

e^* —TRANSITIVITY AS AN EXTENSION TO CHAOS NOTIONS (e^* —Transitiviti sebagai Suatu Perluasan kepada Konsep Kekalutan)

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ABSTRACT

The introduction of e^* —transitivity is motivated by the broader concept of e^* —open sets, representing a generalised extension of various open set concepts. In this paper, we prove that the e^* —transitivity implies the classical transitivity and many other types of transitivity such as b —transitivity, δ —transitivity, α —transitivity, and sr —transitivity. Therefore, exploring the properties of e^* —transitivity holds significance, as these properties extend to other forms of transitivity. Additionally, we introduce several e^* —chaos notions that serve as analogues, to their classical counterparts. These notions include total e^* —transitivity, e^* —mixing, e^* —locally everywhere onto (abbreviated as e^* —l.e.o.), and weak e^* —blending. Subsequently, we demonstrate the interrelations among these newly introduced e^* —chaos notions and illustrate their connections to classical chaos concepts. This comparative analysis not only enriches our understanding of e^* —transitivity but also introduce new notions of chaos based on e^* —transitivity.

Keywords: e^* —transitivity; total e^* —transitivity; e^* —mixing; e^* —locally everywhere onto (e^* —l.e.o.)

ABSTRAK

Pengenalan e^* —transitiviti adalah didorong oleh konsep yang lebih luas iaitu set e^* —terbuka, yang mewakili satu pengitlakan umum daripada pelbagai konsep set terbuka. Dalam kertas ini, kami membuktikan bahawa e^* —transitiviti mengimplikasikan transitiviti klasik dan pelbagai jenis transitiviti lain seperti b —transitiviti, δ —transitiviti, α —transitiviti, dan sr —transitiviti. Oleh itu, penerokaan sifat-sifat e^* —transitiviti adalah penting kerana sifat-sifat ini diperluaskan ke jenis transitiviti yang lain. Tambahan pula, kami memperkenalkan beberapa konsep e^* —kalut yang berperanan sebagai analogi kepada konsep klasik, termasuk e^* —transitiviti menyeluruh, e^* —pencampuran, e^* —tempat di mana-mana dan e^* —percampuran lemah. Seterusnya, kami menunjukkan hubungan antara konsep-konsep e^* —kalut yang baru diperkenalkan ini dan menjelaskan hubungannya dengan konsep kekalutan klasik. Analisis perbandingan ini bukan sahaja memperkayakan pemahaman kita tentang e^* —transitiviti tetapi juga memperkenalkan konsep kekacauan baharu berdasarkan e^* —transitiviti.

Kata kunci: e^* —transitiviti, e^* —transitiviti menyeluruh, e^* —pencampuran, e^* —tempat di mana-mana (e^* —l.e.o.)

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