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MATHEMATICAL APPROACHES FOR UNDERSTANDING HARMFUL ALGAL BLOOMS IN WEST COAST OF SABAH

(*Pendekatan Matematik untuk Memahami dan Mengurus Algal Mekar Berbahaya di Pantai Barat Sabah*)

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ABSTRACT

Massive algal growth at a certain region and period has alarmed the nation for the past years. The algal bloom is said to be harmful due to its damaging effect on marine creatures, economic sectors such as aquaculture and tourism, and human health. Mathematical modelling of Harmful Algal Blooms (HABs) is developed to describe the HAB's dynamic on the West Coast of Sabah for its annual bloom. There are four variables used in the developed model which are nutrient concentration, Toxic-Producing Phytoplankton (TPP) population, Non-Toxic Phytoplankton (NTP) population, and Zooplankton (Z) population. There exist interaction between all the variables where the nutrient is the food source for the TPP and NTP population. TPP population released toxin while the NTP population does not secrete out toxin chemicals, but it gives competition to the TPP population in terms of food. Zooplankton population act as a predator toward TPP and NTP population. The model is modified from Lotka-Volterra prey-predator model and the stability of the model is also examined. TPP species released toxin to the environment and the effects of toxin on the stability of steady states is investigated. The results show that the system is unstable when TPP species released toxin to the environment and lead to the occurrence of HAB. Zooplankton population are harmed if they consumed TPP species if the toxin content is at peak. Besides, the results also shows that interspecies competition exist between NTP species and TPP species in order to get food. Therefore, this research gives information on how HAB could occur with the biological factors involved.

Keywords: harmful algal bloom (HAB), modelling, stability, toxin

ABSTRAK

Pertumbuhan alga yang besar di rantau dalam tempoh tertentu membimbangkan negara sejak beberapa tahun lalu. Alga dikatakan berbahaya kerana kesannya kepada makhluk laut, sektor ekonomi seperti akuakultur dan pelancongan, dan juga kesihatan manusia. Pemodelan matematik Pertumbuhan Alga Berbahaya (HABs) dijalankan untuk menggambarkan dinamik HAB di Pantai Barat Sabah. Terdapat empat pembolehubah yang digunakan dalam model yang dibangunkan iaitu kepekatan nutrien, populasi Fitoplankton Penghasil Toksik (TPP), populasi Fitoplankton Tidak Toksik- (NTP), dan populasi Zooplankton (Z). Terdapat interaksi antara semua pembolehubah di mana nutrien adalah sumber makanan untuk populasi TPP dan NTP. TPP-mengeluarkan toksin manakala NTP tidak merembeskan bahan kimia toksin, tetapi ia memberi saingan kepada penduduk TPP dari segi makanan. Populasi zooplankton bertindak sebagai pemangsa terhadap populasi TPP dan NTP. Model ini diubah suai daripada model pemangsa mangsa Lotka-Volterra, dan seterusnya kestabilan model juga dianalisa. Spesies TPP membebaskan toksin kepada alam sekitar dan kesan toksin terhadap kestabilan dianalisa. Hasil menunjukkan bahawa sistem menjadi tidak stabil apabila spesies TPP melepaskan toksin ke alam sekitar dan membawa kepada kejadian HAB. Populasi zooplankton tergugat jika mereka memakan spesies TPP yang mempunyai kandungan toksin berada di puncak. Selain itu, hasil menunjukkan bahawa persaingan antara spesies wujud antara spesies NTP dan spesies TPP untuk mendapatkan makanan. Oleh itu, kajian ini memberi maklumat bagaimana HAB boleh berlaku dengan faktor biologi yang terlibat.

Kata kunci: pertumbuhan alga memudaratkan (HAB), permodelan, kestabilan, toksin

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