

## HANKEL DETERMINANT FOR FUNCTIONS STARLIKE AND CONVEX WITH RESPECT TO SYMMETRIC POINTS

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### ABSTRACT

Sakaguchi (1959) introduced the class of functions  $S_x^*$ , starlike with respect to symmetric points. In 1977, Das and Singh introduced similar class of functions  $C_x$ , convex with respect to symmetric points. Here, upper bound for the functional  $|a_2a_4 - a_3^2|$  is attained for functions

$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$  belonging to  $S_x^*$  and  $C_x$ . The result obtained is sharp.

**Key words:** Starlike with respect to symmetric points; convex with respect to symmetric points; Hankel determinant

### ABSTRAK

Sakaguchi (1959) telah memperkenalkan kelas fungsi  $S_x^*$ , bak-bintang terhadap titik simetri. Pada tahun 1977, Das dan Singh telah memperkenalkan kelas fungsi  $C_x$ , cembung terhadap titik simetri. Di sini, batas atas bagi fungsian  $|a_2a_4 - a_3^2|$  diperoleh untuk fungsi  $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$  ahli kepada  $S_x^*$  and  $C_x$ . Hasilnya adalah yang terbaik.

**Kata kunci:** bak-bintang terhadap titik simetri; cembung terhadap titik simetri; penentu Hankel

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