

SIFAT PEMBAYANGAN SECARA AKHIRNYA PADA SET RANTAIAN BERULANG JENIS- k UNTUK SUATU TINDAKAN- \mathbb{Z}^d (The Eventually Shadowing Property On k -type Chain Recurrent Set for a \mathbb{Z}^d -action)

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ABSTRAK

Pemerhatian terhadap kajian lepas telah mencetuskan satu ilham menarik untuk mentakrifkan satu konsep sifat pembayangan secara khusus untuk suatu tindakan- \mathbb{Z}^d yang dikenali sebagai sifat pembayangan secara akhirnya pada suatu subset tak varian. Dengan tertumpu kepada kajian terhadap tindakan- \mathbb{Z}^d yang merupakan suatu pemetaan $T : \mathbb{Z}^d \times X \rightarrow X$ pada suatu ruang metrik padat (X, ρ) , kajian ini memerhatikan dua jenis set dengan ciri-ciri yang tertentu iaitu set titik-titik tidak merayau jenis- k , $\Omega^k(T)$ dan set rantaian berulang jenis- k , $CR^k(T)$. Beberapa hubungan tertentu telah dihuraikan dan dibuktikan terlebih dahulu untuk mendekati tujuan utama kajian ini iaitu membuktikan bahawa $\Omega^k(T) = CR^k(T)$ jika pemetaan T tersebut mempunyai sifat pembayangan secara akhirnya pada set $CR^k(T)$. Seterusnya, kajian ini meringkaskan perbandingan antara penemuan hasil di atas bagi tindakan- \mathbb{Z}^d dengan penemuan hubungan implikasi yang sama bagi kasus homeomorfisma yang pernah dibincangkan pada beberapa makalah kajian lepas.

Kata kunci: sifat pembayangan secara akhirnya; set titik-titik tidak merayau jenis- k ; set rantaian berulang jenis- k

ABSTRACT

An observation of the past study has triggered an interesting idea to define a concept of shadowing property specifically for a \mathbb{Z}^d -action which known as eventually shadowing property on an invariant subset. By focusing on the study of the \mathbb{Z}^d -action which is a map $T : \mathbb{Z}^d \times X \rightarrow X$ on a compact metric space (X, ρ) , this study observes the two type of sets with certain characteristics which are the set of k -type nonwandering points, $\Omega^k(T)$ and the k -type chain recurrent set, $CR^k(T)$. Some certain relationships have been described and proven first in order to approach the main purpose of this study which is to prove that $\Omega^k(T) = CR^k(T)$ if the map T has the eventually shadowing property on the set $CR^k(T)$. Next, this study summarizes the comparison between the discovery of the above result for the \mathbb{Z}^d -action with the findings of the similar implication relationships for the case of homeomorphism that has been discussed according to several past research papers.

Keywords: eventually shadowing property; set of k -type nonwandering points; k -type chain recurrent set

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