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3 PROOFS OF 2 WELL-KNOWN THEOREMS ON STARLIKE AND CONVEX FUNCTIONS

(3 Bukti bagi 2 Teorem yang Dikenali bagi Fungsi Bakbintang dan Cembung)

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ABSTRACT

Let f be analytic in $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$, and be given by $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. We give three different proofs for the well-known sharp bounds for the second Hankel determinant $|H_2(2)(f)| = |a_2 a_4 - a_3^2|$ for starlike and convex functions.

Keywords: analytic; univalent; starlike; convex; Hankel determinant; coefficients

ABSTRAK

Andaikan f analisis dalam $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$, dan diberi oleh $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. Tiga pembuktian berbeza bagi batas terbaik bagi penentu kedua Hankel $|H_2(2)(f)| = |a_2 a_4 - a_3^2|$ diberi untuk fungsi bakbintang dan cembung.

Kata kunci: analisis; univalen; bakbintang; cembung; penentu Hankel; pekali

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