartmental mathematical model is proposed to investigate the least step needed to be taken by the government to revitalize agricultural sector in Nigeria.

The model is made up of five compartments *G(t), I(t), L(t), F(t)* and *E(t). G(t)* is used to denote the three tiers of government in Nigeria which are: the federal, state and local governments, *I(t)* represents the proportion of inputs available for agriculture, *L(t)* represents proportion of land allocated to farming practices, *F(t)* denotes proportion of government agencies like financial institutions and research institutes which are saddled with the responsibility of providing aids to the farmers while *E(t)* is the proportion of labour force that engages in agriculture. The flow between the compartments is depicted in the transfer diagram in Figure 1.

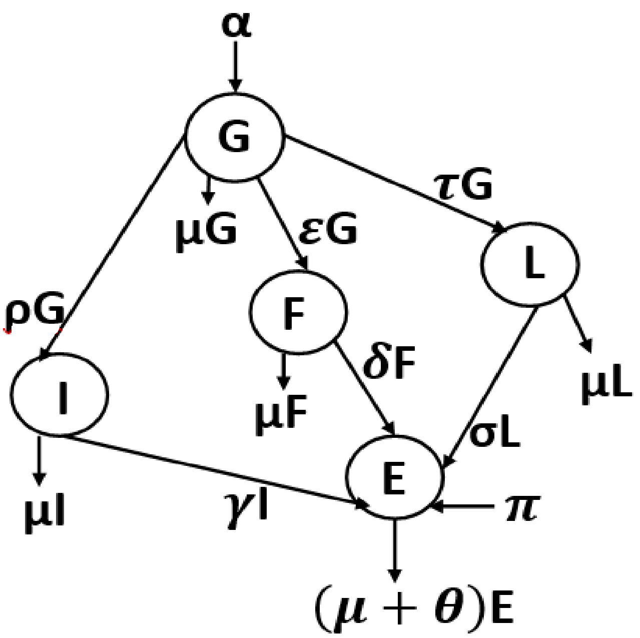

Figure 1: Transfer diagram of the model

In the diagram, α is the recruitment rate into the levels of government, ρ, τ, and ε are the rates at which government contributes to the increase in input availability, land availability and aids to government agencies respectively. The rates γ, σ, and δ are the rates of supply of inputs, land and aids to the farmers respectively. π is the recruitment rate into farmers' population while θ is the rate at which the able-bodied leave farming activities. It is assumed that the following occurs at the same rate μ : diversion of government contribution to *I(t), L(t)* and *F(t)*which is denoted as μG, diversion of inputs to other purposes apart from farming denoted as μI, diversion of aids from government agencies to other areas denoted as μF, diversion of land meant for agriculture to other purposes denoted as μL and diversion of resources made available to farmers for farming purposes to other areas by the farmers denoted as μE in the expansion of $(\mu + \theta)E$. Going by these assumptions, the following set of first order ordinary differential equations is derived from Figure 1.

$$\frac{dG}{dt} = \alpha - \rho G - \varepsilon G - \tau G - \mu G \tag{1}$$

$$\frac{dI}{dt} = \rho G - \gamma I - \mu I \tag{2}$$

$$\frac{dL}{dt} = \tau G - \sigma L - \mu L \tag{3}$$

$$\frac{dF}{dt} = \varepsilon G - \delta F - \mu F \tag{4}$$

$$\frac{dE}{dt} = \pi + \gamma I + \delta F + \sigma L - (\mu + \theta) E \tag{5}$$

Simulation is conducted for the model using the dimensionless hypothetical values in Table 1.

Table 1: Parameters description and values

Parameters	Symbols	Values
Recruitment rate into *G(t)*	α	0.001
Rate of govt contribution into *I(t)*	ρ	0.01
Rate of govt contribution into *L(t)*	ε	0.012
Rate of govt contribution into *F(t)*	τ	0.011
Rate of leakages in all the compartments	μ	0.0009
Rate of supply of input from *I(t)* into *E(t)*	γ	0.002
Rate of supply of land to *E(t)*	σ	0.0021
Rate of supply of aids from *F(t)* to *E(t)*	δ	0.0022
Recruitment rate into *E(t)*	π	0.01
Rate at which individuals leave *E(t)*	θ	0.001

2.1. *Positivity and Boundedness of Solutions*

The initial conditions of the model are assumed nonnegative since the model considers physical phenomenon, and we show that the solutions of the model are also nonnegative.

Theorem 2.1. *The state variables, G(t); I(t); L(t); F(t) and E(t), of the system (1)-(5), whose initial conditions are non-negative, remain non-negative for all* $t \geq 0$.

Proof.

Suppose $\{G(t), I(t), L(t), \mathrm{F}(t), \mathrm{E}(t)\}$ are the solutions of the system for all $t \geq 0$ with positive initial conditions $\{G(0) \geq 0, I(0) \geq 0, L(0) \geq 0, F(0) \geq 0, E(0) \geq 0\}$. From Eq. (1),

$$\frac{dG}{dt} \geq -(\rho + \varepsilon + \tau + \mu) G \tag{6}$$

$$\Rightarrow \quad \ln \mathrm{G} \geq -(\rho + \varepsilon + \tau + \mu) t + k \tag{7}$$

$$\Rightarrow G(t) \geq G(0) e^{-(\rho+\varepsilon+\tau+\mu)t} \geq 0 . \tag{8}$$

Following the same process,

$$\Rightarrow I(t) \geq I(0)e^{-(\gamma+\mu)t} \geq 0 \tag{9}$$

$$\Rightarrow L(t) \geq L(0)e^{-(\sigma+\mu)t} \geq 0 \tag{10}$$

$$\Rightarrow F(t) \geq F(0)e^{-(\delta+\mu)t} \geq 0 \tag{11}$$

$$\Rightarrow E(t) \geq E(0)e^{-\mu t} \geq 0\,. \tag{12}$$

Hence, the solutions of the system remain positive as long as the initial conditions of the state variables are positive since e^{q} is positive for all real values of q. □

Moreover, it is easy to verify that the solutions for the model are bounded meaning that there exists feasible region for the solutions where the solutions are mathematically well-posed.

Theorem 2.2. *The solutions to the model remain bounded in the region* Ω *defined by*

$$\Omega = \left\{ \left(G(t), I(t), L(t), F(t), E(t) : 0 \leq G(t) + I(t) + L(t) + F(t) + E(t) \right) \leq \frac{\pi + \alpha}{\mu} \right\}$$

Ω *is the region of attraction for the model which attracts every solution initiating in the interior of the positive octant.*

Proof.

The sum of Eq.(1) – Eq.(5) i.e.

$$\frac{dG}{dt} + \frac{dI}{dt} + \frac{dL}{dt} + \frac{dF}{dt} + \frac{dE}{dt} \tag{13}$$

$$\Rightarrow \frac{d}{dt}\left(\mathrm{G}(t) + I(t) + L(t) + F(t) + E(t) \right) = \pi + \alpha - \theta E - \mu\left(G + I + L + F + E \right)$$

$$\therefore \frac{d}{dt}\left(\mathrm{G}(t) + I(t) + L(t) + F(\mathrm{t}) + E(\mathrm{t}) \right) \leq \pi + \alpha - \mu\left(G + I + L + F + E \right) \tag{14}$$

By taking limit supremum

$$\lim_{t \to \infty} Sup\left(\mathrm{G}(t) + I(t) + L(t) + F(\mathrm{t}) + E(\mathrm{t}) \right) \leq \frac{\pi + \alpha}{\mu} \tag{15}$$

Hence, the solutions of the model are not only positive invariant but bounded in **Ω**. Therefore, the model is suitable to conduct the study and the analysis of the system dynamics can be considered in **Ω**. □

3. Model Analysis

The equilibrium solutions to a system of first-order differential equations are the points at which the first derivatives are equal to zero. The no-government support equilibrium (NSE) does not exist since there is no time when agricultural sector is totally not aided by the government considering the pivotal position which the sector occupies in the economy. The sector is only suffering from neglect which is the situation with the discovery of crude oil in Nigeria. However, agricultural output will be maximized and turned the economy around when the sector is fully aided by the government. The level which government needs to reach to revive economy through agriculture shall be examined.

3.1. *Existence of Optimum Agricultural output Equilibrium*

The optimum equilibrium solution exists when agricultural sector is fully aided by the government such that all the compartments coexist in the solution of the model. The optimum equilibrium solutions of the model are thus obtained as:

$$G^* = \left(\frac{\alpha}{\rho + \varepsilon + \tau + \mu} \right), \tag{16}$$

$$I^* = \left(\frac{\alpha\rho}{(\gamma + \mu)(\rho + \varepsilon + \tau + \mu)} \right), \tag{17}$$

$$L^* = \left(\frac{\alpha\tau}{(\mu + \sigma)(\tau + \mu + \rho + \varepsilon)} \right), \tag{18}$$

$$F^* = \left(\frac{\alpha\varepsilon}{(\delta + \mu)(\rho + \varepsilon + \tau + \mu)} \right), \tag{19}$$

$$E^* = \frac{\alpha}{(\theta + \mu)(\rho + \varepsilon + \tau + \mu)} \left(\frac{\pi(\rho + \varepsilon + \tau + \mu)}{\alpha} + \frac{\rho\gamma}{(\mu + \gamma)} + \frac{\varepsilon\delta}{(\mu + \delta)} + \frac{\sigma\tau}{(\mu + \sigma)} \right) \tag{20}$$

3.2. *Stability Analysis of the Optimum Agricultural Output Equilibrium*

Theorem 3.1. *The optimum agricultural output equilibrium of the model is locally asymptotically stable if all the eigenvalues of the Jacobian matrix of the model are negative.*

Proof.

To investigate the local stability of the model, the Jacobian matrix of Eq. (1) – Eq. (5) is evaluated thus

$$J = \begin{pmatrix} -(\rho+\varepsilon+\tau+\mu) & 0 & 0 & 0 & 0 \\ \rho & -(\gamma+\mu) & 0 & 0 & 0 \\ \tau & 0 & -(\sigma+\mu) & 0 & 0 \\ \varepsilon & 0 & 0 & -(\delta+\mu) & 0 \\ 0 & \gamma & \sigma & \delta & -(\theta+\mu) \end{pmatrix} \quad (21)$$

The characteristic equation of Eq. (21) is

$$(\theta+\mu+\lambda)(\delta+\mu+\lambda)(\sigma+\mu+\lambda)(\gamma+\mu+\lambda)(\tau+\mu+\rho+\varepsilon+\lambda)=0 \quad (22)$$

$$\Rightarrow$$

$$\left.\begin{aligned} \lambda_1 &= -(\rho+\varepsilon+\tau+\mu) \\ \lambda_2 &= -(\gamma+\mu) \\ \lambda_3 &= -(\sigma+\mu) \\ \lambda_4 &= -(\delta+\mu) \\ \lambda_5 &= -(\theta+\mu) \end{aligned}\right\} \quad (23)$$

Since all the eigenvalues are less than zero then the optimum agricultural output equilibrium of the model is locally asymptotically stable.

3.3. *The Reproductive Impact,* R_I

In disease models, the reproduction number R_0 is a threshold for determining whether an outbreak will take off or not. The quantity can be computed by using the next generational matrix method formulated by Driessche and Watmough (2002).

However, in the present study, the reproductive number R_0 is used to mean reproductive impact R_I which is a threshold for determining the level of impact of the government on the agricultural development. Like in disease models, if R_I <1, the impact of the government on agricultural development is minimum and the sector is underperforming but if R_I >1, the impact of the government on agricultural development is maximum and the performance of the sector is optimum. The quantity R_I is derived by following similar approach as in deriving R_0 by considering Eq. (2) – Eq. (4). These compartments are considered because they are intermediaries between the government and the farmers. Governments reach the farmers through the compartments. The associated next generational matrices are:

$$F = \begin{pmatrix} \rho & 0 & 0 \\ \tau & 0 & 0 \\ \varepsilon & 0 & 0 \end{pmatrix}. \quad (24)$$

$$V = \begin{pmatrix} \mu+\gamma & 0 & 0 \\ 0 & \mu+\sigma & 0 \\ 0 & 0 & \mu+\delta \end{pmatrix}. \tag{25}$$

The inverse of matrix V is obtained as

$$V^{-1} = \begin{pmatrix} \frac{1}{\mu+\gamma} & 0 & 0 \\ 0 & \frac{1}{\mu+\sigma} & 0 \\ 0 & 0 & \frac{1}{\mu+\delta} \end{pmatrix}. \tag{26}$$

The product of matrices F and V^{-1} is:

$$FV^{-1} = \begin{pmatrix} \frac{\rho}{\mu+\gamma} & 0 & 0 \\ \frac{\tau}{\mu+\gamma} & 0 & 0 \\ \frac{\varepsilon}{\mu+\gamma} & 0 & 0 \end{pmatrix} \tag{27}$$

The reproductive impact is thus obtained as the spectral radius (largest eigenvalue) of Eq. (27), which is:

$$R_I = \frac{\rho}{\mu+\gamma} \tag{28}$$

It is observed in Eq. (28) that the impact of the government effort on agricultural development is inversely proportional to the sum of the leakages (μ) and the amount of farm inputs that get to the farmers (γ) while the rate of government contribution to the availability of farm inputs (ρ) is the constant of proportionality. The transformation in agriculture is therefore a function of the dynamics in Eq. (28) because while it is one thing for the government to provide farm inputs (ρ), it is another thing for the inputs to get to those who need them (γ) given high rate of corruption in Nigeria (Kabiru 2019). It is even another thing for the input to be used judiciously if it gets to the farmers given high rate of illiteracy and inordinate desires of many farmers in Nigeria.

4. Numerical Simulation and Discussion

The simulation is implemented via dimensionless variables and parameters values aided by the computer-in-built Runge-Kutta package in Maple. The parameter values in Table 1 are used to determine the initial effort of the government in revitalizing agricultural sector given by Eq. (28). The value of μ is then varied to examine the effect of changes in its values on the impact of government in maximizing agricultural output, the result of which is presented in Table 2. The attention is only on μ because it represents every form of leakages to the efforts directed towards the development of agriculture.

Table 2: Effect of changes in the values of μ on the reproductive impact

S/No	γ	ρ	μ	R_I	Remark
1	0.002	0.01	0.0009	3.448	Optimum
2	0.002	0.01	0.0008	3.571	Optimum
3	0.002	0.01`	0.0006	3.846	Optimum
4	0.002	0.01	0.0004	4.167	Optimum
5	0.002	0.01	0.0002	4.545	Optimum
6	0.002	0.01	0.001	3.333	Optimum
7	0.002	0.01	0.003	2.000	Optimum
8	0.002	0.01	0.005	1.429	Optimum
9	0.002	0.01	0.007	1.111	Optimum
10	0.002	0.01	0.009	0.909	Minimum

The numerical results in Table 2 are supported graphically with Figures 2–3. The initial values assumed for the state variables to plot the graphs are: $G(0)=3$, $I(0)=100$, $L(0)=10\ 000$, $F(0)=20$ and $E(0)=1\ 000$**.** The optimum agricultural output equilibrium of the model has been proved to be locally asymptotically stable in Subsection 3.2. The implication of the stability of the optimum agricultural output equilibrium is that the efforts of the government in stabilizing agricultural output is maximum and the sector is performing at optimum level. The analytical result for stability in subsection 3.2 indicates that the optimum contribution of the sector is guaranteed, and the sector is able to revive the economy within the framework of the model. As for the numerical results, in Table 2, as leakages to the government effort on agricultural development (μ) are reduced while the rates of inputs availability and inputs supply to the farmers ρ and γ respectively are stabilized, the agricultural output is optimum and keeps increasing because R_I is raising (S/No 1–S/No 5) whereas, as leakages to the government effort on agricultural development (μ) are increased while the rates of inputs availability and inputs supply to the farmers ρ and γ respectively are stabilized, the agricultural output is also increasing but at a lower rate because R_I is falling (S/No 6 – S/No 9). However, when leakages to the government effort on agricultural development (μ) are increased ten times and beyond, we get to the region where the impact of the government on the agricultural development is not felt and the output is worst because $R_I < 1$ (S/No 10 and below).

Figure 2 is obtained by using the stated dimensionless initial values for the state variables together with the dimensionless parameter values in Table 1. In Figure 2, the reproductive impact $R_I > 1$ and the population of those who engage in farming increases with time over a period of a decade. The positive relationship between the population of farmers and time shows that every form of leakage from each compartment is minimal so that the resources allocated to agriculture is properly channeled to agriculture. Figure 3 is obtained by using the stated initial values for the state variables as in Figure 2 but with changes in the values of some parameters in Table 1: $\pi = 0.001$, $\gamma = 0.0002$, $\delta = 0.00022$, $\sigma = 0.00021$, $\theta = 0.01$, $\mu = 0.009$. In Figure 3, the reproductive impact $R_I < 1$ and the population of farmers and time are inversely related. The interpretation is that the population of those who engage in agriculture diminishes with time over a period of ten years. This is because the recruitment rate into the population of farmers π, supply rate of inputs to farmers γ, supply rate of land to farmers σ and supply rate of aids to farmers δ reduce ten times while the rate at which the able-bodied leave farming activities θ and the rate at which farmers divert resources meant for agriculture μ to other purposes increase ten times respectively as against the scenario in Figure 2.

The results of the analysis show that to restore the economy and bring back the glory of agriculture by stimulating the optimum performance of the agricultural sector, there is need for the government to stabilize its contribution to the availability of inputs and land and at the same time, ensures proper funding of the agencies which are saddled with the responsibility of providing assistance to the farmers. However, since the resources are scarce, there should be a proper monitoring to block diversion of limited resources which are channeled into agriculture. Lastly, as Nigeria is ranked high among the corrupt nations in the world according to Ahmed *et al*. (2019), government should set all machinery in motion to track diverted resources meant for agriculture and channel it back to agriculture.

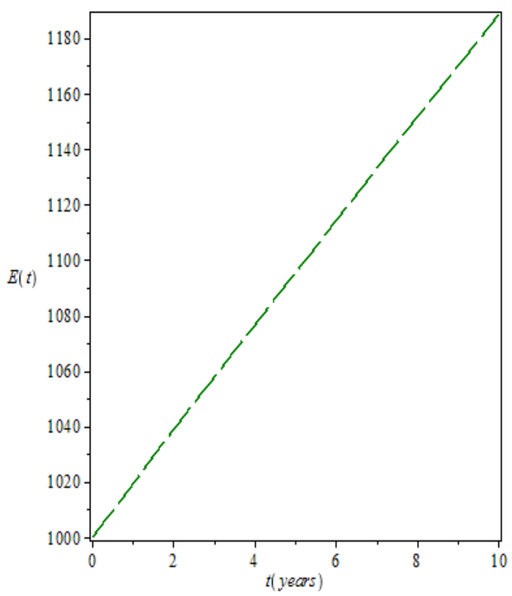

Figure 2: Simulation of *E(t)* for $R_1 > 1$

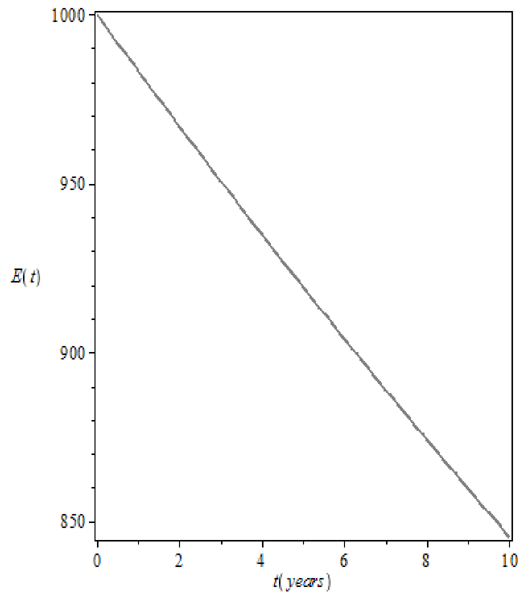

Figure 3: Simulation of *E(t)* for $R_2 < 1$

5. Conclusion

In this paper, we have analyzed the stability of the impact of government effort on the agricultural output growth in Nigeria by formulating a mathematical model to examine the least effort required by the government to revitalize agriculture so as to revive the economy. The positivity and boundedness of solutions of the model were established and the stability of the optimum agricultural output equilibrium of the model was proved. The threshold quantity R_I of the model was derived and the numerical values of the quantity were computed by using hypothetical values for the parameters to locate the regions where the effort of the government was enough to revitalize agricultural sector and where it was not enough. The tabulated results for R_I were finally illustrated graphically to visualize the scenarios of revitalization and non-revitalization of agricultural sector in the presence of governmental effort.

Acknowledgments

Authors are grateful to the anonymous reviewers for their valuable comments and suggestions.

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Received: 19 October 2020
Accepted: 30 April 2021

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