

INVERSE ESTIMATION OF THERMAL CONDUCTIVITY PARAMETERS IN NONLINEAR STEADY HEAT CONDUCTION PROBLEMS

(Anggaran Songsang Parameter Konduktiviti Terma dalam Masalah Pengaliran Haba
Tetap tak Linear)

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ABSTRACT

The precise determination of the thermal conductivity is crucial for material characterization in engineering applications. However, the accurate identification of temperature-dependent thermal conductivity poses a significant challenge. This article proposed an adjoint-based inverse method to identify the thermal conductivity in the context of nonlinear steady heat conduction scenarios. The simulated temperature associated with the nonlinear steady heat conduction problem was quantified by employing the finite element method. The inverse retrieval of the thermal conductivity parameters was performed utilizing the adjoint method. Numerical examples were presented and analyzed to demonstrate the effectiveness of the proposed approach to identify the thermal conductivity parameters. For both numerical examples, three parameters of thermal conductivity were recovered utilizing temperature distribution with around 30 iterations and remarkably low objective function values. These results demonstrated the validity of the proposed method even in cases when the initial guess was far away from the target parameter value. The findings demonstrated the potential of the adjoint method in effectively determining the thermophysical parameters by solving the inverse heat conduction problems.

Keywords: FEM; temperature-dependent thermal conductivity; adjoint method; inverse problem

ABSTRAK

Penentuan yang tepat bagi konduktiviti terma adalah sangat penting untuk pencirian bahan dalam aplikasi kejuruteraan. Namun, mengenal pasti konduktiviti terma yang bergantung kepada suhu merupakan satu cabaran besar. Artikel ini mencadangkan kaedah songsang berasaskan adjoint untuk mengenal pasti konduktiviti terma dalam senario pengaliran haba mantap nonlinier. Suhu simulatif yang berkaitan dengan masalah tersebut diukur menggunakan kaedah elemen terhingga. Pengambilan terbalik parameter konduktiviti terma dilakukan dengan ketepatan dan kepresisian yang tinggi melalui kaedah adjoint, yang terkenal dengan ketepatan dan penumpuan yang cepat. Contoh numerik dibentangkan dan dianalisis untuk menunjukkan keberkesanan pendekatan yang dicadangkan dalam pengenalan parameter konduktiviti terma. Hasil kajian ini membuktikan kesahihan kaedah yang dicadangkan walaupun tekaan awal jauh dari nilai parameter sasaran. Penemuan ini menunjukkan potensi kaedah adjoint dalam menentukan parameter termofizik dengan berkesan melalui penyelesaian masalah pengaliran haba songsang.

Kata kunci: konduktiviti terma bergantung kepada suhu, kaedah adjoint, masalah songsang

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