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EMPLOYING HYPERSINGULAR INTEGRAL EQUATIONS TO OBTAIN STRESS INTENSITY FACTOR FOR TWO SLANTED CRACKS IN THERMOELECTRIC BONDED MATERIALS

(*Menggunakan Persamaan Kamiran Hipersingular untuk Mendapatkan Faktor Keamatan Regangan bagi Dua Retakan Condong dalam Bahan Tercantum Termoelektrik*)

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ABSTRACT

This research focuses on solving hypersingular integral equations (HSIEs) numerically for thermoelectric bonded materials (TEBM) subjected to shear stress and weakened by two slanted cracks. The modified complex variable function (MCVF) method is applied, integrating continuity conditions (CC) for both the electric field effect (EFE) and the displacement electric function (DEF), which are used to formulate the governing HSIEs. Utilizing a curved length coordinate approach, the unknown functions associated with crack opening displacement (COD), as well as constants for current vector field and surface energy load, are expressed in terms of the fracture singularity basis function. These HSIEs are solved numerically using suitable quadrature techniques, with the crack traction as the right-hand term. The solutions for COD, current vector field, and surface energy load are then employed to calculate the dimensionless stress intensity factors (DSIFs), which offer insights into the stability of TEBM with two slanted cracks. Numerical simulations demonstrate that the computed DSIFs at the crack tips are consistent with previous research. Additionally, the results show that DSIFs are significantly affected by factors such as the ratio of elastic constants (ECR), crack geometry, and current vector field coefficients.

Keywords: dimensionless stress intensity factors; bonded materials; hypersingular integral equations; two slanted cracks; thermoelectric

ABSTRAK

Penyelidikan ini mengkaji penyelesaian berangka bagi persamaan kamiran hipersingular (HSIEs) untuk bahan tercantum termoelektrik (TEBM) yang dilemahkan oleh dua retakan condong di bawah tegasan ricih. Kaedah fungsi pembolehubah kompleks yang diubah suai (MCVF) digunakan untuk menangani masalah ini dengan menggabungkan syarat kesinambungan bagi daya elektrik terhasil (EFE) dan fungsi anjakan elektrik (DEF) untuk merumuskan HSIEs. Melalui aplikasi kaedah koordinat panjang lengkung, fungsi tidak diketahui yang berkaitan dengan anjakan pembukaan retakan (COD), serta pemalar bagi ketumpatan arus elektrik dan beban fluks tenaga, diterjemahkan ke dalam rangka kerja fungsi singular punca kuasa dua. Persamaan yang terhasil diselesaikan secara berangka menggunakan formula kuadratur yang sesuai, dengan tarikan retakan sebagai terma sebelah kanan. Fungsi COD, ketumpatan arus elektrik, dan beban fluks tenaga yang dikira digunakan untuk menentukan faktor keamatan regangan tanpa dimensi (DSIF), memberikan pandangan tentang kelakuan kestabilan TEBM yang dilemahkan oleh dua retakan condong. Penyelesaian berangka menunjukkan DSIF pada hujung retakan condong, dengan keserasian yang sangat baik dengan kajian terdahulu. Selain itu, pemerhatian menunjukkan bahawa DSIF dipengaruhi oleh faktor seperti nisbah pemalar keanjalan (ECR), geometri retakan, dan pekali yang berkaitan dengan ketumpatan arus elektrik.

Kata kunci: faktor keamatan regangan tanpa dimensi; bahan tercantum; persamaan kamiran hipersingular; dua retakan condong; termoelektrik

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