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AN MCDM FRAMEWORK USING COMBINED COMPROMISE SOLUTION AND INTEGRATED WEIGHTING METHOD: OPTIMIZING SUSTAINABLE ENERGY OPTIONS

(Rangka Kerja MCDM Menggunakan Penyelesaian Kompromi Gabungan dan Kaedah Pemberat Bersepadu: Mengoptimalkan Pilihan Tenaga Lestari)

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ABSTRACT

The Combined Compromise Solution (CoCoSo) method is a relatively recent multiple criteria decision-making (MCDM) technique designed to address complex MCDM problems. This method evaluates and selects the optimal alternative based on multiple and often conflicting criteria. Given that criteria weights can significantly impact the decision-making process, it is crucial to devote particular attention to these weights. The aim of this study is to develop a framework to evaluate different MCDM evaluation models using the CoCoSo method, incorporating various objective weight methods and Integrated Criteria Objective Weight (ICOW) methods. A case study focusing on ranking sustainable energy options serves as the context for this MCDM problem. The study seeks to employ a comparative approach to identify the most appropriate weighting method for criteria in the CoCoSo framework. Relative Entropy Theory and the method of Lagrange Multipliers were used to derive ICOW. The case study results indicated that the MCDM model employing the ICOW method was identified as the most preferred model. The framework of CoCoSo method coupled with ICOW, along with the systematic approach in using similarity measures for ranking, offers valuable insights for MCDM practitioners and serves as a useful addition to the MCDM literature.

Keywords: multiple criteria decision-making (MCDM); Combined Compromise Solution (CoCoSo) method; objective weight method; Integrated Criteria Objective Weight (ICOW)

ABSTRAK

Kaedah Penyelesaian Kompromi Gabungan (CoCoSo) ialah teknik pembuat keputusan berdasarkan pelbagai kriteria (MCDM) yang agak terkini yang direka untuk menangani masalah MCDM yang kompleks. Kaedah ini menilai dan memilih alternatif yang optimum berdasarkan kriteria berbilang dan sering bercanggah. Memandangkan wajaran kriteria boleh memberi kesan ketara kepada proses membuat keputusan, adalah penting untuk menumpukan perhatian khusus kepada wajaran ini. Matlamat kajian ini adalah untuk membangunkan rangka kerja untuk menilai model penilaian MCDM yang berbeza menggunakan kaedah CoCoSo, menggabungkan pelbagai kaedah pemberat objektif dan kaedah Pemberat Objektif Kriteria Bersepadu (ICOW). Kajian kes yang memfokuskan pada kedudukan pilihan tenaga mampan berfungsi sebagai konteks untuk masalah MCDM ini. Kajian ini menggunakan pendekatan perbandingan untuk mengenal pasti kaedah pemberat yang paling sesuai untuk kriteria dalam rangka kerja CoCoSo. Teori Entropi Relatif dan kaedah Pengganda Lagrange telah digunakan untuk mendapatkan ICOW. Keputusan kajian kes menunjukkan bahawa model MCDM yang menggunakan kaedah ICOW dikenal pasti sebagai model yang paling diutamakan. Rangka kerja kaedah CoCoSo ditambah dengan ICOW, bersama-sama dengan pendekatan sistematik dalam menggunakan ukuran persamaan untuk pemeringkatan, menawarkan pandangan berharga untuk pengamal MCDM dan berfungsi sebagai tambahan berguna kepada literatur MCDM.

Kata kunci: pembuat keputusan berdasarkan pelbagai kriteria (MCDM); Kaedah Penyelesaian Kompromi Gabungan (CoCoSo); kaedah pemberat objektif; Pemberat Objektif Kriteria Bersepadu (ICOW)

1. Introduction

Multiple criteria decision-making (MCDM) is a field within operational research focused on ranking decision alternatives based on multiple, often conflicting criteria (Kumar *et al.* 2017). Various MCDM methods are rapidly developing to address and solve MCDM problems. To date, the widely used MCDM methods that are included in literature are such as the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) by Hwang and Yoon (1981), the multiple criteria optimization compromise solution (VIKOR) by Opricovic (1998), the elimination and choice expressing the reality method (ELECTRE) by Benayoun *et al.* (1966), the weighted sum method (WSM) by Fishburn (1967), the weighted product method (WPM) by Bridgman (1922), the preference ranking organization method for the enrichment of evaluations (PROMETHEE) by Brans and Vincke (1985) and the combined compromise solution (CoCoSo) method by Yazdani *et al.* (2019). The MCDM techniques are helpful in situations involving inherent complex decision-making, where no single criterion can be used as the sole basis for a decision (Villalba *et al.* 2024).

MCDM methods are employed to address a wide range of practical problems across various research fields, including solar photovoltaic (PV) plant investment and site selection (Wang *et al.* 2022; Yılmaz *et al.* 2023; Ilham *et al.* 2024), diet ranking system (Haseena *et al.* 2022; Ng *et al.* 2023), design of nearly zero energy buildings (Lu *et al.* 2024), carbon finance management for sustainable development (Liu *et al.* 2019; Wu & Niu 2024), academic performance ranking in education (Carnia *et al.* 2018) and structural assessment of building (Villalba *et al.* 2024). To improve the accuracy of results for an MCDM problem, Ishizaka and Siraj (2018) recommended employing multiple MCDM techniques in the solving process. Ozernoy (1992) noted that while many MCDM techniques are available for evaluating MCDM problems, no single approach is universally perfect for achieving desired results in all cases. Figure 1 provides an overview of the number of research articles associated with MCDM methods.

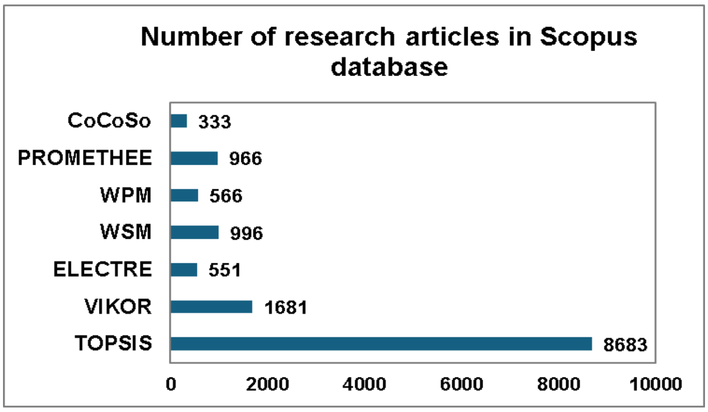


Figure 1: Number of research articles written in English and published between 2020 – 2024 in Scopus database

The results shown in Figure 1 were obtained by searching the Scopus database related to MCDM methods. The results indicate that CoCoSo has the fewest research articles, suggesting significant potential for growth in this area within MCDM.

In this paper, we focus on the CoCoSo method, a relatively recent MCDM technique introduced by Yazdani *et al.* (2019). The CoCoSo method is a combined compromise decision-making algorithm that takes into account a distance measure. It originates from grey relational coefficient and aims to improving the flexibility of the outcomes. The criteria weights are placed in the three equations in the decision-making algorithm. An aggregated multiplication rule is used in the last phase of the decision-making process to determine the rankings of the alternatives (Yazdani *et al.* 2019). Therefore, the criteria weights play a vital role in solving MCDM problem using CoCoSo method.

There have been several recent studies employing the CoCoSo method to address various aspects in MCDM. For example, Ozcalici (2023) integrated queue theory in several MCDM tools including CoCoSo method with random weight simulation. Kacprzak (2024) presented a new similarity measures of rankings to study the impact of various normalization techniques in CoCoSo method. As there is a lack of research focused on the objective weighting methods of criteria within CoCoSo analysis, our study intends to use a comparative approach to evaluate different MCDM evaluation models based on various objective weighting methods within the CoCoSo framework.

In practice, decision makers often rely on subjective weighting methods or objective weighting methods to determine criteria weights in MCDM cases. Subjective weighting methods include best-worst method (BWM) by Rezaei (2015), analytic hierarchy process (AHP) by Saaty (1987), linear programming techniques for multidimensional analysis of preferences (LINMAP) by Srinivasan and Shocker (1973), the simple multi attribute rating technique (SMART) by Edwards and Barron (1994) and direct rating by Bottomley and Doyle (2001). Objective weighting methods include Criteria Importance Through Intercriteria Correlation (CRITIC) Method by Diakoulaki *et al.* (1995), Shannon’s Entropy Method by Shannon (1948), Standard Deviation Method, Mean Weight method and coefficient of variation method.

However, criteria weights determined by subjective methods depend on and are influenced by the decision maker’s evaluations, judgments, and preferences (Wang & Lee 2009). In contrast to subjective weight methods, which rely on decision makers’ evaluations and preferences, objective weight methods use data present in the decision matrix and mathematical models to determine weights. This approach minimizes subjective bias and enhances objectivity. Consequently, this paper focuses exclusively on using objective weight methods for solving MCDM problems with the CoCoSo framework.

This study examines four objective weight methods from the literature: the CRITIC Method, Improved CRITIC Method, Shannon’s Entropy Method, and Standard Deviation Method. In this paper, a theoretical grounded methodology is presented in which criteria weights from two of these objective weight methods are combined to create the Integrated Criteria Objective Weight (ICOW). Overall, the study employs nine different weighting methods to form nine distinct MCDM evaluation models.

The aim of this study is to develop a framework to evaluate different MCDM evaluation models using the CoCoSo method, incorporating various objective weight methods and ICOW methods. A case study focusing on ranking sustainable energy options serves as the context for this MCDM problem. The study seeks to employ a comparative approach to identify the most appropriate weighting method for criteria in the CoCoSo framework. In this study, Relative Entropy Theory and the method of Lagrange Multipliers are used to derive

ICOW. Statistical techniques namely Spearman's rank correlation coefficient, Kendall rank correlation coefficient, Cosine similarity measure and Rank similarity index are utilized as measures of similarity to compare the rankings of alternatives from different MCDM evaluation models.

The remainder of the paper is organized as follows. Section 2 outlines the formulation of the CoCoSo method and describes the analytical procedures for computing weights using the CRITIC Method, Improved CRITIC Method, Shannon's Entropy Method, and Standard Deviation Method. Section 3 details the derivation of the ICOW and presents the framework used in this study. Section 4 includes a case study on sustainable energy, assessing various MCDM evaluation models using the CoCoSo method with different objective weight methods and ICOW methods. Section 5 concludes the work.

2. Preliminaries

The formulation of the CoCoSo method and the analytical procedures for computing weights using objective weight methods are presented in this section.

2.1. *The formulation of CoCoSo*

The combined compromise solution (CoCoSo) method was first proposed by Yazdani *et al.* (2019) in solving MCDM problems. The method produces the ranking of alternatives according to the CoCoSo Index. The procedure of CoCoSo method is presented as follows.

Considering a MCDM problem consists of *m* alternatives that are assessed by *n* criteria. The set of alternatives is denoted as $\mathbf{A} = \{A1, A2, ..., Am\}$. The set of criteria is denoted as $\mathbf{C} = \{C1, C2, ..., Cn\}$. In MCDM problem, the criteria are usually conflicting. Assuming J^+ is associated to the set of positive criteria and J^- is associated to the set of negative criteria. A criterion is characterized as positive criterion when the higher value of the criterion is the better for decision-making. A criterion is characterized as negative criterion when the lower value of the criterion is the better. Therefore, *n* criteria are divided into two sets, i.e. J^+ and J^-.

An MCDM problem is represented by a decision matrix $\mathbf{X} = \left\{ \left\{ x_{ij} \right\}_{i=1}^{m} \right\}_{j=1}^{n}$ with an order of $m \times n$, where $i = 1, 2, ..., m$ and $j = 1, 2, \ldots, n$. The decision matrix is given by

$$\mathbf{X} = \begin{array}{c} \\ \end{array}\begin{array}{c} \begin{array}{cccc} C1 & C2 & \cdots & Cn \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array} \begin{array}{c} \\ A1 \\ A2 \\ \vdots \\ Am \end{array}. \tag{1}$$

The normalized decision matrix is given by $\mathbf{P} = \left\{ \left\{ p_{ij} \right\}_{i=1}^{m} \right\}_{j=1}^{n}$ in which

$$p_{ij}=\begin{cases}\dfrac{x_{ij}-\min\left\{x_{ij}\right\}_{i=1}^{m}}{\max\left\{x_{ij}\right\}_{i=1}^{m}-\min\left\{x_{ij}\right\}_{i=1}^{m}}, & j\in J^{+}\\ \dfrac{x_{ij}-\max\left\{x_{ij}\right\}_{i=1}^{m}}{\min\left\{x_{ij}\right\}_{i=1}^{m}-\max\left\{x_{ij}\right\}_{i=1}^{m}}, & j\in J^{-}\end{cases}. \tag{2}$$

Assuming the weight for each criterion is w_j, where $j=1,2,\ldots,n$. Using weighted sum method (WSM), the WSM score S_i for each alternative is given by

$$S_i=\sum_{j=1}^{n}w_j p_{ij}\ . \tag{3}$$

Using weighted product method (WPM), the WPM score P_i for each alternative is given by

$$P_i=\sum_{j=1}^{n}p_{ij}^{\ w_j}\ . \tag{4}$$

Three characteristics values k_{ai}, k_{bi} and k_{ci} that incorporate the WSM score S_i and the WPM score P_i are computed using the Eq. (5), Eq. (6) and Eq. (7), respectively.

$$k_{ai}=\frac{S_i+P_i}{\sum_{j=1}^{m}\left(S_i+P_i\right)} \tag{5}$$

$$k_{bi}=\frac{S_i}{\min\left\{S_i\right\}_{i=1}^{m}}+\frac{P_i}{\min\left\{P_i\right\}_{i=1}^{m}} \tag{6}$$

$$k_{ci}=\frac{\lambda S_i+(1-\lambda)P_i}{\lambda\max\left\{S_i\right\}_{i=1}^{m}+(1-\lambda)\max\left\{P_i\right\}_{i=1}^{m}}, \tag{7}$$

where $0\le\lambda\le1$ is a parameter chosen by decision maker in which usually $\lambda=0.5$ as suggested in Yazdani *et al.* (2019).

The value of k_{ai} is obtained as the arithmetic mean of the sum of S_i and P_i. The value of k_{bi} is obtained as the sum of relative scores of S_i and P_i compared to the best. The value of k_{ci} is obtained as the balanced compromise of scores from WSM and WPM models.

The ranking of the alternatives using CoCoSo method is according to the CoCoSo Index k_i that employs an aggregated multiplication rule based on the three characteristics values k_{ai}, k_{bi} and k_{ci}. The CoCoSo Index k_i is given by

$$k_i=\left(k_{ai}k_{bi}k_{ci}\right)^{\frac{1}{3}}+\frac{1}{3}\left(k_{ai}+k_{bi}+k_{ci}\right), \tag{8}$$

where $i = 1, 2, \ldots, m$.

The ordered rank of alternatives is in accordance with the descending order of the values of k_i. The best alternative is selected when its CoCoSo Index is the largest.

2.2. *The analytical procedures for computing weights using objective weight methods*

In this paper, four existing objective weight methods are examined: CRITIC Method, Improved CRITIC Method, Shannon's Entropy Method and Standard Deviation Method. This section presents the concepts and formulas used to compute weights with these methods.

2.2.1. *CRITIC Method*

Criteria Importance Through Intercriteria Correlation (CRITIC) Method is an objective weight method that was first introduced by Diakoulaki *et al.* (1995). This method uses correlation analysis to detect contrast among the criteria (Yilmaz & Harmancioglu 2010).

Given an MCDM problem with *m* alternatives and *n* criteria, where the decision matrix is denoted by $\mathbf{X} = \left\{ \left\{ x_{ij} \right\}_{i=1}^{m} \right\}_{j=1}^{n}$. A dimensionless normalized matrix is given by $\mathbf{U} = \left\{ \left\{ u_{ij} \right\}_{i=1}^{m} \right\}_{j=1}^{n}$ in which u_{ij} is obtained through the following process.

$$u_{ij} = \frac{x_{ij} - x_j^{worst}}{x_j^{best} - x_j^{worst}} = \begin{cases} \dfrac{x_{ij} - \min\left\{x_{ij}\right\}_{i=1}^{m}}{\max\left\{x_{ij}\right\}_{i=1}^{m} - \min\left\{x_{ij}\right\}_{i=1}^{m}}, & j \in J^{+} \\ \dfrac{x_{ij} - \max\left\{x_{ij}\right\}_{i=1}^{m}}{\min\left\{x_{ij}\right\}_{i=1}^{m} - \max\left\{x_{ij}\right\}_{i=1}^{m}}, & j \in J^{-} \end{cases}, \tag{9}$$

where x_j^{best} is the best performance in j^{th} criterion, x_j^{worst} is the worst performance in j^{th} criterion, J^{+} is associated to positive criteria while J^{-} is associated to negative criteria.

Using the normalized matrix $\mathbf{U}$, *n* vectors $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n$ corresponding to *n* criteria are generated. Thus, we have $\mathbf{U} = \left\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n\right\}$. The standard deviation for each vector is denoted by s_j. The contrast intensity of the j^{th} criterion is quantified by the standard deviation s_j as given by

$$s_j = \sqrt{\frac{\sum_{i=1}^{m}\left(u_{ij} - \overline{u_j}\right)^2}{m-1}}. \tag{10}$$

The linear correlation coefficient r_{jk} between vector $\mathbf{u}_j$ and vector $\mathbf{u}_k$ is given by

$$r_{jk} = \frac{\sum_{i=1}^{m}\left(u_{ij} - \overline{u_j}\right)\left(u_{ik} - \overline{u_k}\right)}{\sqrt{\sum_{i=1}^{m}\left(u_{ij} - \overline{u_j}\right)^2}\sqrt{\sum_{i=1}^{m}\left(u_{ik} - \overline{u_k}\right)^2}}, \tag{11}$$

where $-1 \le r_{jk} \le 1$.

A lower magnitude of r_{jk} indicates greater discordance between the values of alternatives for the j^{th} criterion and the k^{th} criterion. The sum of $(1 - r_{jk})$ for j^{th} criterion is given by

$$T_j = \sum_{k=1}^{n} (1 - r_{jk}) . \tag{12}$$

The degree of the conflict generated by j^{th} criterion with respect to the decision situation defined by the other criteria is represented by T_j (Diakoulaki *et al.* 1995).

As the information contained in an MCDM problem is related to both contrast intensity and conflict of the criteria, the Eq. (13) gives the multiplicative aggregation formula that is the product of s_j and $(1 - r_{jk})$. The formulation of I_j, as given in Eq. (13), represents the amount of information emitted by the j^{th} criterion that is determined by composing the measures that quantify both contrast intensity and conflict.

$$I_j = s_j \sum_{k=1}^{n} (1 - r_{jk}). \tag{13}$$

The CRITIC Weight (CRW) for each criterion calculated by CRITIC Method is given by

$$CRW_j = \frac{I_j}{\sum_{j=1}^{n} I_j} , \tag{14}$$

where $\sum_{j=1}^{n} CRW_j = 1$ and $j = 1, 2, \ldots, n$.

2.2.2. *Improved CRITIC Method*

Paramanik *et al.* (2022) proposed the Improved CRITIC Method as another objective weight method to compute weights of criteria in MCDM problem. The Improved CRITIC Method is developed based on the approach of reducing the maximum-to-minimum ratio of the weight.

Let's denote $g_j = I_j \big/ \sum_{j=1}^{n} I_j$ as given by Eq. (14). The Improved CRITIC Weight (ICW) for each criterion calculated by Improved CRITIC Method is given by

$$ICW_j = \frac{1 + \sqrt{g_j}}{\sum_{j=1}^{n} \left(1 + \sqrt{g_j}\right)} , \tag{15}$$

where $\sum_{j=1}^{n} ICW_j = 1$ and $j = 1, 2, \ldots, n$.

2.2.3. *Shannon's Entropy Method*

Shannon's Entropy Method was first introduced by Shannon (1948) and it is an objective weight method. Assuming there are m alternatives to be evaluated by n criteria. The decision matrix is denoted by $\mathbf{X} = \left\{ \left\{ x_{ij} \right\}_{i=1}^{m} \right\}_{j=1}^{n}$.

The normalized matrix is given by $\mathbf{V} = \left\{ \left\{ v_{ij} \right\}_{i=1}^{m} \right\}_{j=1}^{n}$ in which

$$v_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}. \tag{16}$$

The entropy for each criterion is computed using the following formula.

$$e_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} v_{ij} \ln v_{ij}. \tag{17}$$

The Shannon's Entropy Weight (SEW) for each criterion calculated by Shannon's Entropy Method is given by

$$SEW_j = \frac{1 - e_j}{n - \sum_{j=1}^{n} e_j}, \tag{18}$$

where $\sum_{j=1}^{n} SEW_j = 1$ and $j = 1, 2, \ldots, n$.

2.2.4. *Standard Deviation Method*

The Standard Deviation Method is an objective approach used to determine criteria weights in MCDM problems. The method uses standard deviation s_j of each criterion to quantify the contrast intensity of the criterion.

The Standard Deviation Weight (SDW) for each criterion calculated by Standard Deviation Method is given by

$$SDW_j = \frac{s_j}{\sum_{j=1}^{n} s_j}, \tag{19}$$

where $\sum_{j=1}^{n} SDW_j = 1$ and $j = 1, 2, \ldots, n$.

3. Methodology

In this section, the Integrated Criteria Objective Weight (ICOW) is derived by combining two objective weighting methods. The derivation of ICOW is detailed in Section 3.1. Section 3.2 constructs nine MCDM evaluation models using the CoCoSo method, incorporating both objective weight methods and ICOW methods. Section 3.3 outlines the framework for model selection using CoCoSo coupled with ICOW.

3.1. *Derivation of the ICOW*

Relative Entropy Theory and the Lagrange Multipliers Method were used to derive the ICOW. Initially, Relative Entropy is defined in Definition 3.1, followed by an explanation of the Lagrange Multipliers Method. Theorem 3.1 then details the derivation of ICOW.

Definition 3.1. Relative entropy is known as Kallback-Leiber divergence. It is used to measure the degree of similarity of two discrete probability distributions (Post & Poti 2016; Zhou & Xu 2011; Barchielli *et al.* 2018). The smaller value in relative entropy means the better consistency of the two probability distributions. Suppose $p(x)$ and $q(x)$ are two discrete probability distributions, the relative entropy is defined as

$$\begin{aligned} D\left(p(x) \,\|\, q(x)\right) &= E_p\left[\ln\left(\frac{p(x)}{q(x)}\right)\right] \\ &= \sum_{x\in X} p(x)\ln\left(\frac{p(x)}{q(x)}\right) \end{aligned}. \tag{20}$$

Suppose a function f has variables x_j, $j = 1, 2, \ldots, n$. The method of Lagrange Multipliers is used to solve the unknown x_j that maximize or minimize f subject to constraint $g(x_j) = k$ (Stewart *et al.* 2021). Assuming $\nabla g(x_j) \neq 0$ and the maximum or minimum values of f exist on the surface $g(x_j) = k$, the unknown values of x_j can be found such that

$$\begin{cases} \nabla f(x_j) = \lambda \nabla g(x_j) \\ g(x_j) = k \end{cases}, \tag{21}$$

where λ is the Lagrange Multiplier.

Using **Definition 3.1** and the method of Lagrange Multipliers, ICOW that combines two objective weighting methods is derived in **Theorem 3.1**.

Theorem 3.1. *Assuming β_j and γ_j are the weights of criteria determined by two objective weight methods. The Integrated Criteria Objective Weight (ICOW) w_j is given by*

$$w_j = \frac{\left(\beta_j\right)^{\frac{1}{2}}\left(\gamma_j\right)^{\frac{1}{2}}}{\sum_{j=1}^{n}\left(\beta_j\right)^{\frac{1}{2}}\left(\gamma_j\right)^{\frac{1}{2}}}, \tag{22}$$

where $j = 1, 2, \ldots, n$.

Proof. A discrete probability distribution function has two characteristics: (a) Each probability is between 0 and 1 (inclusive). (b) The sum of the probabilities is 1. Thus, weights of criteria are regarded as discrete probability distribution since it has these two characteristics: (a) The value of each weight is between 0 and 1 (inclusive). (b) The sum of the weights is 1. Assuming β_j and γ_j are the criteria weights determined by two objective weight methods, and w_j is denoted as the Integrated Criteria Objective Weight (ICOW). Hence, β_j, γ_j and w_j are regarded as discrete probability distributions. The relative entropy is given by

$$f(w_j) = \sum_{j=1}^{n} \left[w_j \ln\left(\frac{w_j}{\beta_j}\right) + w_j \ln\left(\frac{w_j}{\gamma_j}\right) \right]. \quad (23)$$

By minimizing the relative entropy $f(w_j)$ and subject to constraint $\sum_{j=1}^{n} w_j = 1$, we solve ICOW w_j as follows.

$$w_j = \arg\min \sum_{j=1}^{n} \left[w_j \ln\left(\frac{w_j}{\beta_j}\right) + w_j \ln\left(\frac{w_j}{\gamma_j}\right) \right] \text{ subject to } \sum_{j=1}^{n} w_j = 1 \quad (24)$$

To adopt the method of Lagrange Multipliers in solving unknown w_j, rewrite $f(w_j)$ in Eq. (23) into Eq. (25). Hence, we obtain $\nabla f(w_j)$ in Eq. (26).

$$\begin{aligned} f(w_j) &= \sum_{j=1}^{n} \left[w_j \ln\left(\frac{w_j^{2}}{\beta_j \gamma_j}\right) \right] \\ &= \sum_{j=1}^{n} \left[w_j \ln\left(w_j^{2}\right) - w_j \ln\left(\beta_j \gamma_j\right) \right] \\ &= \sum_{j=1}^{n} \left[2 w_j \ln\left(w_j\right) - w_j \ln\left(\beta_j \gamma_j\right) \right] \end{aligned} \quad (25)$$

$$\nabla f(w_j) = \begin{bmatrix} f_{w_1} \\ f_{w_2} \\ \vdots \\ f_{w_n} \end{bmatrix} = \begin{bmatrix} 2 + 2\ln(w_1) - \ln(\beta_1 \gamma_1) \\ 2 + 2\ln(w_2) - \ln(\beta_2 \gamma_2) \\ \vdots \\ 2 + 2\ln(w_n) - \ln(\beta_n \gamma_n) \end{bmatrix} \quad (26)$$

Define another function $g(w_j)$ in Eq. (27). Hence, we obtain $\nabla g(w_j)$ in Eq. (28).

$$g(w_j) = \sum_{j=1}^{n} w_j - 1 \quad (27)$$

$$\nabla g(w_j)=\begin{bmatrix} g_{w_1} \\ g_{w_2} \\ \vdots \\ g_{w_n} \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{28}$$

The unknown w_j can be found such that

$$\begin{cases} \nabla f(w_j)=\lambda \nabla g(w_j) \\ g(w_j)=0 \end{cases}, \tag{29}$$

where λ is the Lagrange Multiplier.

Hence, the system of equations in Eq. (29) can be expressed in Eq. (30), where the first n equations are formulated from $\nabla f(w_j)=\lambda \nabla g(w_j)$ and the last equation is formulated from $g(w_j)=0$.

$$\begin{cases} 2+2\ln\left(w_1\right)-\ln\left(\beta_1\gamma_1\right)-\lambda=0 \\ 2+2\ln\left(w_2\right)-\ln\left(\beta_2\gamma_2\right)-\lambda=0 \\ \vdots \\ 2+2\ln\left(w_n\right)-\ln\left(\beta_n\gamma_n\right)-\lambda=0 \\ \sum_{j=1}^{n} w_j-1=0 \end{cases} \tag{30}$$

The equation $2+2\ln\left(w_j\right)-\ln\left(\beta_j\gamma_j\right)-\lambda=0$, where $j=1, 2, \ldots, n$, can be expressed as Eq. (31).

$$2\ln\left(w_j\right)-\ln\left(\beta_j\gamma_j\right)=\lambda-2$$

$$\ln\left(\frac{w_j^2}{\beta_j\gamma_j}\right)=\lambda-2$$

$$w_j=e^{\frac{\lambda}{2}-1}\left(\beta_j\right)^{\frac{1}{2}}\left(\gamma_j\right)^{\frac{1}{2}} \tag{31}$$

Substitute w_j in Eq. (31) into $\sum_{j=1}^{n} w_j-1=0$, we obtain λ in Eq. (32).

$$\sum_{j=1}^{n} e^{\frac{\lambda}{2}-1}\left(\beta_j\right)^{\frac{1}{2}}\left(\gamma_j\right)^{\frac{1}{2}}-1=0$$

$$\lambda=-2\ln\left[\sum_{j=1}^{n}\left(\beta_j\right)^{\frac{1}{2}}\left(\gamma_j\right)^{\frac{1}{2}}\right]+2 \tag{32}$$

Replacing λ in Eq. (31) with Eq. (32), the w_j is solved in Eq. (33).

$$w_j = e^{\frac{1}{2}\left\{-2\ln\left[\sum_{j=1}^{n}(\beta_j)^{\frac{1}{2}}(\gamma_j)^{\frac{1}{2}}\right]+2\right\}-1}(\beta_j)^{\frac{1}{2}}(\gamma_j)^{\frac{1}{2}} = \frac{(\beta_j)^{\frac{1}{2}}(\gamma_j)^{\frac{1}{2}}}{\sum_{j=1}^{n}(\beta_j)^{\frac{1}{2}}(\gamma_j)^{\frac{1}{2}}} \quad (33)$$

The proof is complete. □

3.2. *Formulas for weights computed using objective weight and ICOW methods*

Table 1 summarizes the formulas for objective weights and ICOW used in the MCDM evaluation models. This study includes four existing objective weight methods: the CRITIC method (CR), Improved CRITIC method (IC), Shannon's Entropy method (SE) and Standard Deviation method (SD). Additionally, five ICOW variants are derived: CR*SE, CR*SD, IC*SE, IC*SD and SE*SD. Models M1 – M4 represent the objective weight methods while M5 – M9 represent ICOW methods.

Table 1: Formulas for weights computed using objective weight methods (models M1 – M4) and ICOW methods (models M5 – M9)

Model	Weighting methods	Notation of weights	Formulas for weights
M1	CRITIC method (CR)	CRITIC Weight (CRW)	$CRW_j = \frac{I_j}{\sum_{j=1}^{n} I_j}$ in Eq. (14)
M2	Improved CRITIC method (IC)	Improved CRITIC Weight (ICW)	$ICW_j = \frac{1+\sqrt{g_j}}{\sum_{j=1}^{n}\left(1+\sqrt{g_j}\right)}$ in Eq. (15)
M3	Shannon's Entropy method (SE)	Shannon's Entropy Weight (SEW)	$SEW_j = \frac{1-e_j}{n-\sum_{j=1}^{n} e_j}$ in Eq. (18)
M4	Standard Deviation method (SD)	Standard deviation Weight (SDW)	$SDW_j = \frac{s_j}{\sum_{j=1}^{n} s_j}$ in Eq. (19)
M5	CR*SE	ICOW(CR*SE)	$ICOW(CR*SE)_j = \frac{(CRW_j)^{\frac{1}{2}}(SEW_j)^{\frac{1}{2}}}{\sum_{j=1}^{n}(CRW_j)^{\frac{1}{2}}(SEW_j)^{\frac{1}{2}}}$
M6	CR*SD	ICOW(CR*SD)	$ICOW(CR*SD)_j = \frac{(CRW_j)^{\frac{1}{2}}(SDW_j)^{\frac{1}{2}}}{\sum_{j=1}^{n}(CRW_j)^{\frac{1}{2}}(SDW_j)^{\frac{1}{2}}}$

Table 1 (Continued)			
M7	IC*SE	ICOW(IC*SE)	$ICOW(IC*SE)_j = \dfrac{\left(ICW_j\right)^{\frac{1}{2}}\left(SEW_j\right)^{\frac{1}{2}}}{\sum_{j=1}^{n}\left(ICW_j\right)^{\frac{1}{2}}\left(SEW_j\right)^{\frac{1}{2}}}$
M8	IC*SD	ICOW(IC*SD)	$ICOW(IC*SD)_j = \dfrac{\left(ICW_j\right)^{\frac{1}{2}}\left(SDW_j\right)^{\frac{1}{2}}}{\sum_{j=1}^{n}\left(ICW_j\right)^{\frac{1}{2}}\left(SDW_j\right)^{\frac{1}{2}}}$
M9	SE*SD	ICOW(SE*SD)	$ICOW(SE*SD)_j = \dfrac{\left(SEW_j\right)^{\frac{1}{2}}\left(SDW_j\right)^{\frac{1}{2}}}{\sum_{j=1}^{n}\left(SEW_j\right)^{\frac{1}{2}}\left(SDW_j\right)^{\frac{1}{2}}}$

3.3. *The framework of CoCoSo method coupled with ICOW*

Figure 2 presents the proposed framework for the CoCoSo method, incorporating both objective weight methods and ICOW methods. The framework is structured into three phases. Phase 1 defines the decision problem, alternatives and criteria. Phase 2 involves constructing MCDM evaluation models using various criteria weighting methods and analyzing them with the CoCoSo method. Phase 3 utilizes similarity measures to compare the results of the MCDM evaluation models and suggests the optimal alternative based on the most preferred model.

4. Case Study

This section presents a case study involving an MCDM problem focused on ranking sustainable energy options.

4.1. *Phase 1: Define the decision problem, alternatives and criteria*

Global climate change poses a significant threat to the sustainable development of humanity. To ensure a healthier environment for future generations, policymakers need to make decisions that have a minimal negative impact on our planet. As development progresses, electricity demand rises, which in turn contributes significantly to global carbon dioxide emissions. Therefore, the choices made regarding electricity generation are crucial in addressing global climate change. It is essential to select sustainable energy options that not only benefit the environment but also support national growth and development.

In this case study, six electricity generation options in Turkiye were evaluated based on a range of conflicting criteria from the aspects of technical, environmental, economic and socioeconomic (Sahin 2021; Edenhofer *et al.* 2011; Bacon & Kojima 2011; Evans *et al.* 2017). The electricity generation options are designated as alternatives A1 through A6: solar photovoltaic (A1), hydro (A2), natural gas (A3), coal (A4), onshore wind (A5) and geothermal (A6).

Ten attributes for evaluation of sustainable energy options are designated as criteria C1 through C10: efficiency (C1), capacity factor (C2), lifetime (C3), job creation (C4), electricity mix share (C5), levelized cost of electricity (C6), land use (C7), water use (C8), greenhouse gas (GHG) emission (C9) and accident-related fatality (C10). As illustrated in Figure 3, the criteria include four attributes from the technical aspect, three from the environmental aspect, one from the economic aspect, and two from the socioeconomic aspect.

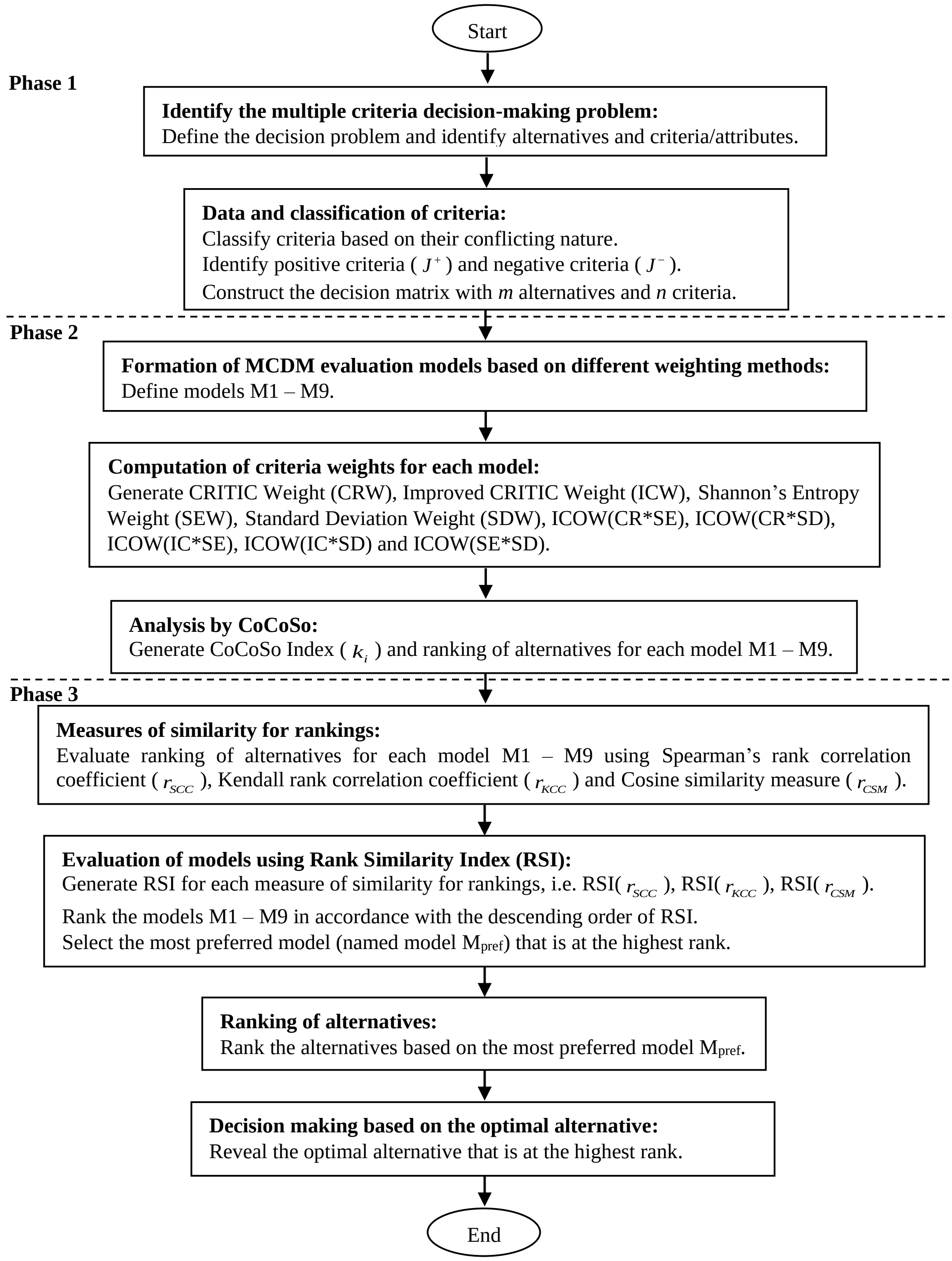


Figure 2: Framework of CoCoSo method coupled with ICOW in solving MCDM problem

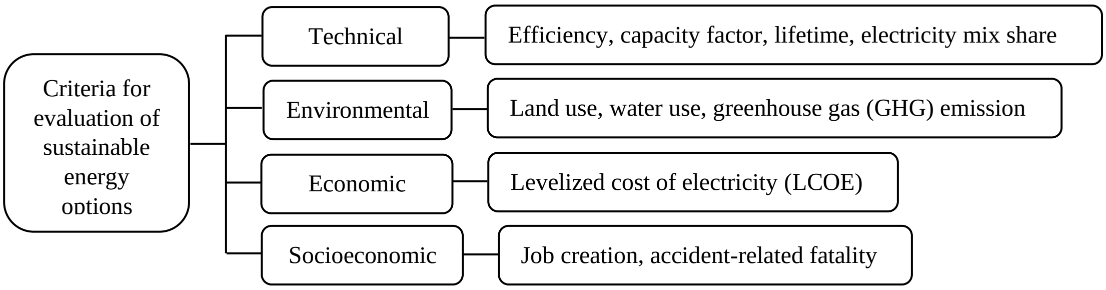


Figure 3: Criteria for evaluation of sustainable energy options

The criteria as shown in Figure 3 are categorized into two types: positive criteria (J^+) and negative criteria (J^-). Five criteria (C1 – C5) are classified as positive criteria while the remaining five criteria (C6 – C10) are classified as negative criteria as shown in Table 2. The data and units were sourced from published documents (Sahin 2021; Edenhofer *et al.* 2011; Bacon & Kojima 2011; Evans *et al.* 2017).

Table 2: List of positive criteria (C1 – C5) and negative criteria (C6 – C10)

Category	Criteria	Descriptions	Units
Positive criteria (J^+)	C1	Efficiency	%
	C2	Capacity factor	%
	C3	Lifetime	Year
	C4	Job creation	Avg job years/GWh
	C5	Electricity mix share	%
Negative criteria (J^-)	C6	Levelized cost of electricity (LCOE)	USD/MWh
	C7	Land use	m^2/kWh
	C8	Water use	L/kWh
	C9	Greenhouse gas (GHG) emission	gCO2-e/kWh
	C10	Accident-related fatality	Fatalities/GW_eyr

The criteria descriptions are outlined as follows.

- Efficiency (C1): The ratio of output energy to input energy.
- Capacity factor (C2): The ratio of the actual output of the plant to its maximum potential output.
- Lifetime (C3): Overall lifespan of the electricity generation option.
- Job creation (C4): Job years of full-time employment created over the overall lifetime of the electricity generation option.
- Electricity mix share (C5): The electricity generation share of the option.
- Levelized cost of electricity (C6): The average cost of electricity generation for a plant throughout its lifetime, including expenses for capital construction, operation and maintenance, fuel, carbon management, decommissioning and waste management costs.
- Land use (C7): Land area needed for the generation technology.
- Water use (C8): Water that is consumed and cannot be returned to its source.
- Greenhouse gas (GHG) emission (C9): The lifetime GHG emissions from the electricity generation option.

- Accident-related fatality (C10): Deaths resulting from accidents over the entire lifespan of the electricity generation option.

Table 3 displays the data for the case study.

Table 3: Data for the case study

	Sustainable energy options (alternatives)					
	Solar PV	Hydro	Natural gas	Coal	Onshore wind	Geothermal
	A1	A2	A3	A4	A5	A6
C1	13	90	49	38.5	34	15
C2	18	35	85	85	33	90
C3	25	80	30	40	25	40
C4	0.87	0.27	0.11	0.11	0.17	0.25
C5	2.56	19.7	30.34	37.2	6.54	2.44
C6	160	41.34	156	92.5	73.19	116.33
C7	0.0003	0.004	0.0003	0.0004	0.015	0.05
C8	0.01	20	1.6	1.6	0.001	156
C9	85	26	499	888	26	170
C10	0.0002	0.0027	0.0721	0.12	0.0019	0.0017

In this study, the goal of the decision problem was to identify the optimal sustainable energy option by maximizing the positive criteria and minimizing the negative criteria. A 6×10 decision matrix, comprising $m = 6$ alternatives and $n = 10$ criteria, was constructed as shown in Eq. (34). The elements of decision matrix $\mathbf{X}$ are extracted from the data presented in Table 3.

$$\mathbf{X} = \begin{array}{c} \begin{matrix} C1 & C2 & \cdots & C10 \end{matrix} \\ \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,10} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,10} \\ \vdots & \vdots & \ddots & \vdots \\ x_{6,1} & x_{6,2} & \cdots & x_{6,10} \end{bmatrix} \begin{matrix} A1 \\ A2 \\ \vdots \\ A6 \end{matrix} \end{array}. \tag{34}$$

4.2. *Phase 2: The analysis of CoCoSo and the MCDM evaluation models*

Nine MCDM evaluation models (M1 – M9) using CoCoSo were developed, each based on a distinct objective weight and ICOW method as shown in Table 4.

Table 4: MCDM evaluation models (models M1 – M9) based on distinct weighting methods

Category	Model	Weighting method	Criteria weights
Objective Weight Method	M1	CRITIC method (CR)	CRITIC Weight (CRW)
	M2	Improved CRITIC method (IC)	Improved CRITIC Weight (ICW)
	M3	Shannon's Entropy method (SE)	Shannon's Entropy Weight (SEW)
	M4	Standard Deviation method (SD)	Standard Deviation Weight (SDW)
Integrated Criteria Objective Weight (ICOW) Method	M5	CR*SE	ICOW(CR*SE)
	M6	CR*SD	ICOW(CR*SD)
	M7	IC*SE	ICOW(IC*SE)
	M8	IC*SD	ICOW(IC*SD)
	M9	SE*SD	ICOW(SE*SD)

Table 5 displays the criteria weights calculated using the CRITIC method, Improved CRITIC method, Shannon's Entropy method, and Standard Deviation method for models M1 – M4. The computation procedures are detailed previously in Section 2.

Table 5: Weights of criteria using objective weight methods (models M1 – M4)

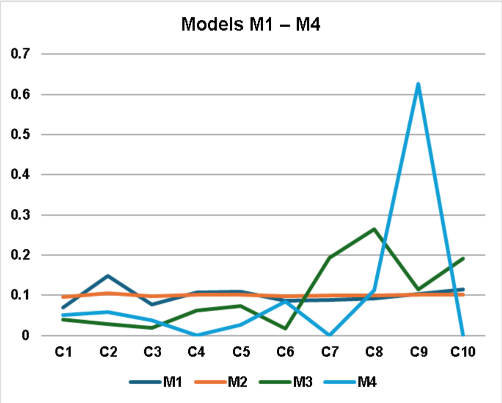
	M1	M2	M3	M4
	CRW	ICW	SEW	SDW
C1	0.07037	0.09625	0.03869	0.05114
C2	0.14742	0.10528	0.02795	0.05871
C3	0.07751	0.09725	0.01911	0.03762
C4	0.10777	0.10104	0.06132	0.00052
C5	0.10878	0.10116	0.07358	0.02717
C6	0.08728	0.09854	0.01691	0.08494
C7	0.08908	0.09877	0.19308	0.00004
C8	0.09290	0.09926	0.26387	0.11298
C9	0.10360	0.10055	0.11490	0.62678
C10	0.11529	0.10190	0.19059	0.00009

Table 6 shows the criteria weights calculated using ICOW methods (CR*SE, CR*SD, IC*SE, IC*SD, SE*SD) for models M5 – M9. The computation algorithm is detailed earlier in Section 3.

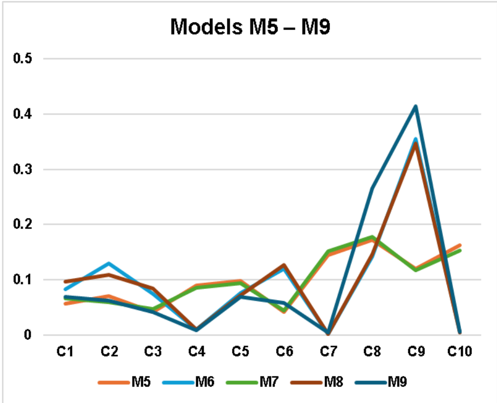
Table 6: Weights of criteria using ICOW methods (models M5 – M9)

	M5	M6	M7	M8	M9
	ICOW(CR*SE)	ICOW(CR*SD)	ICOW(IC*SE)	ICOW(IC*SD)	ICOW(SE*SD)
C1	0.05740	0.08363	0.06699	0.09713	0.06865
C2	0.07061	0.12969	0.05955	0.10883	0.06252
C3	0.04234	0.07528	0.04733	0.08374	0.04139
C4	0.08942	0.01048	0.08641	0.01007	0.00875
C5	0.09841	0.07579	0.09470	0.07258	0.06901
C6	0.04226	0.12003	0.04481	0.12666	0.05850
C7	0.14426	0.00248	0.15159	0.00260	0.00405
C8	0.17223	0.14282	0.17765	0.14660	0.26648
C9	0.12001	0.35524	0.11799	0.34754	0.41417
C10	0.16306	0.00456	0.15298	0.00425	0.00649

Figure 4 illustrates the varying distribution of criteria weights obtained from models M1 – M4 and models M5 – M9. The variability in criteria weights is presented in the box plot diagrams in Figure 5, which reveal differences in maximum values, minimum values, and ranges across all models. The CoCoSo Index (k_i) values and the rankings of alternatives for models M1 – M4 are provided in Table 7, while those for models M5 – M9 are presented in Table 8.



(a)



(b)

Figure 4: Criteria weights for (a) models M1 – M4 and (b) models M5 – M9

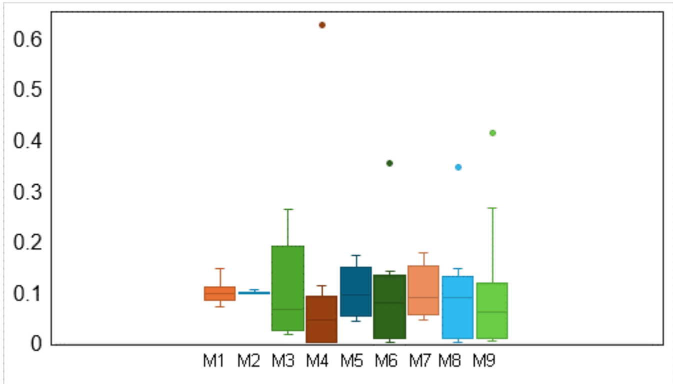


Figure 5: Box plot diagrams of criteria weights for models M1 – M9

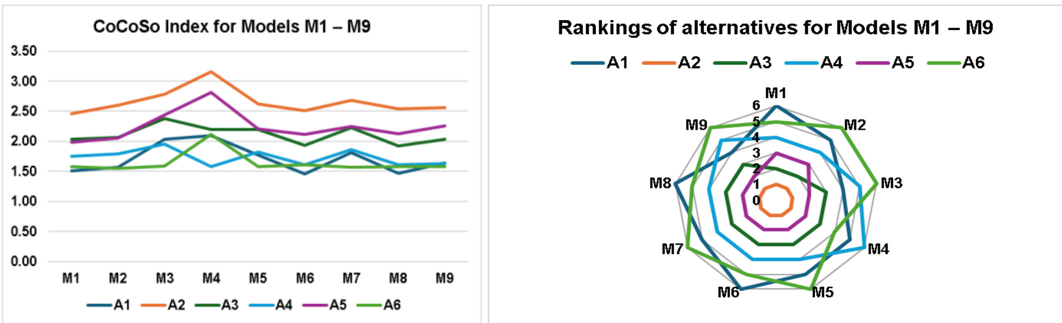
Table 7: CoCoSo Index (k_i) and rankings of alternatives for models M1 – M4

	M1		M2		M3		M4	
	k_i	Rank	k_i	Rank	k_i	Rank	k_i	Rank
A1	1.511	6	1.567	5	2.036	4	2.097	5
A2	2.458	1	2.603	1	2.779	1	3.156	1
A3	2.037	2	2.063	2	2.378	3	2.192	3
A4	1.748	4	1.787	4	1.949	5	1.581	6
A5	1.982	3	2.051	3	2.443	2	2.814	2
A6	1.577	5	1.550	6	1.585	6	2.114	4

Table 8: CoCoSo Index (k_i) and rankings of alternatives for models M5 – M9

	M5		M6		M7		M8		M9	
	k_i	Rank	k_i	Rank	k_i	Rank	k_i	Rank	k_i	Rank
A1	1.775	5	1.459	6	1.815	5	1.463	6	1.642	4
A2	2.617	1	2.511	1	2.682	1	2.544	1	2.558	1
A3	2.195	3	1.938	3	2.229	3	1.923	3	2.039	3
A4	1.823	4	1.614	4	1.858	4	1.606	4	1.630	5
A5	2.206	2	2.112	2	2.246	2	2.122	2	2.255	2
A6	1.582	6	1.607	5	1.572	6	1.581	5	1.574	6

The formulation of the CoCoSo Index (k_i) was influenced by the criteria weights as described earlier. The variations in criteria weights based on different weighting methods, result in different CoCoSo Index values and different rankings of alternatives, as shown in Figure 6. It is observed that not all wheels corresponding to alternatives have the same radius in the radar chart (Figure 6b). This indicates that the rankings of alternatives generated from different MCDM evaluation models are different.



(a) (b)

Figure 6: An overview of (a) CoCoSo Index (k_i) and (b) rankings of alternatives for models M1 – M9

4.3. *Phase 3: Similarity measures of rankings*

Measures of similarity are used to compare the rankings of alternatives obtained from CoCoSo with different weighting methods. Table 9 summarizes the rankings of alternatives for the MCDM evaluation models M1 – M9.

Table 9: Rankings of alternatives for models M1 – M9

	M1	M2	M3	M4	M5	M6	M7	M8	M9
A1	6	5	4	5	5	6	5	6	4
A2	1	1	1	1	1	1	1	1	1
A3	2	2	3	3	3	3	3	3	3
A4	4	4	5	6	4	4	4	4	5
A5	3	3	2	2	2	2	2	2	2
A6	5	6	6	4	6	5	6	5	6

Assuming that the rankings of m alternatives for two different MCDM evaluation models resulted in sequences $H = (h_1, h_2, \cdots, h_m)$ and $T = (t_1, t_2, \cdots, t_m)$. Spearman's rank correlation coefficient (r_{SCC}), Cosine similarity measure (r_{CSM}) and Kendall rank correlation coefficient

(r_{KCC}) are used to generate the measures of similarity. The formulas for these measures of similarity are given in Eq. (35), Eq. (36) and Eq. (37).

$$r_{SCC} = 1 - \frac{6\sum_{i=1}^{m}\left(h_i - t_i\right)^2}{m(m^2-1)} \tag{35}$$

$$r_{CSM} = \frac{\sum_{i=1}^{m} h_i t_i}{\sqrt{\sum_{i=1}^{m} h_i^2}\sqrt{\sum_{i=1}^{m} t_i^2}} \tag{36}$$

$$r_{KCC} = \frac{n_c - n_d}{\frac{1}{2}m(m-1)} \ , \tag{37}$$

where n_c is the number of concordant pairs and n_d is the number of discordant pairs. Any pair of rankings $\left(h_i, t_i\right)$ and $\left(h_j, t_j\right)$ where $i < j$ are said to be concordant if either both $h_i > h_j$ and $t_i > t_j$ holds or both $h_i < h_j$ and $t_i < t_j$. Otherwise, the pair of rankings are said to be discordant.

Using the rankings of alternatives for models M1 – M9 as shown in Table 9, the results of Spearman's rank correlation coefficient, Kendall rank correlation coefficient and Cosine similarity measure are presented in Table 10, Table 11 and Table 12, respectively.

Table 10: Spearman's rank correlation coefficient (r_{SCC})

	M1	M2	M3	M4	M5	M6	M7	M8	M9
M1	1.000	0.943	0.771	0.771	0.886	0.943	0.886	0.943	0.771
M2	0.943	1.000	0.886	0.714	0.943	0.886	0.943	0.886	0.886
M3	0.771	0.886	1.000	0.829	0.943	0.829	0.943	0.829	1.000
M4	0.771	0.714	0.829	1.000	0.771	0.829	0.771	0.829	0.829
M5	0.886	0.943	0.943	0.771	1.000	0.943	1.000	0.943	0.943
M6	0.943	0.886	0.829	0.829	0.943	1.000	0.943	1.000	0.829
M7	0.886	0.943	0.943	0.771	1.000	0.943	1.000	0.943	0.943
M8	0.943	0.886	0.829	0.829	0.943	1.000	0.943	1.000	0.829
M9	0.771	0.886	1.000	0.829	0.943	0.829	0.943	0.829	1.000

Table 11: Kendall rank correlation coefficient (r_{KCC})

	M1	M2	M3	M4	M5	M6	M7	M8	M9
M1	1.000	0.867	0.600	0.600	0.733	0.867	0.733	0.867	0.600
M2	0.867	1.000	0.733	0.467	0.867	0.733	0.867	0.733	0.733
M3	0.600	0.733	1.000	0.733	0.867	0.733	0.867	0.733	1.000
M4	0.600	0.467	0.733	1.000	0.600	0.733	0.600	0.733	0.733
M5	0.733	0.867	0.867	0.600	1.000	0.867	1.000	0.867	0.867
M6	0.867	0.733	0.733	0.733	0.867	1.000	0.867	1.000	0.733
M7	0.733	0.867	0.867	0.600	1.000	0.867	1.000	0.867	0.867
M8	0.867	0.733	0.733	0.733	0.867	1.000	0.867	1.000	0.733
M9	0.600	0.733	1.000	0.733	0.867	0.733	0.867	0.733	1.000

Table 12: Cosine similarity measure (r_{CSM})

	M1	M2	M3	M4	M5	M6	M7	M8	M9
M1	1.000	0.989	0.956	0.956	0.978	0.989	0.978	0.989	0.956
M2	0.989	1.000	0.978	0.945	0.989	0.978	0.989	0.978	0.978
M3	0.956	0.978	1.000	0.967	0.989	0.967	0.989	0.967	1.000
M4	0.956	0.945	0.967	1.000	0.956	0.967	0.956	0.967	0.967
M5	0.978	0.989	0.989	0.956	1.000	0.989	1.000	0.989	0.989
M6	0.989	0.978	0.967	0.967	0.989	1.000	0.989	1.000	0.967
M7	0.978	0.989	0.989	0.956	1.000	0.989	1.000	0.989	0.989
M8	0.989	0.978	0.967	0.967	0.989	1.000	0.989	1.000	0.967
M9	0.956	0.978	1.000	0.967	0.989	0.967	0.989	0.967	1.000

Rank similarity index (RSI) is a measure of decision outcome similarity for an MCDM method with other MCDM methods (Chakraborty & Yeh, 2012). The value of RSI indicates the relative closeness of a method with other methods in terms of ranking outcome similarity. RSI for each method is computed by averaging the correlation values in each row of the correlation matrix. The method with the highest RSI value is considered the most preferred.

In this case study, nine MCDM evaluation models were constructed based on CoCoSo using nine distinct objective weighting methods. Hence, the model with the largest value of RSI indicates the most preferred model. The RSI values for r_{SCC} , r_{KCC} and r_{CSM} are given in Table 13.

Table 13: Rank Similarity Index (RSI) for r_{SCC} , r_{KCC} and r_{CSM} and rank of MCDM evaluation models

	Spearman's rank correlation coefficient		Kendall rank correlation coefficient		Cosine similarity measure	
	RSI(r_{SCC})	Rank of model	RSI(r_{KCC})	Rank of model	RSI(r_{CSM})	Rank of model
M1	0.879	8	0.763	8	0.977	8
M2	0.898	5	0.778	7	0.980	5
M3	0.892	6	0.807	5	0.979	6
M4	0.816	9	0.689	9	0.965	9
M5	**0.930**	1	**0.852**	1	**0.987**	1
M6	0.911	3	0.837	3	0.983	3
M7	**0.930**	1	**0.852**	1	**0.987**	1
M8	0.911	3	0.837	3	0.983	3
M9	0.892	6	0.807	5	0.979	6

It was observed that Model 5 and Model 7 consistently exhibit the highest RSI values, regardless of the similarity measures used. Therefore, Model 5 and Model 7 are selected as the most preferred models as both are ranked highest among the nine MCDM evaluation models.

Model 5 employed ICOW (CR*SE) to compute criteria weights while Model 7 used ICOW(IC*SE). It is worth noting that both models used ICOW methods. Table 14 displays the rankings of alternatives generated by Model 5 and Model 7.

The ranking results indicate that hydro power emerges as the most suitable sustainable energy option in this case study. The second most favorable option is onshore wind, followed by natural gas in third place. Geothermal energy is the least preferred option. These conclusions were reached by maximizing positive criteria and minimizing negative criteria across technical, environmental, economic and socioeconomic aspects in this case study.

Table 14: Rankings of sustainable energy options

Sustainable energy option	Solar PV	Hydro	Natural gas	Coal	Onshore wind	Geothermal
Alternative	A1	A2	A3	A4	A5	A6
Rank	5	1	3	4	2	6

5. Conclusion

The challenge that regularly arise up in MCDM applications is the determination of criteria weights. Considering that criteria weights had the potential to substantially influence the result of the decision-making process, it was crucial to devote particular attention to these weights. In this study, we have derived Integrated Criteria Objective Weight (ICOW) and a framework of CoCoSo method coupled with ICOW in solving MCDM problem.

Weights of criteria computed by two objective weight methods were combined to obtain ICOW by the method of Lagrange Multipliers and Relative Entropy Theory. Different objective weighting methods often relied on different assumptions for determining weights. These methods could result in varying weight distributions. Relative Entropy Theory provided a framework to reconcile these different weight distributions by quantifying the similarity or difference between them.

This paper presented a case study on ranking sustainable energy options using the framework developed in this study. Criteria weights were generated using nine different weighting methods. The distributions of criteria weights varied across these methods. The criterion with the highest weight has the greatest impact on the evaluation process. However, there was no consensus among the different weighting methods regarding this matter. The criterion "capacity factor" (C2) was found to have the largest weight by CRITIC method (CR) and Improved CRITIC method (IC). The weight for criterion "water use" (C8) was the highest when the criteria weights were computed by Shannon's Entropy method (SE), ICOW(CR*SE) method and ICOW(IC*SE) method. The weight of criterion "greenhouse gas emission" (C9) was the largest when Standard Deviation method (SD), ICOW(CR*SD) method, ICOW(IC*SD) method and ICOW(SE*SD) method were used to compute the criteria weights.

The rankings of alternatives for nine MCDM evaluation models obtained using CoCoSo also varied. Several measures of similarity (Spearman's rank correlation coefficient, Kendall rank correlation coefficient, Cosine similarity measure and Rank similarity index) were used to compare the rankings from these nine models. The similarity measures consistently led to the same conclusion. Specifically, the results indicated that Model 5 and Model 7, both utilizing the ICOW method for computing criteria weights, were the most preferred models according to the similarity measures. Model 5 utilized ICOW(CR*SE) to determine criteria weights, while Model 7 used ICOW(IC*SE). Both models produced identical values for the Rank Similarity Index (RSI). However, Model 5 had an advantage over Model 7 due to its simpler and faster computation algorithm for obtaining CR compared to IC. The resulting rankings of sustainable energy options in this case study are as follows:

$$\text{Hydro} \succ \text{Onshore wind} \succ \text{Natural gas} \succ \text{Coal} \succ \text{Solar PV} \succ \text{Geothermal}$$

In this study, hydro was chosen as the most suitable electricity generation option, having secured the top rank. The outcome of this study was compared with the models evaluated by

Sahin (2021). Based on the 42 different models utilized TOPSIS, VIKOR, ELECTRE, WSM, WPM, PROMETHEE, CRITIC, Shannon's Entropy, AHP, BWM, standard deviation weight and mean weight, Sahin (2021) concluded that the integrated rankings of sustainable energy options are:

$$\text{Hydro} \succ \text{Onshore wind} \succ \text{Solar PV} \succ \text{Geothermal} \succ \text{Natural gas} \succ \text{Coal}$$

Although the rankings of the current study are not exactly same as the rankings obtained in Sahin (2021), both studies concluded that hydro was the best energy generation option, followed by onshore wind as the second-best option. These findings are important for policy maker to make informed decision. To summarize, the results obtained from the framework of CoCoSo method coupled with ICOW in the current study show similar results with previous study that used different MCDM models. In addition, combining different weighting methods had the advantage of getting more reliable rankings of alternatives, as concluded in Alemi-Ardakani *et al.* (2016) and Al-Aomar (2010). Overall, this study indicates the robustness and effectiveness of the CoCoSo method and ICOW model.

The primary contributions of this study were summarized as follows:

- A framework of CoCoSo method coupled with ICOW was developed and demonstrated through a case study focused on selecting the optimal sustainable energy option.
- The study presented a systematic approach for comparing rankings of alternatives by utilizing similarity measures across different MCDM models.
- The study presented a theoretically grounded methodology that combined two objective weight methods to create ICOW.

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References

Al-Aomar R. 2010. A combined AHP-entropy method for deriving subjective and objective criteria weights. *International Journal of Industrial Engineering* **17**(1): 12-24.

Alemi-Ardakani M., Milani A.S., Yannacopoulos S. & Shokouhi G. 2016. On the effect of subjective, objective and combinative weighting in multiple criteria decision making: A case study on impact optimization of composites. *Expert Systems with Applications* **46**: 426-438.

Bacon R. & Kojima M. 2011. Issues in estimating the employment generated by energy sector activities. Washington, DC: World Bank Group.

Barchielli A., Gregoratti M. & Toigo A. 2018. Measurement uncertainty relations for discrete observables: Relative entropy formulation. *Communications in Mathematical Physics* **357**(3): 1253-1304.

Benayoun R., Roy B. & Sussman B. 1966. ELECTRE: Une méthode pour guider le choix en présence de points de vue multiples. *Note de travail* **49**: 2-120.

Bottomley P.A. & Doyle J.R. 2001. A comparison of three weight elicitation methods: good, better, and best. *Omega* **29**(6): 553-560.

Brans J.P. & Vincke Ph. 1985. A preference ranking organisation method: The PROMETHEE method for multiple criteria decision-making. *Management Science* **31**(6): 647-656.

Bridgman P.W. 1922. *Dimensional Analysis*. New Haven: Yale University Press.

Carnia E., Amarti Z. & Supriatna A.K. 2018. Academic performance ranking by the use of the analytic hierarchy process (AHP) and TOPSIS methods. *AIP Conference Proceedings* **2043**(1): 020005.

Chakraborty S. & Yeh C.H. 2012. Rank similarity based MADM method selection. *2012 International Conference on Statistics in Science, Business and Engineering (ICSSBE)* in Langkawi, Malaysia.

Diakoulaki D., Mavrotas G. & Papayannakis L. 1995. Determining objective weights in multiple criteria problems: The critic method. *Computers & Operations Research* **22**(7): 763-770.

Edenhofer O., Pichs-Madruga R., Sokona Y., Seyboth K., Kadner S., Zwickel T., ... & Matschoss P. 2011. *Renewable energy sources and climate change mitigation: Special report of the intergovernmental panel on climate change.* New York: Cambridge University Press.

Edwards W. & Barron F.H. 1994. SMARTS and SMARTER: Improved simple methods for multiattribute utility measurement. *Organizational Behavior and Human Decision Processes* **60**(3): 306-325.

Evans A., Strezov V. & Evans T.J. 2017. Sustainability concepts of energy generation technologies. In Abraham M.A. (ed.). *Encyclopedia of sustainable technologies*: 3-10. Amsterdam, The Netherlands: Elsevier.

Fishburn P.C. 1967. Additive utilities with incomplete product sets: Application to priorities and assignments. *Operations Research* **15**(3): 537-542.

Haseena S., Saroja S. & Revathi T. 2022. A fuzzy approach for multi criteria decision making in diet plan ranking system using cuckoo optimization. *Neural Computing and Applications* **34**(16): 13625-13638.

Hwang C.L. & Yoon K. 1981. *Multiple Attribute Decision Making: Methods and Applications A State-of-the-Art Survey*. Berlin, Heidelberg: Springer-Verlag.

Ilham N.I., Dahlan N.Y. & Hussin M.Z. 2024. Optimizing solar PV investments: A comprehensive decision-making index using CRITIC and TOPSIS. *Renewable Energy Focus* **49**: 100551.

Ishizaka A. & Siraj S. 2018. Are multi-criteria decision-making tools useful? An experimental comparative study of three methods. *European Journal of Operational Research* **264**(2): 462-471.

Kacprzak D. 2024. A new similarity measure for rankings obtained in MCDM problems using different normalization techniques. *Operations Research and Decisions* **34**(2): 47-64.

Kumar A., Sah B., Singh A.R., Deng Y., He X., Kumar P. & Bansal R. C. 2017. A review of multi criteria decision making (MCDM) towards sustainable renewable energy development. *Renewable and Sustainable Energy Reviews* **69**: 596-609.

Liu X., Zhou X., Zhu B., He K. & Wang P. 2019. Measuring the maturity of carbon market in China: An entropy-based TOPSIS approach. *Journal of Cleaner Production* **229**: 94-103.

Lu M., Sun Y. & Ma Z. 2024. Multi-objective design optimization of multiple energy systems in net/nearly zero energy buildings under uncertainty correlations. *Applied Energy* **370**: 123620.

Ng S.F., Tukiman N., Ismail A., Khalid A.K., Yunos N.M. & Norizan N.A. 2023. A framework of nutrient analyser model for comparing food nutrients. *Current Nutrition & Food Science* **19**(7): 745-754.

Opricovic S. 1998. Multicriteria optimization of civil engineering systems. PhD Thesis. University of Belgrade.

Ozcalici M. 2023. Integrating queue theory and multi-criteria decision-making tools for selecting roll-over car washing machine. *Operations Research and Decisions* **33**(2): 99-119.

Ozernoy V.M. 1992. Choosing the "best" multiple criterlv decision-making method. *INFOR: Information Systems and Operational Research* **30**(2): 159-171.

Paramanik A.R., Sarkar S. & Sarkar B. 2022. OSWMI: An objective-subjective weighted method for minimizing inconsistency in multi-criteria decision making. *Computers & Industrial Engineering* **169**: 108138.

Post T. & Potì V. 2016. Portfolio analysis using stochastic dominance, relative entropy, and empirical likelihood. *Management Science* **63**(1): 153-165.

Rezaei J. 2015. Best-worst multi-criteria decision-making method. *Omega* **53**: 49-57.

Saaty R.W. 1987. The analytic hierarchy process—what it is and how it is used. *Mathematical Modelling* **9**(3-5): 161-176.

Sahin M. 2021. A comprehensive analysis of weighting and multicriteria methods in the context of sustainable energy. *International Journal of Environmental Science and Technology* **18**(6): 1591-1616.

Shannon C.E. 1948. A mathematical theory of communication. *The Bell System Technical Journal* **27**(3): 379-423.

Srinivasan V. & Shocker A.D. 1973. Linear programming techniques for multidimensional analysis of preferences. *Psychometrika* **38**(3): 337-369.

Stewart J., Clegg D.K. & Watson S. 2021. *Calculus*. 9th Ed. Toronto: Cengage Learning.

Villalba P., Sánchez-Garrido A.J. & Yepes V. 2024. A review of multi-criteria decision-making methods for building assessment, selection, and retrofit. *Journal of Civil Engineering and Management* **30**(5): 465-480.

Wang C.N., Dang T.T., Nguyen N.A.T. & Wang J.W. 2022. A combined Data Envelopment Analysis (DEA) and Grey Based Multiple Criteria Decision Making (G-MCDM) for solar PV power plants site selection: A case study in Vietnam. *Energy Reports* **8**: 1124-1142.

Wang T.C. & Lee H.D. 2009. Developing a fuzzy TOPSIS approach based on subjective weights and objective weights. *Expert Systems with Applications* **36**(5): 8980-8985.

Wu S. & Niu R. 2024. Development of carbon finance in China based on the hybrid MCDM method. *Humanities and Social Sciences Communications* **11**(1): 156.

Yazdani M., Zarate P., Kazimieras Zavadskas E. & Turskis Z. 2019. A combined compromise solution (CoCoSo) method for multi-criteria decision-making problems. *Management Decision* **57**(9): 2501-2519.

Yılmaz K., Dinçer A.E. & Ayhan E.N. 2023. Exploring flood and erosion risk indices for optimal solar PV site selection and assessing the influence of topographic resolution. *Renewable Energy* **216**: 119056.
Yilmaz B. & Harmancioglu N. 2010. Multi-criteria decision making for water resource management: A case study of the Gediz River Basin, Turkey. *Water SA* **36**(5): 563-576.
Zhou Y. & Xu J. 2011. Evaluation of enterprises liquidity value based on the relative entropy-topsis. *2011 International Conference on E-Business and E-Government (ICEE)* in Shanghai, China.

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