

ENERGY AND LAPLACIAN ENERGY OF THE RELATIVE CO-PRIME GRAPH FOR SYMMETRIC GROUP OF ORDER SIX

(Tenaga dan Tenaga Laplacian bagi Graf Ko-Perdana secara Relatif bagi Kumpulan Simetrik Peringkat Enam)

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ABSTRACT

The energy of a graph is calculated as the sum of the modulus of eigenvalues of an adjacency matrix for a graph. The Laplacian energy refers to the eigenvalues of the graph's Laplacian matrix. In graph theory, the co-prime graph of a group is a graph where the vertices represent the elements of a group, and two distinct vertices are connected if and only if their orders are relatively prime. This study is then extended to the relative co-prime graph of a group, where the vertices are the elements of the group, and two distinct vertices are adjacent if and only if their orders are co-prime and any of them is in the subgroup. By using the definition of the relative co-prime graph of a group with respect to a subgroup, some graphs of symmetric group of order six are constructed. Some energies and Laplacian energies of the relative co-prime graph related to the symmetric group of order six are also obtained.

Keywords: adjacency matrices; energy; Laplacian energy; relative co-prime graph; symmetric group

ABSTRAK

Tenaga graf dikira sebagai jumlah modulus nilai eigen bagi matriks bersebelahan untuk graf. Tenaga Laplacian merujuk kepada nilai eigen bagi graf matriks Laplacian. Dalam teori graf, graf ko-perdana bagi suatu kumpulan ialah graf di mana bucu-bucu mewakili unsur-unsur bagi suatu kumpulan, dan dua bucu yang berbeza adalah bersambung jika dan hanya jika peringkat bagi unsur-unsur berkenaan adalah perdana secara relatif. Kajian ini kemudiannya diperluaskan selanjutnya kepada graf ko-perdana secara relatif bagi suatu kumpulan, di mana bucu-bucu adalah unsur-unsur bagi suatu kumpulan, dan dua bucu yang berbeza adalah bersebelahan jika dan hanya jika peringkat bagi unsur-unsur berkenaan adalah ko-perdana dan mana-mana antara kedua-dua bucu ini ada di dalam subkumpulan. Dengan menggunakan takrifan bagi graf ko-perdana secara relatif bagi suatu kumpulan, beberapa graf kumpulan simetrik peringkat enam dibina. Beberapa tenaga dan tenaga Laplacian bagi graf ko-perdana secara relatif yang berkaitan dengan kumpulan simetrik peringkat enam juga diperoleh.

Kata kunci: nilai eigen bagi matriks; tenaga; tenaga Laplacian; graf ko-perdana secara relatif; kumpulan simetrik

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