

## SOLVING FRACTIONAL DIFFERENTIAL EQUATIONS USING FRACTIONAL EXPLICIT METHOD (Menyelesaikan Persamaan Pembezaan Pecahan Menggunakan Kaedah Tak Tersirat Pecahan)

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### ABSTRACT

This research is focusing in solving the fractional differential equations (FDEs) for linear and non-linear type using fractional explicit method (FEM) with constant step-size. Most of the numerical methods for solving FDEs involved the interpolating points of step size  $h$ . Some modifications were implemented in the derivation technique, where the step size  $2h$  are considered in the formula of the proposed method. The main goal of this research is to derive FEM by considering the implementation of second-order Adam-Bashforth method using Lagrange interpolation for fractional case. Besides, the order and convergence analysis of the developed method will also be investigated in this study. The algorithm of the proposed method is written in C language. Based on the numerical results obtained, it is clearly ratified that the proposed method converges as the step size,  $h$  is getting smaller in solving the FDEs.

*Keywords:* fractional differential equations; linear FDE; nonlinear FDE; fractional Riccati differential equation; single order FDE

### ABSTRAK

Kajian ini memberi tumpuan dalam menyelesaikan persamaan pembezaan pecahan (PPP) bagi jenis linear dan bukan linear menggunakan kaedah tak tersirat pecahan (KTTP) untuk langkah malar. Kebanyakan kaedah berangka bagi penyelesaian PPP melibatkan titik interpolasi saiz langkah  $h$ . Beberapa pengubahsuaian telah dilaksanakan dalam teknik derivasi, di mana saiz langkah  $2h$  dipertimbangkan dalam formula kaedah yang dicadangkan. Matlamat utama kajian ini adalah untuk menerbitkan KTTP dengan mempertimbangkan pelaksanaan kaedah Adam-Bashforth peringkat kedua menggunakan interpolasi Lagrange untuk kes pecahan. Selain itu, peringkat dan analisis konvergen bagi kaedah yang dibangunkan juga akan disiasat dalam kajian ini. Algoritma bagi kaedah yang dibangunkan ditulis dalam bahasa C. Berdasarkan hasil berangka yang diperolehi, jelas menunjukkan bahawa kaedah yang dicadangkan menumpu apabila saiz langkah  $h$  semakin kecil dalam menyelesaikan PPP.

*Kata kunci:* persamaan perbezaan pecahan; linear PPP; bukan linear PPP; persamaan perbezaan pecahan Riccati; PPP peringkat satu

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