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MATHEMATICAL MODEL AND ANALYSIS OF POPULATION DYNAMICS FOR HEART FAILURE AND HEART TRANSPLANT

(Model Matematik dan Analisis Dinamik Populasi Kegagalan Jantung dan Pemindahan Jantung)

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ABSTRACT

Heart failure is a non-communicable disease that causes significant morbidity and mortality worldwide. Heart transplant is a vital treatment option for prolonging the lives of patients with heart failure. This study aims to develop a mathematical model that describes the population dynamics of heart failure and examines its stability. This model consists of three nonlinear ordinary differential equations that describe the interaction between individuals at risk of heart failure, heart failure patients, and heart transplant patients. The positivity of the solution is proven. The Routh-Hurwitz criteria are used to analyze the local stability of the equilibrium point. The global stability of the equilibrium point is established using a suitable Lyapunov function. The analysis reveals that the model has only one non-negative equilibrium point, which is found to be locally and globally asymptotically stable. Numerical simulations of the model using MATLAB are provided to verify and illustrate the analytical results. The findings of the study show that the parameters that should be controlled are the rate of acquired risk factors later in life, the probability of reversing modifiable risk factors, the progression rate from at-risk individuals to heart failure patients, the availability of heart transplant resources, the success rate of transplants, the rate of failed transplants, and the saturation factor. Control measures include implementing educational and vaccination programs, promoting lifestyle changes, conducting regular screenings, and expanding heart transplant resources. Controlling these parameters can reduce the at-risk population, decrease progression to heart failure, and improve transplant outcomes, thereby contributing to a lower overall prevalence of heart failure.

Keywords: heart failure; mathematical model; qualitative analysis; local stability; global stability

ABSTRAK

Kegagalan jantung adalah penyakit tidak berjangkit yang menyebabkan morbiditi dan kematian yang ketara di seluruh dunia. Pemindahan jantung adalah pilihan rawatan penting untuk memanjangkan hayat pesakit yang mengalami kegagalan jantung. Kajian ini bertujuan untuk membangunkan model matematik yang menerangkan dinamik populasi kegagalan jantung dan mengkaji kestabilannya. Model ini terdiri daripada tiga persamaan pembezaan biasa tak linear yang menerangkan interaksi antara individu yang berisiko mengalami kegagalan jantung, pesakit kegagalan jantung dan pesakit pemindahan jantung. Kepositifan penyelesaian model juga dibuktikan. Kriteria Routh-Hurwitz digunakan untuk menganalisis kestabilan setempat bagi titik keseimbangan. Kestabilan global bagi titik keseimbangan dibuktikan menggunakan fungsi Lyapunov yang sesuai. Analisis menunjukkan bahawa model hanya mempunyai satu titik keseimbangan bukan negatif, yang didapati stabil secara tempatan dan global secara asimptotik. Simulasi berangka model ini dijalankan menggunakan MATLAB untuk mengesahkan dan menggambarkan keputusan analitik. Dapatan kajian ini menunjukkan bahawa parameter yang perlu dikawal ialah kadar faktor risiko yang diperoleh pada akhir hayat, kebarangkalian untuk membalikkan faktor risiko yang boleh diubah, kadar perkembangan dari individu yang berisiko kepada pesakit kegagalan jantung, ketersediaan sumber pemindahan jantung, kadar kejayaan pemindahan kadar pemindahan yang gagal, dan faktor ketepuan. Langkah kawalan termasuk melaksanakan program pendidikan dan vaksinasi, mempromosikan perubahan gaya hidup, menjalankan saringan secara berkala, dan memperluas sumber pemindahan jantung. Mengubah parameter ini boleh mengurangkan populasi yang berisiko, menurunkan kadar perkembangan kepada kegagalan jantung, dan memperbaiki hasil pemindahan, dengan itu menyumbang kepada pengurangan keseluruhan prevalens kegagalan jantung.

Kata kunci: kegagalan jantung; model matematik; analisis kualitatif; kestabilan setempat; kestabilan global

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