

DIAGNOSING STUDENTS' DIFFICULTIES IN LEARNING MATHEMATICS: THE CASE OF BINARY OPERATION

ENVER TATAR & RAMAZAN DİKİCİ

ABSTRACT

The aim of this study is to diagnose students' difficulties in learning binary operation and its properties. A test consisting of 16 open-ended questions is applied to 74 students studying in the Department of primary school education of Agri Education Faculty at Ataturk University, Turkey. In addition to this, a semi-structured interview is applied to fifteen students to understand their learning difficulties clearly. The results indicate that the distributive and associative properties of binary operations have not been learned in the conceptual level and so students have more difficulty in these properties. It is also found that students have more difficulties in operations with the rules than those with the table.

Keywords: Abstract algebra; binary operation; learning difficulty; properties of binary operation.

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Appendix: Examination Questions

- Let $A = \{1, 2, 3\}$ and let Δ be defined as a function sending any pair of $A \times A$ to the first term of pair. Point Δ operation as a scheme and table.
- The operation of $*$ is defined by $x * y = x.y + 4.(x + y)$ on the set of integers. According to this, what is the result of the operation of $2 * 5$?
- Let $A = \{-1, 0, 1, 2\}$ and the operation of \diamond be defined by $x \diamond y = 3x - y^2$. According to this, what is the result of the operation of $(-1) \diamond 2$?
- The operations of Δ and $*$ are defined in the form of $a \Delta b = a + b$ and $a * b = ab + 2$ on the set of real numbers. According to this, what is the result of the operation of $(2 \Delta 3) * (4 \Delta 5) = ?$
- The operation of $\frac{3}{a} \Delta \frac{b}{4} = 2.a + b - a.b$ is defined for each a, b element on the set of positive real numbers. According to this, what is the result of the operation of $2 \Delta 1$?
- The set of positive odd numbers $T = \{1, 3, 5, 7, \dots\}$ and the operation of \circ defined in the form of $x \circ y = x + y$ are given. Is the set of T closed according to the operation of \circ ? Explain.
- The set of $A = \{x, y, z, t, e\}$ and the operation of Δ are given. The operation of Δ is defined as in the table on the right. According to this, is the set of A closed according to the operation of Δ ?

Δ	x	y	z	t	e
x	x	y	z	t	e
y	e	x	y	z	t
z	t	e	x	y	z
t	z	t	e	x	y
e	y	z	t	e	x

- The operation of Δ defined on the set of integers, is given in the form of $x \Delta y = 2x + y$. According to this,

- a. Is there associative properties of the operation of Δ ? Explain.
 b. Is there commutative properties of the operation of Δ ? Explain.
9. The operation of $*$ defined on the set of $A = \{a, b, c, d\}$, is given with the table on the side. Is there commutative property of the operation $*$? Explain.

*	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

10. The operations $x \Delta y = x$ and $x * y = 5xy$ are given on the set of real numbers. Search if there is distributive property of the operation $*$ on the operation of Δ .
11. The operation of Δ defined on the set of real numbers is given by the rule of $x \Delta y = x + y + xy$. Search the identity element of the operation Δ .

12. The $*$ operation defined on the set of $A = \{1, 2, 3, 4\}$ is given by the table on the side. What is the identity element of $*$ operation? Explain.

*	1	2	3	4
1	3	4	1	2
2	4	1	2	3
3	1	2	3	4
4	2	3	4	1

13. The \circ operation defined on the set of integers is given by the rule of $x \circ y = x - y + 3$. Search the identity element of the operation \circ .
14. The \otimes operation defined on the set of positive integers is given by the rule of $a \otimes b = 3a + 3b$. Search the identity element of the operation \otimes .
15. The Δ operation defined on the set of real numbers is defined in the form of $x \Delta y = x + y + 2xy$. According to this operation, find the inverse of the number 5.

16. The $*$ operation defined on the set of $A = \{a, b, c, d\}$ is given by the table on the side. Finding the identity element of the operation $*$, determine the inverse of every elements of the set of A.

*	a	b	c	d
a	d	a	b	c
b	a	b	c	d
c	b	c	d	a
d	c	d	a	b

Department Primary Education,
 Agri Education Faculty,
 Atatürk University, 04100-Agri, Turkey;
 E-mail: entatar@gmail.com

Department of Secondary Education,
 Kazım Karabekir Education Faculty,
 Atatürk University, 25240-Erzurum, Turkey;
 E-mail: rdikici@atauni.edu.tr

* Corresponding author.