

SECOND HANKEL DETERMINANT FOR NEW SUBCLASS OF ANALYTIC FUNCTIONS ASSOCIATED WITH q -DERIVATIVE OPERATOR

(Penentu Hankel Kedua untuk Subkelas Baharu bagi Fungsi Analisis Bersekutu dengan
Operator Pembeza- q)

TSEU SUET YIE & AINI JANTENG*

ABSTRACT

This paper utilizes the concept of subordination and q -derivative operator to introduce a new subclass of analytic functions in the open unit disk. The primary objective of this paper is to obtain the upper bounds for the second Hankel determinant for the functions belonging to this class.

Keywords: analytic functions; q -derivative operator; Hankel determinant

ABSTRAK

Kajian ini menggunakan konsep subordinasi dan operator pembeza- q untuk memperkenalkan satu subkelas baharu bagi fungsi analisis dalam cakera unit terbuka. Objektif utama adalah untuk memperoleh had atas bagi penentu Hankel kedua bagi fungsi yang tergolong dalam kelas ini.

Kata kunci: fungsi analisis; operator pembeza- q ; penentu Hankel

References

- Aral A., Gupta V. & Agarwal R.P. 2013. *Applications of q -Calculus in Operator Theory*. New York, NY: Springer.
- Cheng Y., Srivastava R. & Liu J-L. 2022. Applications of the q -derivative operator to new families of bi-univalent functions related to the Legendre polynomials. *Axioms* **11**(11): 595.
- Duren P.L. 1983. *Univalent Functions*. New York, NY: Springer-Verlag.
- Grenander U. & Szegő G. 1958. *Toeplitz Forms and Their Applications*. Berkeley, CA: University of California Press.
- Hayman W.K. 1968. On the second Hankel determinant of mean univalent functions. *Proceedings of the London Mathematical Society* **18**(1): 77-94.
- Hu Q., Shaba T.G., Younis J., Khan B., Mashwani W.K. & Çağlar M. 2022. Applications of q -derivative operator to subclasses of bi-univalent functions involving Gegenbauer polynomials. *Applied Mathematics in Science and Engineering* **30**(1): 501-520.
- Hern A.L.P., Janteng A. & Omar R. 2022. Subclasses of analytic functions with negative coefficients involving q -derivative operator. *Science and Technology Indonesia* **7**(3): 327-332.
- Jackson F.H. 1910. On q -definite integrals. *Quarterly Journal of Pure and Applied Mathematics* **41**(15): 193-203.
- Janteng A., Halim S.A. & Darus M. 2006. Coefficient inequality for a function whose derivative has a positive real part. *Journal of Inequalities in Pure and Applied Mathematics* **7**(2): 50.
- Khan M.G., Ahmad B., Murugusundaramoorthy G., Mashwani W.K., Yalçın S., Shaba T.G. & Salleh Z. 2022. Third Hankel determinant and Zalcman functional for a class of starlike functions with respect to symmetric points related with sine function. *J. of Math. and Computer Sci.* **25**: 29-36.
- Lasode A.O. & Opoola T.O. 2022. Hankel determinant of a subclass of analytic and bi-univalent functions defined by means of subordination and q -differentiation. *Int. J. Nonlinear Analytic Appl.* **13**(2): 3105-3114.
- Olatunji S.O. & Panigrahi T. 2022. Hankel determinant for certain subclasses of analytic functions with respect to q -difference operator associated with generalized telephone number. *Acta Universitatis Apulensis* **70**: 87-100.
- Pommerenke C. 1966. On the coefficients and Hankel determinants of univalent functions. *J. London Math. Soc.* **41**(1): 111-122.
- Srivastava H.M., Rath B., Kumar K.S. & Krishna D.V. 2023a. The sharp bound of the third Hankel determinant of k th-root transformation for analytic functions. *Acta Et Commentationes Universitatis Tartuensis De Mathematica* **27**(2): 185-210.

- Srivastava H.M., Shaba T.G., Murugusundaramoorthy G., Wanas A.K. & Oros G.I. 2023b. The Fekete-Szegő functional and the Hankel determinant for a certain class of analytic functions involving the Hohlov operator. *AIMS Mathematics* **8**(1): 340-360.
- Srivastava H.M., Khan B., Khan N., Tahir M., Ahmad S. & Khan N. 2021. Upper bound of the third Hankel determinant for a subclass of q -starlike functions associated with the q -exponential function. *Bulletin des Sciences Mathématiques* **167**: 102942.
- Srivastava R. & Zayed H.M. 2019. Subclasses of analytic functions of complex order defined by q -derivative operator. *Stud. Univ. Babeş-Bolyai Math.* **64**(1): 71-80.
- Yang Y., Srivastava R. & Liu J-L. 2024. A new subclass of analytic functions associated with the q -derivative operator related to the Pascal distribution series. *Symmetry* **16**(3): 280.
- Yie T.S. & Janteng A. 2024. Fekete-Szegő functional for classes $X_q^n(\varphi)$ and $Y_q^n(\varphi)$. *Malaysian Journal of Fundamental and Applied Sciences* **20**(2): 435-443.
- Yie T.S., Janteng A & Abbas M. 2024. Some coefficient problems for subclasses of holomorphic functions in complex order associating Sălăgean q -differential operator. *Science and Technology Indonesia* **9**(4): 981-988.

Faculty of Science and Natural Resources

Universiti Malaysia Sabah

88450 Kota Kinabalu

Sabah, MALAYSIA

*E-mail: snowieeeee0209@gmail.com, aini_jg@ums.edu.my**

Received: 21 February 2025

Accepted: 28 April 2025

*Corresponding author