

NON-REGULAR QUADRATIC STOCHASTIC OPERATORS WITH COUNTABLE STATE SPACE

(Ketidaksekataan Operator Stokastik Kuadratik dengan Set Terbilang)

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ABSTRACT

In this paper we present an example of non-regular extremal Volterra quadratic stochastic operator defined on infinite-dimensional simplex. We show that the construction of this operator to odd dimensional face of infinite-dimensional simplex is non-regular transformation. We employ method of approximation to prove non-regularity of initial operator.

Keywords: ergodic hypothesis; regular transformation; extremal Volterra quadratic stochastic operator with infinite state space.

ABSTRAK

Dalam kertas kerja ini, kami memperkenalkan satu contoh operator stokastik kuadratik Volterra yang ekstrim pada simplex berdimensi tak terhingga. Kami menunjukkan bahawa pembentukan operator ini pada muka dimensi ganjil daripada simpleks dimensi tak terhingga adalah penjelmaan yang tidak sekata. Kami menggunakan kaedah penghampiran untuk membuktikan ketidaksekataan pada operator awal.

Kata kunci: hipotesis ergodik; transformasi sekata; operator stokastik kuadratik Volterra yang ekstrim pada set tak terhingga.

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