Journal of Quality Measurement and Analysis *JQMA* 18(2) 2022, 29-44
e-ISSN: 2600-8602 *http://www.ukm.my/jqma*

COVID-19 OUTBREAK AND MITIGATION BY MOVEMENT RESTRICTIONS: A MATHEMATICAL ASSESSMENT OF ECONOMIC IMPACT ON NIGERIAN HOUSEHOLDS

ABAYOMI AYOTUNDE AYOADE* & OLUWASEYI ALIU

ABSTRACT

As COVID-19 continues to spread across the globe, governments all over the world have been finding ways of mitigating the pandemic. Various measures including lockdown and social distancing have been enacted to curb the spread of the virus. While the epidemiological trends of the disease have received considerable attention, the economic impacts of the outbreak on the households have been largely understudied. A compartmental model was developed and an epidemiological modelling approach was adopted to examine the economic impact of lockdown and social distancing on Nigerian households. The basic epidemiological features of the model were examined before the model was studied analytically and numerically. Both the analytical and numerical results showed that the economies of every household in Nigeria apart from the rich was badly affected by the mitigation measures of COVID-19 when there was an impediment to the payment of salaries and distribution of palliatives.

Keywords: COVID-19; households; lockdown; social distancing

ABSTRAK

Disebabkan COVID-19 masih merebak di serata dunia, kerajaan di seluruh dunia telah cuba mencari jalan bagi mengurangkan pandemik ini. Pelbagai langkah telah diambil termasuk sekatan pergerakan dan penjarakan sosial untuk mengawal penyebaran virus ini. Meskipun trend epidemiologi wabak ini telah menerima perhatian, impak ekonomi disebabkan oleh wabak ini kepada isi rumah masih kurang dikaji. Model kompartmen telah dibangunkan dan pendekatan pemodelan epidemiologi telah diterapkan untuk meneliti impak ekonomi kesan daripada sekatan pergerakan dan penjarakan sosial kepada isi rumah di Nigeria. Ciri-ciri asas epidemiologi model tersebut telah diteliti sebelum dikaji secara analitik dan berangka. Kedua-dua hasil analitik dan berangka menunjukkan ekonomi setiap isi rumah di Nigeria, selain daripada yang kaya, sangat terkesan dengan langkah-langkah mitigasi COVID-19 apabila terdapatnya halangan kepada pembayaran gaji dan pengedaran paliatif.

Kata kunci: COVID-19; isi rumah; sekatan pergerakan; penjarakan sosial

1. Introduction

The outbreak of coronavirus disease 2019 (COVID-19) is not a new phenomenon to human race as man has been struggling with disease epidemic before the Stone Age. Indeed, human account has documented pestilence of diseases like the Black Death and influenza that spread in Europe in the fourteenth and twentieth century respectively, cholera and typhus that ravaged the globe in the seventeenth to the early nineteenth century, with destructive implications on human population (Ayoade 2013). However, COVID-19 has become one of the largest pandemics in the last millennium with at least one hundred thousand fatalities.

Table 1: Fifteen major pandemic episodes with minimum of 100,000 fatalities

Event	Start	End	Deaths
Black Death	1347	1352	75,000,000
Italian Plague	1623	1632	280,000
Great Plague of Sevilla	1647	1652	2,000,000
Great Plague of London	1665	1666	100,000
Great Plague of Marseille	1720	1722	100,000
First Asia Europe Cholera Pandemic	1816	1826	100,000
Second Asia Europe Cholera Pandemic	1829	1851	100,000
Russia Cholera Pandemic	1852	1860	1,000,000
Global Flu Pandemic	1889	1890	1,000,000
Sixth Cholera Pandemic	1899	1923	800,000
Encephalitis Lethargica Pandemic	1915	1926	1,500,000
Spanish Flu	1918	1920	100,000,000
Asian Flu	1957	1958	2,000,000
Hong Kong Flu	1968	1969	1,000,000
H1N1 Pandemic	2009	2009	203,000

[a]Source: Alfani and Murphy (2017), Taleb and Cirillo (2020) cited in Jordà *et al.* (2020)

As of 1st October, 2020, 10:31 GMT, the figure of reported cases of COVID-19 globally is 34, 192, 734and no fewer than 1, 019, 242 fatalities have been attributed to the pandemic since the first case of unidentified etiology was reported in China in 2019 December (Chan *et al*. 2020; Ayoade *et al*. 2021a). As coronavirus disease 2019 spreads to new places, the virus transmits quickly and regional explosions continue. The mitigation of the epidemic is compounded by high infectivity, fairly low disease-induced death rate, and prolong asymptomatic period of infection (Wu *et al*. 2020). Presently, restrictions on physical interactions and travels have been the most effective strategies to contain the rapid spread of COVID-19, a policy known as lockdown and social distancing (Fang *et al*. 2020). Lockdown and social distancing have prevented millions of Nigerians from earning an income and the economic damage has increased mortality due to the connection between income and mortality (Chetty *et al*. 2016).

The first case of COVID-19 in Nigeria was discovered in Lagos on February 27, 2020 from an Italian businessman who returned from his country on a business trip. The government of Nigeria, in the bid to contain the spread of the virus, institutes lockdown on March 30, 2020. The enforcement of lockdown necessitates closure of schools, worship centres, markets, offices, leisure spots and businesses. There have also been restrictions on movements and large gatherings including social and religious gatherings. The consequence on the already battered economy, where the figure of the citizens falling into absolute poverty increases by nearly six persons per minute, is the economic loss particularly for the individuals in the informal sector of the economy (Adebayo 2018). The informal sector accommodates over 80% working Nigerians. The people in the sector include drivers, street traders, tradesmen, food vendors, hairdressers, artisans, commercial motorcyclists etc., whose incomes are generally low with no savings (Alamba 2020).

Before the enforcement of lockdown, Nigeria has become the world poverty capital as the country is ahead of India, a country which is seven times as populous as Nigeria, in the ranking of countries with the highest figures of individuals living in extreme poverty, as about 87 million Nigerians, almost half of the total population are believed to be living below poverty line (i.e. on less than World Bank benchmark of $1.90 per day) (Adebayo 2018). However, massive job loss due to the enforcement of lockdown and social distancing has worsened the situation for many households. Recent reports indicate that the population of Nigerians living in extreme poverty has risen from 86.71 million in 2019 to 89.5 million in

2020 with more Nigerians expected to fall below the poverty line (Ayoade & Farayola 2021b).

A household is made up of a group of individuals (usually, a family), who purchases goods to satisfy wants. A household is the smallest economic unit. In Nigeria, before the outbreak of COVID-19, households can be categorised into three namely: the upper class, the middle class and the lower class. The upper class households consist of the well-to-do families who are living far above the poverty line and whose economies can withstand economic storm. The middle class households comprise daily income earners from petty trading and low salary earners. Individuals in the middle class are just living around the poverty line and are vulnerable to economic storm. The lower class households are extremely poor Nigerians who are living below the poverty line. The outbreak of COVID-19 and the enforcement of lockdown order have lead to a new class of households termed the rescued class. Households in the middle class whose economies are affected by lockdown enforcement but who are able to secure borrowing to keep body and soul together as well as households in the lower class who are fortunate to benefit from occasional poverty alleviation programmes of the government are rescued from the lockdown economic misfortunes. These households who are rescued from the lockdown economic misfortunes formed the rescued class.

The enforcement of lockdown order instigated by COVID-19 outbreak has changed the narratives of many households in Nigeria and to cope with the economic new normal imposed by the lockdown enforcement, many households resort to borrowing when their incomes are not forthcoming while some others depended on government palliative distributions for survival. Those individuals who are able to secure borrowing or palliatives are rescued from the lockdown economic misfortunes and moved to the rescued class while others, who are unable to secure borrowing or palliatives, either moved from the middle class to the lower class or remained in the lower class. Before the lockdown and even after, many states in Nigeria have been unable to pay their workers while several months of unpaid salaries have thrown workers in the affected states into serious economic disarray. The situation has forced some households in the middle class to the rescued class and many others to the lower class.

Epidemiological models have been used extensively to forecast the sequence and progression of COVID-19 (Wrapp *et al*. 2020; Zhu *et al.* 2020; Gralinski & Menachery 2020; Yang & Wang, 2020; Fang *et al*. 2020; Ashleigh *et al.* 2020; Liu *et al.* 2020; Jia *et al*. 2020; Anastassopoulou *et al*. 2020; Cakir & Savas 2020). These and many other models on COVID-19 focus on the epidemiological aspect of the pandemic and their results are significant as they guide the policy makers and health practitioners in the control of the pandemic. These models quantify the figure of infected persons, predict the figure of patients who are in need of serious treatment, forecast the mortality toll, and evaluate the results of various policies. However, while the models have played tremendous roles, they are limited by a major setback: they do not integrate and quantify the economic implications of the epidemic on the households.

To precisely forecast the transmission of disease and quantify implications beyond the disease itself, there is a need for models to incorporate the impact of mitigation measures on the economy. With the exception of a few studies in (Jordà *et al*. 2020; Goldsztejn *et al*. 2020; Ruiz Estrada *et al*. 2020), existing epidemiological models lack economic consequences of policies implemented to check the transmission of COVID-19. Till now, no models of epidemiological nature focus solely on the economic impact of mitigating COVID-19 outbreak through lockdown and social distancing on the Nigerian households especially the economically challenged households. Besides, compartmental modelling method has not been applied to economies of any other countries. It is completely a new approach towards modelling the economic impact of movement restrictions in checking the spread of COVID-19. The remaining part of the article is arranged thus; section 2 consists of the formulation and the model basic properties. Section 3 contains the analysis, while section 4 deals with simulations and discussion. Finally, the study is concluded in section 5.

2. Model Formulation

A compartmental model MRLG is proposed and an epidemiological modelling approach is used to analysethe economic effects of lockdown and social distancing in checking the spread of COVID-19 on the economically challenged Nigerian households. The transfer diagram for the model is in Figure 1.

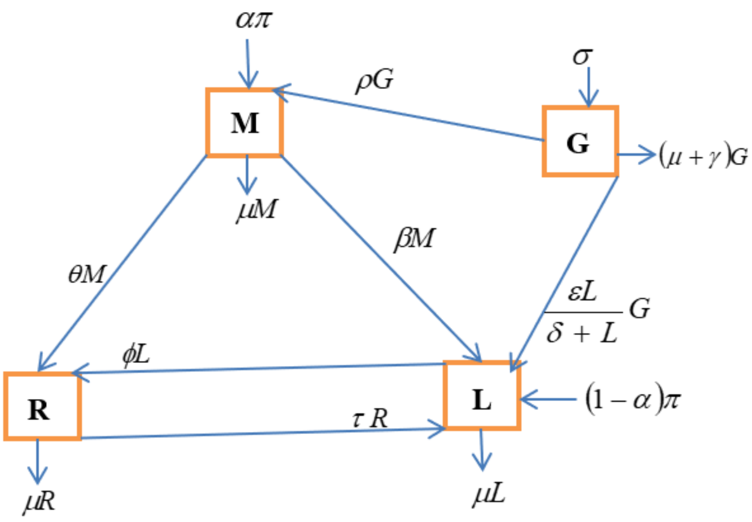

Figure 1: Transfer diagram of the model

In Figure 1, MRLG stands for the middle class *M(t),* the rescued class *R(t),* the lowerclass *L(t)* and the government compartment *G(t).* Households in the middle, the rescued and the lower classes are the economically challenged households in the society because while the middle and the rescued classes are living around the poverty line, the lower class households are living in extreme poverty below the poverty line. The rate of income generation to the government is σ. While the recruitment rate into the middle class is $\alpha\pi$, the recruitment rate into the lower class is $(1-\alpha)\pi$. When the lockdown was enforced on March 30, 2020, the government of Nigeria promised to sustain the lower class of the society with palliatives while the salary earners were expected to rely on their fixed salaries to keep body and soul together. Some households in the middle class are receiving salaries at rate ρ. The government releases palliatives to the lower class but since human beings are involved in the management and distribution and given the high level of corruption in Nigeria, the effect of palliatives cannot be linear and is therefore expressed as $\frac{\varepsilon}{\delta+L}$. However, households in the lower class who are fortunate to be benefiting from the government palliatives move to the rescued class at rate ϕ.

Some state governments in Nigeria have been having issues with the payment of salaries before the lockdown while the enforcement of lockdown has forced the salaries of some workers especially those in private establishments (e.g. private schools, hotels, etc.) to cease.

When the income ceases, households in the middle class who are able to secure borrowing to keep their heads above water move to the rescued class at rate θ while those who cannot manage the situation move to the lower class at rate β. Since the households in the rescued class are at the mercy of borrowing and palliatives which are not certain given the economic condition imposed by lockdown, some of them move back to the lower class at rate τ when their hopes are dashed. The cost of lockdown on the households is the time forfeited to engage in both productive and pleasurable activities while the cost on the government is the decline in revenues both from the export of crude oil and some government workers who are being paid salaries but who do not work due to the enforcement of lockdown.

Nigeria is the largest oil producer in Africa and about 90% of Nigeria's foreign exchange earnings come from the export of crude oil (Okpi 2018).The price of crude oil in November 2019 before the outbreak of COVID-19 was above \$60 per barrel but the price declined below \$20 per barrel before the end of March 2020 due to the outbreak of COVID-19 pandemic (Olawoyin 2020).While γ is the rate of loss in government revenue due to instability in crude oil prices, μ is the rate of loss in government revenue due to stoppage in the services of some government workers (e.g. civil servants). It is assumed that the cost of lockdown, on the households in terms of the time forfeited to engage in both productive and pleasurable activities and on the government in terms of revenues lost due to inability of some of her workers to render services, is the same and is represented by μ . μ is a monetary term because time is a measurable space where things happen to man. Besides, pleasurable activities (e.g. leisure) are included in the measurement of national income. Bringing these assumptions, formulations as well as the transfer diagram together, the following set of differential equations are derived.

$$\frac{dM}{dt} = \alpha\pi + \rho G - \theta M - \beta M - \mu M, \tag{1}$$

$$\frac{dG}{dt} = \sigma - \rho G - \frac{\varepsilon L}{\delta + L} G - \mu G - \gamma G, \tag{2}$$

$$\frac{dL}{dt} = (1-\alpha)\pi + \beta M + \tau R + \frac{\varepsilon L}{\delta + L} G - \phi L - \mu L, \tag{3}$$

$$\frac{dR}{dt} = \theta M + \phi L - \tau R - \mu R. \tag{4}$$

The model parameters are restated in Table 2 for ease of reference.

2.1. *Basic features*

The model must satisfy basic conditions before it could be suitable to conduct the study. It must be well-posed mathematically. It is well-posed if all its solutions are bounded and positive.

Table 2: The model parameters and their definitions

Parameters	**Definitions**
π	Human recruitment rate
α	Proportion of individuals recruited into the middle class
ρ	Rate of payment of salary
θ	Rate at which individuals in the middle class who are able to secure borrowing when their incomes cease remain around the poverty line
β	Rate at which individuals in the middle class who are unable to secure borrowing when their incomes cease fall below the poverty line
σ	Rate of government income generation
ε	Rate of supply of COVID-19 palliatives
δ	Amount of palliatives that get to the individuals below the poverty line that bring the possibility of 50% relief to the individuals
ϕ	Rate at which individuals below the poverty line are moving towards the line due to the effect of COVID-19 palliatives
τ	Rate at which individuals who are being sustained by borrowing and palliatives fall below the poverty line when their hopes are dashed
γ	Rate of loss in govt revenues due to instability in crude oil prices
μ	Rate of loss in govt revenues due to inability of some workers to work

Theorem 2.1. *The solutions of the model system (1)-(4) are bounded in the region* Ω *defined by*

$$\Omega = \left\{ (M, R, L, G) : 0 \le M + R + L + G \le \frac{\alpha + \sigma}{\mu} \right\}.$$

Proof. Assuming the total human population is *N* then

$$N = M + R + L + G \Rightarrow,$$

$$\frac{dN}{dt} = \frac{dM}{dt} + \frac{dR}{dt} + \frac{dL}{dt} + \frac{dG}{dt} \Rightarrow,$$

$$\frac{dN}{dt} = \pi + \sigma - \mu N - \gamma G \Rightarrow,$$

$$\frac{dN}{dt} \le \pi + \sigma - \mu N \Rightarrow,$$

$$\frac{dN}{(\pi + \sigma - \mu N)} \le dt. \tag{5}$$

Following the theorem of Birkhoff and Rota in (Ayoade *et al*. 2019b; 2019c),

$$\ln(\pi + \sigma - \mu N) \ge t + q \Rightarrow,$$

$$(\pi + \sigma - \mu N) \ge we^{-\mu t} \tag{6}$$

where $w = e^q$.

When $t = 0$, $N(0) = N_0$ and (6) becomes

$$(\pi + \sigma - \mu N) \geq w. \tag{7}$$

On substituting (7), (6) becomes

$$(\pi + \sigma - \mu N) \geq [(\pi + \sigma - \mu N)]e^{-\mu t} \Rightarrow, \tag{8}$$

$$N(t) \leq \frac{\pi + \sigma}{\mu} - \left(\frac{\pi + \sigma - \mu N}{\mu}\right)e^{-\mu t}. \tag{9}$$

As $t \to \infty$ then,

$$N(t) \leq \frac{\pi + \sigma}{\mu}. \tag{10}$$

Therefore, the solutions for $N(t)$ are feasible within the region

$$\Omega = \left\{(M, R, L, G) \in \Re_+^4 ; N(t) \leq \frac{\pi + \sigma}{\mu}\right\}.$$

This completes the proof. □

Theorem 2.2 *The solutions to the system (1)-(4) are nonnegative for* $t \geq 0$.

Proof. Consider (1),

$$\frac{dM}{dt} = \alpha\pi + \rho G - \theta M - \beta M - \mu M \Rightarrow,$$

$$\frac{dM}{dt} \geq -(\theta + \beta + \mu)M \Rightarrow,$$

$$\frac{dM}{M} \geq -(\theta + \beta + \mu)dt \Rightarrow,$$

$$\int \frac{dM}{dt} \geq -(\theta + \beta + \mu)\int dt, \Rightarrow$$

$$\ln M \geq -(\theta + \beta + \mu)t + c, \Rightarrow$$

$$M(t) \geq Ae^{-(\theta + \beta + \mu)t}, \tag{11}$$

where $A = e^c$.

If $t = 0$, $M(0) = M_0$ and (11) reduces to

$$A = M_0 .$$

Therefore, (11) changes to

$$M(t) \geq M_0 e^{-(\theta+\beta+\mu)t} \; . \tag{12}$$

Using the same approach for (2) – (4), the following are obtained:

$$G(t) \geq G_0 e^{-(\mu+\gamma)t}, \tag{13}$$

$$L(t) \geq L_0 e^{-(\mu+\phi)t} \; , \tag{14}$$

$$R(t) \geq R_0 e^{-(\mu+\tau)t} \; . \tag{15}$$

where M_0, G_0, L_0 and R_0 are the initial conditions of the system variables. Since the model monitors physical phenomenon, the initial conditions to the variables must be positive. Hence, the solutions to the system given by $M(t), G(t), L(t)$ and $R(t)$ in (12)-(15) are positive for all $t \geq 0$ since $e^p > 0$ for all real values of p .Therefore, the model is well posedand suitable to conduct the study since the solutions to it are bounded and positive. □

3. Model Analysis

3.1. *Equilibria*

Two equilibria are associated with epidemic models. Disease-free equilibrium when there is no infection in the population and endemic equilibrium when the population is invaded with infection.In the same manner, two equilibria are associated with the present model. COVID-19-free equilibrium (CFE) when the outbreak of COVID-19 has not occurred and COVID-19 endemic equilibrium (CEE) when the country has been struggling with COVID-19 outbreak. At CFE, there are no COVID-19 palliatives, nobody in the middle class joins the lower class due tolockdown, nobody in the lower class moves to the rescued class and nobody from the rescued class comes to the lower class. That is, $\varepsilon = \beta = \phi = \tau = 0$. On the other hand, at CEE, each of the parameter is greater than zero. Hence, the model (1)-(4) permits a COVID-19 free equilibrium (CFE) $\sum_0 = \left(M_0, G_0, L_0, R_0\right)$ with coordinates

$$M_0 = \frac{1}{\mu+\theta}\left[\alpha\pi + \frac{\sigma\rho}{(\mu+\rho+\gamma)}\right],$$

$$G_0 = \frac{\sigma\rho}{(\mu+\rho+\gamma)},$$

$$L_0 = \frac{(1-\alpha)\pi}{\mu},$$

$$R_0 = \frac{\theta}{\mu(\mu+\theta)}\left[\alpha\pi + \frac{\sigma\rho}{(\mu+\rho+\gamma)}\right],$$

and a COVID-19 endemic equilibrium $\sum^{*} = \left(M^{*}, G^{*}, L^{*}, R^{*}\right)$ with coordinates

$$M^{*} = \frac{1}{\mu+\beta+\theta}\left[\alpha\pi + \frac{\sigma\rho}{(\mu+\rho+\gamma+\lambda)}\right],$$

$$G^{*} = \frac{\sigma\rho}{(\mu+\rho+\gamma+\lambda)},$$

$$L^{*} = \frac{1}{\mu+\phi}\left[\frac{\tau}{(\mu+\phi)(\mu+\tau)}(A_1+A_2)+A_3\right],$$

$$R^{*} = \frac{1}{(\mu+\phi)(\mu+\tau)}(A_1+A_2),$$

where

$$\lambda = \frac{\varepsilon L^{*}}{\delta + L^{*}},$$

$$A_1 = \theta\tau + (1-\alpha)\pi\theta + \frac{\theta\beta}{\mu+\beta+\theta}\left(\alpha\pi + \frac{\sigma\rho}{(\mu+\rho+\gamma+\lambda)}\right),$$

$$A_2 = \frac{\lambda\sigma\theta}{\mu+\rho+\gamma+\lambda} + \frac{\theta(\mu+\phi)}{\mu+\beta+\theta}\left(\alpha\pi + \frac{\sigma\rho}{(\mu+\rho+\gamma+\lambda)}\right),$$

$$A_3 = (1-\alpha)\pi + \frac{\lambda\sigma}{\mu+\rho+\gamma+\lambda} + \frac{\beta}{\mu+\beta+\theta}\left(\alpha\pi + \frac{\sigma\rho}{(\mu+\rho+\gamma+\lambda)}\right).$$

3.2. *COVID-19 impact ratio,* $R_{COVID-19} > 1$

In disease models, the reproduction ratio is a parameter that measures the transmission potential of a disease. If the parameter is greater than one, the disease will spread in the population from an initial infection otherwise it will die out or fail to spread. Reducing the parameter below unity is always the motive of epidemiologists. Since the parameter quantifies the rate of spread of a disease, it may be derived from an infectious compartment in simple epidemic models. In the present analysis, the economic impact of COVID-19 outbreak on the Nigerian households is a function of rate of government income generation, stability rate of oil prices, rate of income lost due to the inability of some government workers to work and the rate of equitable distribution of the COVID-19 palliatives. Therefore, COVID-19 impact ratio, $R_{COVID-19}$ is defined as

$$R_{COVID-19} = \frac{\sigma\delta}{\mu + \gamma}. \tag{16}$$

Unlike in disease models, if $R_{COVID-19} > 1$, the economic impact of lockdown and social distancing on the households is not severe but it is severe if otherwise. In (16), $R_{COVID-19}$ will be increasing if there is a decrease in μ and γ but an increase in δ while σ is fixed. On the other hand, $R_{COVID-19}$ will be decreasing if σ is fixed while μ , γ and δ are increasing. Mathematically, the implication is that a decrease in μ and γ with an increase in δ while σ is fixed will enhance ρ, ε and ϕ but limit θ and τ . Whereas, an increase in μ , γ and δ while σ is fixed will limit ρ, ε and ϕ but increase θ and τ .

3.3. *Stability of COVID-19-free equilibrium,* Σ_0

Theorem 3.1. *The COVID-19-free equilibrium of the model is stable if* $R_{COVID-19} > 1$ *otherwise it is unstable.*

Proof. The COVID-19-free equilibrium is stable and $R_{COVID-19} > 1$ if it is established that all the roots of the Jacobian matrix of the model (1)-(4) are less than zero. If the system (1)-(4) is linearised then it becomes

$$J = \begin{pmatrix} -(\mu+\theta+\beta) & \rho & 0 & 0 \\ 0 & -\left(\frac{\varepsilon L}{\delta + L} + \mu + \gamma + \rho\right) & -\left(\frac{\varepsilon G}{\delta + L} - \frac{\varepsilon LG}{(\delta + L)^2}\right) & 0 \\ \beta & \frac{\varepsilon L}{\delta + L} & \left(\frac{\varepsilon G}{\delta + L} - \frac{\varepsilon LG}{(\delta + L)^2}\right) - (\mu + \phi) & \tau \\ \theta & 0 & \phi & -(\mu + \tau) \end{pmatrix} \tag{17}$$

Evaluating (17) about COVID-19-free equilibrium i.e., when $\varepsilon = \beta = \phi = \tau = 0$ then (17) becomes

$$J(\Sigma_0) = \begin{pmatrix} -(\mu+\theta) & \rho & 0 & 0 \\ 0 & -(\mu+\gamma+\rho) & 0 & 0 \\ 0 & 0 & -\mu & 0 \\ \theta & 0 & 0 & -\mu \end{pmatrix}. \tag{18}$$

In (18),

$$|J(\Sigma_0 - \lambda I)| = 0 \Rightarrow (\mu + \lambda)(\mu + \lambda)(\mu + \gamma + \rho + \lambda)(\mu + \theta + \lambda) = 0, \Rightarrow \tag{19}$$

$$\lambda_1 = \lambda_2 = -\mu,$$
$$\lambda_3 = -(\mu + \gamma + \rho),$$
$$\lambda_4 = -(\mu + \theta).$$

Since all the roots are less than zero, the COVID-19-free equilibrium is stable. □

3.4. *Stability of COVID-19 endemic equilibrium,* Σ^*

Theorem 3.2. *The COVID-19 endemic equilibrium is stable if* $R_{COVID-19} < 1$ *otherwise it is unstable.*

Proof. COVID-19 endemic equilibrium is stable and $R_{COVID-19} < 1$ if it is shown that all the roots of the Jacobian matrix of the system is negative. At CEE when each of the parameters is greater than zero, (17) reduces to

$$J = \begin{pmatrix} -c_1 & \rho & 0 & 0 \\ 0 & -(X+c_2) & -Y & 0 \\ \beta & X & Y-c_3 & \tau \\ \theta & 0 & \phi & -c_4 \end{pmatrix}, \tag{20}$$

where,

$$X = \frac{\varepsilon L^*}{\delta + L^*}, Y = \frac{\varepsilon G^*}{\delta + L^*} - \frac{\varepsilon L^* G^*}{(\delta + L^*)^2}, c_1 = (\mu + \theta + \rho),$$

$$c_2 = (\mu + \gamma + \rho), c_3 = (\mu + \theta), c_4 = (\mu + \tau).$$

In (20),

$$\left| J(\Sigma^* - \lambda I) \right| = 0 \Rightarrow$$

$$k_4\lambda^4 + k_3\lambda^3 + k_2\lambda^2 + k_1\lambda + k_0 = 0, \tag{21}$$

where,

$$k_4 = 1,$$

$$k_3 = c_1 + c_2 + c_3 + c_4 + X - Y,$$

$$k_2 = c_1\left[(X + c_2)(c_3 + c_4 - Y)\right] + (X + c_2)(c_3 + c_4 - Y) + c_4(c_3 - Y) - \phi\tau,$$

$$k_1 = \beta\rho\gamma + c_1\left[(X + c_2)(c_3 + c_4 - Y) + c_4(c_3 - Y) - \phi\tau\right] + (X + c_2)\left[c_4(c_3 - Y) - \phi\tau\right],$$

$$k_0 = c_1c_4(X + c_2)(c_3 - Y) + \beta\rho\gamma c_4 + \rho\theta\tau\gamma - \phi\tau.$$

By Routh-Hurtwitz stability criterion in Ayoade *et al*. (2019a**),** all the roots in (21) are negative if

$$k_3 > 0, k_1 > 0, k_0 > 0 \text{ and } k_3k_2k_1 - (k_1^2 + k_3^2k_0) > 0 \tag{22}$$

Hence, the COVID-19 endemic equilibrium $\sum^*$ is stable if the results in (22) are true. □

3.5. *Sensitivity analysis*

The effect of lockdown order on the Nigerian households are quantified by deriving the relative contributions of the major parameters to the impacts of lockdown on households using the normalised forward sensitivity index formula.

$$\Gamma_n^{R_{COVID-19}} = \frac{\partial R_{COVID-19}}{\partial n} \times \frac{n}{R_{COVID-19}} . \tag{23}$$

With (23), the sensitivity values of the major parameters to the COVID-19 impact ratio are derived in (24)-(25).

$$\frac{\partial R_{COVID-19}}{\partial \mu} = \frac{-\mu}{(\mu+\gamma)} \tag{24}$$

$$\frac{\partial R_{COVID-19}}{\partial \gamma} = \frac{-\gamma}{(\mu+\gamma)} \tag{25}$$

4. Numerical Simulation and Discussion

Numerical simulation is required to examine the influence of parameter perturbations on the structure of the model and to validate theoretical results of the study in section 3. Using the software maple together with a set of logical parameter values, we are able to study the economic impact of COVID-19 mitigation measures on the rate of government income generation which has been quantified in terms of the quantity $R_{COVID-19}$ computed in (16). Besides, the relative contributions of the major parameters to the economic impact of mitigation measures on Nigerian households are also revealed quantitatively via sensitivity indices of major parameters. Now, in system (1)-(4), set $\sigma = 0.6$, $\delta = 0.5$, $\mu = 0.1$, $\gamma = 0.5$, $\alpha = 0.1$, $\pi = 0.5$, $\rho = 0.6$, $\theta = 0.2$, $\beta = 0.8$ $\varepsilon = 0.2$, $\tau = 0.7$, $\phi = 0.1$, $M(0) = 5{,}000{,}000$, $G(0) = 3$, $L(0) = 45{,}000{,}000$, $R(0) = 15\ 000\ 000$

Then, from (16), $R_{COVID-19} = 0.500$ and the sensitivity indices of the key parameters in (24)-(25) are put in Table 3.

Table 3: Sensitivity indices of some parameters to $R_{COVID-19}$

Parameters	Sensitivity indices
μ	-0.167
γ	-0.833

Based on the interpretation of $R_{COVID-19}$ in subsection 3.2, the result $R_{COVID-19} = 0.500$ indicates that, within the parameter space of consideration, the COVID-19 mitigation measures have adverse effect on the government income generation and bring untold hardship to many households in Nigeria. The values of the key parameters σ, δ, μ and γ are then varied to verify the stability conditions of the model outlined in theorems 3 and 4, the results of which are put in Table 4. Table 4 reveals the changes in the structure of the model at various points of parameter considerations.

Table 4: Stability results of COVID-19 impact model

S/No	σ	δ	μ	γ	$R_{COVID-19}$	**Nature of Stability**
1	0.6	0.5	0.1	0.5	0.500	Unstable
2	0.6	0.6	0.09	0.4	0.735	Unstable
3	0.6	0.7	0.08	0.3	1.105	Stable
4	0.6	0.8	0.07	0.2	1.778	Stable
5	0.6	0.9	0.06	0.1	3.375	Stable
6	0.6	0.6	0.2	0.6	0.450	Unstable
7	0.6	0.7	0.3	0.7	0.420	Unstable
8	0.6	0.8	0.4	0.8	0.400	Unstable
9	0.6	0.9	0.5	0.9	0.386	Unstable

The value of each of the parameters is reduced by 10 and a graph is plotted with *M(0)* = 50, *L(0)* = 450, *R(0)* = 150 and *G(0)* = 3 in Figure 2 to visualise the economic effect of lockdown and social distancing order on the population of the poor Nigerian households.

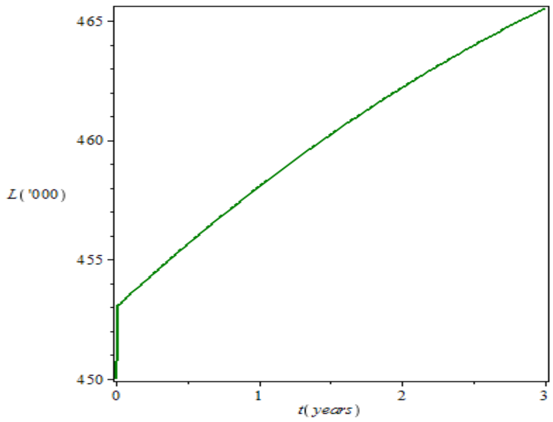

Figure 2: Effect of lockdown on the population of poor Nigerians

Figure 2 is the testimony to the recent upsurge in the figure of the Nigerians living in absolute poverty from 86.71 million in 2019 to 89.5 million in 2020 and the confirmation of the prediction that COVID-19 pandemic will sink more Nigerians below the poverty line (Ayoade & Farayola 2021b). The upward trend of Figure 2 from left to right indicates that the population of individuals in the lower class of the society continues to rise as more and more people from the middle and the rescued classes are falling below the poverty line due to massive job loss and nonpayment of salaries (especially for people who are working in private organisations, e.g., private schools, hotels and other private establishments) occasioned by lockdown and social distancing order.

The negative signs of sensitivity of the two parameters in Table 3 show that an increase or a decrease in the values of these parameters will result in a corresponding decrease or increase in the economic effect of mitigating COVID-19 outbreak through lockdown and social distancing on the economically challenged Nigerian households. For instance, the index -0.167 for μ implies that a decrease in government revenue due to the inability of some government workers to render services increases the economic woes of the poor households as well as the government inability to provide COVID-19 palliatives by 16.7%. In the same way, the index -0.833 for γ indicates that a fall in government revenue due to the instability in crude oil prices aggravates the misery of the poor households and the government inability to provide COVID-19 palliative by 83.3%.

As for the stability results in Table 4, the COVID-19-free equilibrium is stable when the economic impact of mitigating COVID-19 outbreak through movement restriction is minimal on the poor households which is observable within (S/No 3 – S/No 5 in Table 4). COVID-19 mitigation measures do not exert much adverse economic pressures on the poor households when the rates of loss in government revenues due to the instability in crude oil prices (γ) and the inability of some government workers to work (μ) are the least as well as when the rate of distribution of COVID-19 palliatives (δ) to the poor Nigerians is steady. On the other hand, COVID-19-free equilibrium is unstable (invariably, COVID-19 endemic equilibrium is stable) when the economic impact of mitigating COVID-19 outbreak through movement restriction is maximum on the poor households which is observable within (S/No 1 – S/No 2 & S/No 6 – S/No 9 in Table 4). COVID-19 mitigation measures increase the misery of the poor Nigerians when the rates of loss in government revenues due to the instability in crude oil prices (γ) and the inability of some government workers to work (μ) are maximum as well as when there is an impediment to the distribution of COVID-19 palliatives (δ).

Truly, the misery of the poor Nigerians has been increased by the mitigation measures of COVID-19 through the enactment of lockdown and movement restriction and there are strong evidences to support the claim that the enactment of lockdown order has pushed millions of Nigerians into the region of unstable COVID-19-free equilibrium (i.e., S/No 1 – S/No 2 & S/No 6 – S/No 9 in Table 4). The Nigeria budget for 2020 was planned on $57 per barrel of crude oil but was forced to be set at $30 due to the outbreak of COVID-19 which necessitated massive change in the budget to check the impact of the pandemic on the economy (Subair & Adekunle 2020). The government meant well for the poor as it ordered the country customs service to supply seized bags of rice to the states of the federation as parts of lockdown palliatives. However, there were allegations from some states that the rice supplied to them was infested with weevils and unsuitable for consumption. For instance, 1 800 bags of rice were sent to Oyo state of Nigeria whose population according to 2006 census figure was 5 580 894 as lockdown palliatives. The whole bags of rice were declared unfit for consumption and subsequently rejected and returned (Okonkwo 2020).

Furthermore, COVID-19 palliative distributions to the poor Nigerians have been generally insufficient and there have been instances to confirm that the distributions are accompanied with tragic stories. The most pathetic was the news of the death of a 62-year-old woman at Elele kingdom in Rivers state who was trampled to death in a stampede that followed the struggle to get COVID-19 food palliatives. Other four women, including a 91-year-old woman, were also left with various degrees of wounds at the sad event (Ogunsile 2020). Inadequate provision of COVID-19 palliative measures due to high rates of parameters μ and γ, rising public debt profile and dwindling oil prices coupled with corruption and distributive injustice inevitably decline parameters ε and δ and eventually aggravate the suffering of the poor Nigerians in the face of lockdown enactment to curb the spread of COVID-19 pandemic.

5. Conclusion

In this paper, a mathematical model that characterised the economic implications of COVID-19 mitigation measures on the Nigerians households had been designed and analysed. The model solutions were established to be bounded and nonnegative and the equilibria points of the model were derived. An analytic threshold that determined the severity of the economic effect of COVID-19 mitigation measures on the Nigerian households was also derived and used to analyse the stability of the model equilibria. The sensitivity analysis of the key system parameters was conducted and the numerical simulations were performed to locate the region where the economic effect of COVID-19 mitigation measures is minimum and where it is maximum on the Nigerian households.

Above all, the outbreak of COVID-19 pandemic has exposed the level of unpreparedness of Nigeria for emergencies. All academic activities are halted and many workers could not render services. All these are instigated by poor internet know-how and erratic power supply. For Nigeria to prepare adequately for future occurrence, government must diversify economy, prioritise internet know-how and revitalise power supply. If all these are strengthened, parameter σ will be stabilised while parameters μ and γ will be reduced to the barest minimum so that the hope for the individuals in the lower class of the society will continue to rise as parameters ρ, ε and are ϕ improving.

Acknowledgments

The authors acknowledged the insightful comments and valuable suggestions of the anonymous reviewers.

References

Adebayo B. 2018. Nigeria overtakes India in extreme poverty ranking. https://edition.cnn.com/2018/06/26/Africa/Nigeria-overtakes-india-extreme-poverty-intl/index.html (14 July 2021).

Alamba S. 2020. Nigeria: Protect most vulnerable in COVID-19 response. hrw.org/news/2020/04/14/Nigeria-protect-most-vulnerable-covid-19-response (10 Dec. 2020).

Alfani G. & Murphy T. 2017. Plague and lethal epidemics in the pre-industrial world. *Journal of Economic History* **77**(1)**:** 314–343.

Anastassopoulou C., Russo L., Tsakris A. & Siettos C. 2020. Data-based analysis, modelling and forecasting of the COVID-19 outbreak. *PLoS ONE* **15**(3): e0230405.

Ashleigh R.T., David N.F. & Amy L.G. 2020. Mathematical modelling of COVID-19 transmission and mitigation strategies in the population of Ontario, Canada. *CMAJ* **192**(19): E497-E505.

Ayoade A.A. 2013. On modified SEIR epidemic model with vaccination control. M.Sc. Dissertation (unpublished). University of Ilorin, Nigeria.

Ayoade A.A., Latunde T. & Folaranmi R.O. 2021a. Comparative analysis of transmissibility and case fatality ratio of SARS, MERS and COVID-19 via a mathematical modeling approach. *J. Fundam. Appl. Sci.* **13**(3): 1262-1274.

Ayoade A.A. & Farayola P.I. 2021b. A mathematical modelling of economic restoration through agricultural revitalisation in Nigeria. *Journal of Quality Measurement and Analysis* **17**(1): 79-91.

Ayoade A.A., Farayola P.I. & Lamidi T.O. 2019a. Dynamics and stability analysis of party switching in politics of Nigeria: A mathematical approach. *Daffodil International University Journal of Science and Technology* **14**(1): 53-60.

Ayoade A.A., Ibrahim M.O., Peter O.J. & Amadiegwu S. 2019b. On validation of an epidemiological model. *J. Fundam. Appl.Sci.* **11**(2): 578-586.

Ayoade A.A., Peter O.J., Ayoola T.A., Amadiegwu S.& Victor A.A. 2019c. A saturated treatment model for the transmission dynamics of rabies. *Malaysian Journal of Computing* **4**(1): 201-213.

Cakir Z. & Savas H.B. 2020. A mathematical modelling approach in the spread of the novel 2019 coronavirus SARS-CoV-2 (COVID-19) pandemic. *Electronic Journal General Medicine* **17**(4): em205.

Chan J.F.W., Yuan S., Kok K,H., To K.K.W., Chu H., Yang J., Xing F., Liu J., Yip C.C.Y., Poon R.W.S. & Tsoi H.W. 2020. A familiar cluster of pneumonia associated with the 2019 novel coronavirus indicating person-to-person transmission: A study of a family cluster. *The Lancet* **395**(10223): 514–523.

Chetty R., Stepner M., Abraham S., Lin S. & Scuderi B. 2016. The association between income and life expectancy in the United States, 2001-2014. *JAMA,* **315**(16):1750–1766.

Fang Y., Nie Y. & Penny M. 2020. Transmission dynamics of the COVID-19 outbreak and effectiveness of government interventions: A data-driven analysis. *Journal of Medical Virology* **92**(6): 645-659.

Goldsztejn U., Schwartzman D. & Nehorai A. 2020. Public policy and economic dynamics of COVID-19 spread: A mathematical modelling study. *PLoS ONE* **15**(12): e0244174.

Gralinski L.E. & Menachery V.D. 2020. Return of the coronavirus: 2019-nCoV. *Viruses* **12**(2): 135.

Jia J., Ding J., Liu S., Liao G. & Li J. 2020. Modeling the control of COVID-19: Impact of policy interventions and meteorological factors. *Electronic Journal of Differential Equations* **2020**(23): 1–24.

Jordà Ò., Singh S.R. & Taylor A.M. 2020. Longer-run economic consequences of Pandemic. Federal Reserve Bank of San Francisco Working Paper 2020-09.

Liu Y., Gayle A.A., Wilder-Smith A. & Rocklöv J. 2020. The reproductive number of COVID-19 is higher compared to SARS coronavirus. *Journal of Travel Medicine* **27**(2):1–4.

Ogunsile R. 2020. 62-year-old woman dies in struggle to get COVID-19 palliative in Rivers. https://www.naijanews.com/2020/05/18/62-year-old-woman-dies/ (5 March 2021).

Okonkwo S. 2020. Coronavirus: Why Nigeria's rice handouts aren't going down well. https://www.bbc.com/news/world-52488923 (22 February 2021).

Okpi A. 2018. Nigeria's economy: Services drive GDP, but oil still dominates exports. https://africacheck.org/reports/nigerias-economy-services-drive-gdp-but-oil-still-dominates-exports/ (20 August 2021).

Olawoyin O. 2020. Coronavirus: More troubles for Nigeria as oil pricecrashes below $20 - lowest in 18 years. https://www.premiumtimesng.com/business/385029-coronavirus-more-troubles-for-nigeria-as-oil-price-crashes-below-20-lowest-in-18-years.html (December 20, 2020).

Ruiz Estrada M.A., Park D., Konfronas E., Khan A. & Tahir M. 2020.The economic impact of massive infectious and contagious diseases: The case of Wuhan coronavirus. https://ssrn.com/abstract=3533771 (19 July 2021).

Subair K. & Adekunle I. 2021. Forecasting COVID-19 pandemic and capital market efficiency in Africa. *Journal of Quantitative Methods* **5**(1): 136-153.

Wrapp D., Wang N. & Corbett K.S., Goldsmith J.A., Hsieh C.L., Abiona O., Graham B.S. & McLellan J.S. 2020. Cryo-EM structure of the 2019-nCoV spike in the prefusion conformation. *Science* **367**(6483): 1260-1263.

Wu J.T., Leung K. & Leung G.M. 2020. Nowcasting and forecasting the potential domestic and international spread of the 2019-nCoV outbreak originating in Wuhan, China: A modelling study. *The Lancet* **395**(10225): 689–697.

Yang C. & Wang J. 2020. A mathematical model for the novel coronavirus epidemic in Wuhan, China. *Math. Biosci. Engineer*. **17**(3): 2708-2724.

Department of Mathematics
Faculty of Science
University of Lagos
Akoka, Yaba, Lagos,
Lagos State, NIGERIA
E-mail: ayoayoade@unilag.edu.ng[*]

Department of Fundamental and Applied Sciences
Universiti Teknologi PETRONAS
Perak, MALAYSIA
Prototype Engineering Development Institute (PEDI)
Ilesa, NIGERIA
E-mail: oluwaseyialiu@gmail.com

Received: 1 February 2022
Accepted: 8 May 2022

[*]Corresponding author