

PERFECT CODES IN GRAPH THEORY: A RING-THEORETIC PERSPECTIVE

(Kod-kod Sempurna dalam Teori Graf: Perspektif Teori Gelanggang)

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ABSTRACT

A graph is a mathematical structure that represents a network described between lines and points. Due to the graph's many fascinating features, its characteristics are widely studied by many researchers. One of the popular topics studied on graphs is perfect code. A set V of vertices in a graph G is called a perfect code if every vertex in G is either in V or is adjacent to exactly one vertex in V . Originally, perfect codes were used in coding theory. It is then extended to other fields including graph theory. In this paper, mathematical concepts of perfect codes of graphs are bridged using ring theory. The perfect codes are determined for the zero divisor graph of some finite rings of matrices with dimension two. First, the zero divisor graph of the finite rings of matrices is constructed where its vertices are all zero divisors of the ring and two distinct vertices are adjacent if their product is zero. Then, from the graph's vertex set, the vertices' neighborhood elements are determined to compute the graph's perfect codes.

Keywords: perfect codes; graph theory; ring theory

ABSTRAK

Graf merupakan sebuah struktur matematik yang mewakili rangkaian yang digambarkan menggunakan garisan dan titik. Oleh kerana graf mempunyai banyak sifat yang menarik, sifat-sifat tersebut dikaji secara meluas oleh ramai penyelidik. Salah satu topik popular yang dikaji ke atas graf ialah kod sempurna. Sebuah set bucu V dalam sebuah graf G digelar kod sempurna jika setiap bucu di dalam G adalah sama ada berada dalam set V atau bersebelahan secara tepatnya dengan satu bucu dalam V . Pada asalnya, kod sempurna digunakan dalam teori pengekodan. Kemudian, ianya telah diluaskan ke bidang-bidang yang lain, termasuk teori graf. Di dalam naskhah ini, konsep-konsep matematik bagi kod sempurna dalam graf dihubungkan menggunakan teori gelanggang. Kod sempurna tersebut ditentukan untuk graf pembahagi sifar bagi beberapa gelanggang matriks terbatas dengan dua dimensi. Pertama sekali, graf pembahagi sifar bagi gelanggang tersebut telah dibina di mana bucu-bucunya adalah semua pembahagi sifar bagi gelanggang tersebut dan dua bucu yang berbeza adalah bersebelahan jika hasil darab mereka adalah sifar. Kemudian, dari set bucu graf tersebut, unsur-unsur kejiranan bagi setiap bucu telah ditentukan bagi mengira kod-kod sempurna graf tersebut.

Kata kunci: kod sempurna; teori graf; teori gelanggang

References

- Akhtar R. & Lee L. 2016. Connectivity of the zero-divisor graph for finite rings. *Involve, a Journal of Mathematics* **9**(3): 415–422.
- Al-Janabi H. & Bacsó G. 2023. Chromatic uniqueness of zero-divisor graphs. *The Art of Discrete and Applied Mathematics* **6**(1): P1.08.
- Ali N., Siddiqui H.M.A., Riaz M.B., Qureshi M.I. & Akgül A. 2024. A graph-theoretic approach to ring analysis: Dominant metric dimensions in zero-divisor graphs. *Heliyon* **10**(10): e30989.
- Alsarahead M. & Ahmad M.K. 2017. Fully right pure group rings. *Journal of Quality Measurement and Analysis* **13**(1): 25–34.

- Anderson D.F. & Livingston P.S. 1999. The zero-divisor graph of commutative ring. *Journal of Algebra* **217**: 434–447.
- Banerjee S. 2023. Perfect codes and universal adjacency spectra of commuting graphs of finite groups. *Journal of Algebra and Its Applications* **22**(04): 2350097.
- Beck I. 1988. Coloring of commutative rings. *Journal of Algebra* **116**(1): 208–226.
- Bennis D., Mikram J. & Taraza F. 2016. On the extended zero divisor graph of commutative rings. *Turkish Journal of Mathematics* **40**(2): 376–388.
- Biggs N. 1973. Perfect codes in graphs. *Journal of Combinatorial Theory, Series B* **15**(3): 289–296.
- Erfanian A. & Tolue B. 2012. Conjugate graphs of finite groups. *Discrete Mathematics, Algorithms and Applications* **4**(2): 1250035.
- Fraleigh J.B. 2003. *A First Course in Abstract Algebra*. 7th Ed. Boston, MA: Addison-Wesley.
- Hamming R.W. 1950. Error detecting and error correcting codes. *Bell System Technical Journal* **29**(2): 147–160.
- Han J.C. 2008. The zero-divisor graph under group actions in a noncommutative ring. *Journal of the Korean Mathematical Society* **45**(6): 1647–1659.
- Ma X. 2020. Perfect codes in proper reduced power graphs of finite groups. *Communications in Algebra* **48**(9): 3881–3890.
- Mudaber M.H., Sarmin N.H. & Gambo I. 2024. Perfect codes in unity product graphs of some commutative rings. *Southeast Asian Bulletin of Mathematics* **48**(3): 401–409.
- Nikmehr M.J., Azadi A. & Nikandish R. 2021. The weakly zero-divisor graph of a commutative ring. *Revista de la Unión Matemática Argentina* **62**(1): 105–116.
- Omidi G.R. & Vatandoost E. 2011. On the commuting graph of rings. *Journal of Algebra and Its Applications* **10**(3): 521–527.
- Redmond S.P. 2002. The zero-divisor graph of a non-commutative ring. *International Journal of Commutative Rings* **1**(4): 203–211.
- Redmond S.P. 2003. An ideal-based zero-divisor graph of a commutative ring. *Communications in Algebra* **31**(9): 4425–4443.
- Rosen K.H. 2011. *Discrete Mathematics and Its Applications*. 7th Ed. New York: McGraw-Hill Education.
- Spiroff S. & Wickham C. 2011. A zero divisor graph determined by equivalence classes of zero divisors. *Communications in Algebra* **39**(7): 2338–2348.
- Tola K.A., Repalle V.N.S.R. & Ashebo M.A. 2024. Theory and application of interval-valued neutrosophic line graphs. *Journal of Mathematics* **2024**: 5692756.
- Van Lint J.H. 1975. A survey of perfect codes. *Rocky Mountain Journal of Mathematics* **5**(2): 199–224.
- Wu T. 2005. On directed zero-divisor graphs of finite rings. *Discrete Mathematics* **296**(1): 73–86.
- Zai N.A.F.O., Sarmin N.H., Khasraw S.M.S., Gambo I. & Zaid N. 2022. The total perfect codes of the non-zero divisor graph of ring of integers modulo 10. *Proceedings of Science and Mathematics* **6**: 90–93.
- Zaid N., Sarmin N.H. & Khasraw S.M.S. 2020. Probabilistic characterizations of some finite ring of matrices and its zero divisor graph. *Southeast Asian Bulletin of Mathematics* **44**: 859–866.
- Zaid N., Sarmin N.H. & Khasraw S.M.S. 2021. The applications of zero divisors of some finite rings of matrices in probability and graph theory. *Jurnal Teknologi* **83**(1): 127–132.

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