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EXPLORING SIXTH-ORDER COMPACT FINITE DIFFERENCE SCHEMES FOR HYPERBOLIC PDE: A CASE STUDY ON THE NONLINEAR GOURSAT PROBLEM

(Meneroka Skema Beza Terhingga Padat Tertib Keenam untuk Persamaan Pembezaan Separa Hiperbolik: Kajian Kes Mengenai Masalah Goursat Tak Linear)

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ABSTRACT

This study underscores the significant potential of using sixth-order compact finite difference method to address the nonlinear Goursat problem, a major challenge in mathematical modeling. This approach paves the way for advancement in numerical studies, particularly through the linearization of the nonlinear Goursat problem, which facilitates the implementation of linear scheme. The research employs numerical analysis to assess the accuracy of this new scheme and discusses the theoretical outcomes of multiple series of numerical experiments. Through error analysis, the Von Neumann method, and Taylor series expansion, the results demonstrate that the proposed method achieves superior accuracy compared to traditional approaches and is unconditionally stable and consistent. Additionally, the study confirms the convergence of the new scheme through the Lax equivalence theorem. Moreover, the research highlights the cost-effectiveness of this new method, showcasing its ability to achieve high levels of accuracy with fewer computational resources. By thoroughly examining the underlying principles and real-world applications, the significant benefits of utilizing advanced high-order compact finite difference methods are emphasized.

Keywords: higher-order compact finite difference method; nonlinear Goursat problem; hyperbolic partial differential equation; accuracy; convergence

ABSTRAK

Kajian ini menekankan potensi besar penggunaan kaedah beza terhingga padat tertib keenam untuk menangani masalah Goursat tak linear, yang merupakan cabaran utama dalam pemodelan matematik. Pendekatan ini membuka jalan untuk kemajuan dalam kajian berangka, terutamanya melalui proses linearisasi masalah Goursat tak linear yang memudahkan pelaksanaan skema linear. Penyelidikan ini menggunakan analisis berangka untuk menilai ketepatan skema baru ini dan membincangkan hasil teori daripada pelbagai siri eksperimen berangka. Melalui analisis ralat, kaedah Von Neumann, dan pengembangan siri Taylor, hasil menunjukkan bahawa kaedah yang dicadangkan mencapai ketepatan yang lebih tinggi berbanding pendekatan tradisional dan bersifat stabil tanpa syarat serta konsisten. Tambahan pula, kajian ini mengesahkan penumpuan skema baru melalui teorem kesetaraan Lax. Selain itu, penyelidikan ini menekankan kos keberkesanan kaedah baru ini, dengan menunjukkan keupayaannya untuk mencapai tahap ketepatan yang tinggi dengan sumber pengiraan yang lebih sedikit. Dengan meneliti prinsip asas dan aplikasi dunia sebenar, manfaat penting penggunaan kaedah perbezaan terhingga padat tertib tinggi yang maju turut ditekankan.

Kata kunci: kaedah beza terhingga padat tertib tinggi; masalah Goursat tak linear; persamaan pembezaan separa hiperbolik; ketepatan; penumpuan

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