

BATHTUB HAZARD MODEL WITH COVARIATE AND RIGHT CENSORED DATA

(Model *Bathtub Hazard* Berkovariat dengan Data Tertapis Kanan)

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ABSTRACT

Lifetime distributions that present bathtub-shaped failure rates are becoming increasingly important especially when dealing with medical data. In this research, a two-parameter bathtub hazard model was extended to incorporate covariates in the presence of right censored data. The parameter estimates were computed based on maximum likelihood estimation (MLE) method. A simulation study was then executed to assess the performance of parameter estimates based on their bias, standard error (SE) and root mean square error (RMSE) at various censoring proportions and sample sizes. The results suggested that the performance of the estimator is better at larger sample size as it lower the standard error and root mean square error (RMSE). A decrease in censoring proportions yield smaller SE and RMSE values, whereas the values of bias decrease as the sample sizes and censoring proportions increase. Finally, the extended model was applied to a real medical data. Also, three confidence interval estimation (Wald, bootstrap-p, bootstrap-t) were obtained for each of the parameters of the model. The results suggested that the real data fitted the bathtub hazard model well.

Keywords: bathtub hazard; covariates; right censored

ABSTRAK

Taburan jangka hayat yang mempunyai bentuk fungsi kadar kegagalan seperti tab mandi menjadi kian penting terutamanya apabila melibatkan data perubatan. Dalam kajian ini, model *bathtub hazard* dengan dua parameter dikembangkan dengan menambah kovariat bersama data tertapis kanan. Anggaran parameter diperoleh melalui kaedah penganggaran kebolehdan maksimum (MLE). Seterusnya, kajian simulasi dibuat untuk menilai prestasi parameter berdasarkan nilai kecondongan, ralat piawai (SE), dan punca min kuasa ralat (RMSE) pada nilai perkadaran tapisan dan saiz sampel yang berbeza. Hasil analisis menunjukkan bahawa prestasi penganggar adalah lebih baik pada saiz sampel yang lebih besar, kerana ianya menghasilkan nilai SE dan RMSE yang lebih rendah. Penyusutan dalam nilai perkadaran tapisan menghasilkan nilai SE dan RMSE yang lebih rendah, namun nilai kecondongan didapati semakin menurun dengan peningkatan saiz sampel dan nilai perkadaran tapisan. Akhir sekali, model yang telah dikembangkan ini diaplikasikan kepada data perubatan yang sebenar. Tiga kaedah ukuran kebolehppercayaan anggaran (Wald, bootstrap-p, bootstrap-t) juga telah didapatkan untuk setiap parameter dalam model tersebut. Keputusan analisis mencadangkan model *bathtub hazard* sesuai diaplikasikan kepada data sebenar.

Kata kunci: bathtub hazard; kovariat; tertapis kanan

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