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MATHEMATICAL MODEL AND ANALYSIS OF THE EFFECT OF AWARENESS PROGRAM ON DIABETES DYNAMIC

(*Model Matematik dan Analisis Kesan Program Kesedaran Terhadap Diabetes*)

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ABSTRACT

The prevalence of diabetes worldwide is increasingly concerning, affecting individuals of all ages due to genetic predisposition and environmental factors. Diabetes can be prevented or delayed through lifestyle interventions such as a balanced diet, regular physical activity, and weight management. However, a lack of awareness often leads to late diagnosis and progression to diabetes, increasing the burden on the healthcare system. Public awareness plays a crucial role in promoting early interventions and encouraging individuals to adopt healthier behaviours. Considering the benefits of awareness, this study explores its impact on the development of diabetes through a nonlinear mathematical model. This model consists of six subpopulations representing different health statuses and the cumulative of media-driven awareness campaigns. We developed a flow diagram and made several assumptions to describe population dynamics, which led to a system of nonlinear ordinary differential equations. Local stability is analysed using the Jacobian matrix and the Lienard-Chipart criterion, while global stability is examined through the Lyapunov function. Numerical simulations of the model are carried out to verify and illustrate the analytical results. The results indicate that controlling the growth of media-driven awareness programs, increasing the transition of unaware healthy individuals gaining awareness or increasing the transition of unaware prediabetes individuals gaining awareness can reduce the burden of diabetes. Furthermore, exposure to media-driven awareness campaigns targeting unaware individuals may help in reducing the progression of prediabetes and diabetes when they begin adopting a healthy lifestyle.

Keywords: mathematical model; diabetes; stability analysis; numerical simulation

ABSTRAK

Prevalens diabetes di seluruh dunia semakin membimbangkan, menjejaskan individu dari semua peringkat umur disebabkan oleh predisposisi genetik dan faktor persekitaran yang menyumbang kepada penyakit ini. Diabetes boleh dicegah atau ditangguhkan melalui intervensi gaya hidup, seperti diet seimbang, aktiviti fizikal yang kerap, dan pengurusan berat badan. Namun, kekurangan kesedaran sering menyebabkan lewat didiagnosis dan perkembangan diabetes, meningkatkan beban ke atas sistem penjagaan kesihatan. Kesedaran awam memainkan peranan penting dalam mempromosikan intervensi awal dan menggalakkan individu untuk mengamalkan tingkah laku yang lebih sihat. Mengambil kira kelebihan kesedaran, kajian ini meneroka kesannya terhadap perkembangan diabetes melalui model matematik bukan linear. Model ini terdiri daripada enam subpopulasi yang mewakili status kesihatan yang berbeza dan jumlah terkumpul program kesedaran yang didorong oleh media. Kami membangunkan sebuah diagram aliran dan membuat beberapa andaian yang menerangkan dinamik populasi, yang membawa kepada sistem persamaan pembezaan biasa bukan linear. Kestabilan tempatan dianalisis menggunakan matriks Jacobian dan kriteria Lienard-Chipart, manakala kestabilan global diperiksa melalui fungsi Lyapunov. Simulasi berangka dijalankan untuk mengesahkan dan menggambarkan keputusan analitik. Keputusan menunjukkan bahawa adalah penting untuk mengawal pertumbuhan program kesedaran yang didorong oleh media, peralihan daripada individu sihat yang tidak sedar kepada sedar atau peralihan daripada pradiabetes yang tidak sedar kepada sedar boleh mengurangkan beban diabetes. Tambahan pula, pendedahan kepada program kesedaran yang didorong media yang menyasarkan individu yang tidak sedar mungkin dapat membantu mengurangkan perkembangan pradiabetes dan diabetes apabila mereka mula mengamalkan gaya hidup sihat.

Kata kunci: model matematik; diabetes; analisis kestabilan; simulasi berangka

1. Introduction

Diabetes mellitus is categorised as one of non-communicable diseases (Institute for Public Health 2023). The likelihood of developing diabetes increases with positive family history and environmental factors (American Diabetes Association Professional Practice Committee 2024). Diabetes refers to a condition in which an individual has high blood sugar levels (hyperglycaemia), which result from defective insulin secretion and/or action (Sapra & Bhandari 2023). Persistently high blood sugar levels can lead to long-term damage, dysfunction and failure of internal organs. Individuals who struggle to regulate their blood sugar levels may develop macrovascular and microvascular complications (Sapra & Bhandari 2023). Prediabetes, a precursor to type 2 diabetes, is an intermediate state between normoglycemia and diabetes (Chen *et al*. 2024). At this stage, adopting a healthy lifestyle can help reverse blood sugar levels and return to normal by adopting a healthy lifestyle.

According to the National Health and Morbidity Survey 2023, 15.6% of Malaysians had diabetes in 2023, yet 84% of young adults aged 18-29 were unaware of their condition (Institute for Public Health 2023). Globally, the International Diabetes Federation reported that 537 million adults had diabetes in 2021, and this number is projected to increase to 783 million by 2045 (International Diabetes Federation 2021). Managing diabetes and its of diabetes and its complications comes with a high financial burden. In Malaysia alone, the estimated annual cost of diabetes is around treatment 600 million USD (Ganasegeran *et al.* 2020). One major challenge is the high number of undiagnosed diabetes cases, largely due to a lack of awareness. This unawareness often leads to serious complications, increasing the risk of premature death (Khan *et al.* 2019).

Raising diabetes awareness, early diagnosis, and effective management can help prevent or delay the onset of diabetes and its complications. Increasing awareness enables individuals to be more vigilant and take necessary precautions against the disease. Preventing prediabetes and diabetes is achievable through a healthy lifestyle, including proper nutrition and regular physical activity. To address this, several organisations have launched awareness campaigns to educate the public about diabetes risks. World Diabetes Day is one such campaign conducted worldwide to promote diabetes awareness (International Diabetes Federation 2019). Currently, mass media plays a crucial role in delivering health-related information to the public, making it a powerful tool for spreading awareness.

Mat Daud and Toh (2021) introduced a mathematical model of non-communicable disease which is linear and analysed it using qualitative analysis. Also, in the previous works, Auni Mat Daud pointed out various issues associated with mathematical models of population dynamics that related to some cases of non-communicable disease (Mat Daud 2020; Mat Daud & Toh 2021), emphasizing the definition and advantages of qualitative approaches to the models (Mat Daud 2021a), and highlight the significant of Lienard-Chipart criterion (Mat Daud 2021b). Some of the population models involving non-communicable diseases have been investigated, including those for diabetes (Mat Daud *et al.* 2020; Nasir & Mat Daud 2022), hypertension in pregnancy (Mat Daud *et al.* 2019), thyroid disorders in pregnancy and postpartum (Mat Daud 2019) and anaemia in pregnancy and postpartum (Mat Daud *et al.* 2021).

Numerous researchers have investigated the mathematical model of population dynamics of diabetic mellitus in linear and nonlinear cases using the ordinary differential equation

(Boutayeb *et al.* 2004; Boutayeb *et al*. 2006; Boutayeb & Chetouani 2007; Adamu *et al.* 2012; Ardiansah & Kharis 2012; Akinsola & Oluyo 2014; Ulfah *et al*. 2014; Abraham & San 2015; Effendi *et al*. 2015; de Oliveira *et al*. 2017; Asmaidi & Suryanto, 2018; Widyaningsih *et al.* 2018; Dhara *et al*. 2018; Side *et al*. 2020; Andima *et al*. 2022). Additionally, several studies have developed optimal control model for diabetes, incorporating control strategies for diabetes management (Boutayeb *et al*. 2015a; Derouich *et al*. 2014; Henindya *et al*. 2015; Yusuf 2015; Permatasari *et al*. 2018; Kouidere *et al*. 2020).

Boutayeb *et al*. (2015b) proposed a mathematical model of diabetes progression with optimal control by introducing a control variable at the healthy stage to prevent the development of prediabetes. The study concludes that the presence of optimal control strategies can reduce the number of prediabetic and diabetic cases compared to scenarios without control measures. The strategies outlined in the paper include raising awareness about healthy diets, physical activity, smoking reduction, and metabolic risks. These strategies can be implemented through awareness campaigns and educational programs to encourage behavioural changes toward healthier lifestyles. The effectiveness of such awareness campaigns has been demonstrated by Agarwal and Pathak (2014) and Mollah and Biswas (2021). Their studies show that diabetes cases can be reduced through media-driven awareness campaigns targeting prediabetic (Agarwal & Pathak 2014) and susceptible (Mollah & Biswas 2021) populations. Then, Mollah and Biswas (2023) proposed a mathematical model with optimal control considering the effect of awareness by media and treatment. Furthermore, Misra and Kumari (2023) formulated a mathematical model by considering TV and social media advertisements and word-of-mouth communication on diabetes. Rai *et al*. (2024) examine how social media and word-of-mouth influence the spread of communicable and non-communicable diseases. Collectively, these studies highlight that awareness initiatives and media-driven campaigns play a crucial role in diabetes prevention. With the support of media, these campaigns can reach a wider audience, increase knowledge, and promote healthier lifestyles.

Motivated by these findings, we proposed a nonlinear mathematical model that expands on previous studies. Our model differs slightly from those proposed by Agarwal & Pathak (2014), Mollah & Biswas (2021), and Misra & Kumari (2023) in several aspects. According to Agarwal & Pathak (2014), diabetes-related deaths are assumed to occur only among diabetics with complications. However, in this study, we extend this by considering that diabetes-related deaths can occur in both diabetics with and without complications. Additionally, Agarwal & Pathak (2014) allow diabetics with complications to recover, though they continue to have diabetes, whereas those who develop disabilities are removed from the population. In contrast, our study assumes that diabetics with complications do not recover due to irreversible damage to blood vessels, and diabetics with disabilities remain in the diabetes with complication class. This ensures the long-term effects of complications are properly accounted for. Mollah & Biswas (2021) and Misra & Kumari (2023) both consider aware and unaware susceptible individuals, as well as individuals with diabetes. However, our model offers a more precise understanding of the progression from a healthy state to a diabetes state by incorporating the influence of an awareness campaign. Furthermore, Misra & Kumari (2023) focus on broadcasting advertisements through television and social media, whereas our study assumes that any form of media—including print, electronic, and social media—serves as a medium for spreading awareness campaigns. Additionally, the previous studies primarily target prediabetic individuals (Agarwal & Pathak 2014), diabetes individuals (Mollah *et al.* 2024) and susceptible individuals (Misra & Kumari 2023), our model assumes that media-driven campaigns provide information and knowledge that encourage both healthy and prediabetic individuals to adopt a healthier lifestyle.

The present study aims to develop a compartmental model of the impact of awareness programs by media on the population dynamics of diabetics and investigate the local and global behaviour using stability analysis.

2. Model Formulation

In this section, we develop a mathematical model for diabetes, incorporating several assumptions, defining variables and parameters and presenting a flow diagram alongside a system of differential equations. We construct the model based on the following assumptions,

- Diabetes causes organ dysfunction and coma (Antar *et al.* 2023), leading to diabetes-related death. Diabetes-related death occurs in both diabetes without complications and diabetes with complications compartments,
- At the first diagnosis, individuals with prediabetes or diabetes are assumed to have no complications,
- Unaware individuals can become aware through awareness campaigns they may also transition back to an unaware state over time,
- The transition between compartments is governed by a differential equation,
- All the processes in the model are described as an instantaneous rate per unit time,
- Changes in variables are considered with respect to the time,
- The total population are divided into homogenous compartments based on health status at a given time,
- All state variables are continuous,
- All parameters are non-negative,
- Interaction between compartments follows mass action law.

The description of the variables and parameters of the model of diabetes are presented in Table 1 and Table 2.

Table 1: Description of state variables of the model

State variables	Description	Unit
H_U	Number of unaware healthy individuals	Individual
H_A	Number of aware healthy individuals	Individual
P_U	Number of unaware prediabetes individuals	Individual
P_A	Number of aware prediabetes individuals	Individual
D	Number of diabetes individuals without complications	Individual
C	Number of diabetes individuals with complications	Individual
M	Cumulative awareness campaigns driven by the media	Campaign

The population dynamics of diabetes consist of six subpopulations representing different health conditions and awareness levels related to the disease. Furthermore, this study considers the explicit inclusion of awareness campaigns in the modelling process (Agarwal & Pathak 2014; Misra & Kumari 2023). The progression of type 2 diabetes, begins with healthy individuals who may develop prediabetes and, eventually, diabetes (Gong *et al.* 2023) if appropriate preventives are not taken. Healthy individuals (H_U and H_A) have a normal blood glucose level and show no signs of diabetes. Unaware healthy individuals (H_U) lack awareness about diabetes and are more likely to engage in unhealthy behaviours such as a sedentary lifestyle and poor dietary habits. In contrast, aware healthy individuals H_A are informed about diabetes risks and take proactive measures to maintain a healthy lifestyle, reducing their likelihood of developing prediabetes or diabetes. Similarly, prediabetes (P_U and P_A) defined as having impaired glucose tolerance, impaired fasting glucose or both (Buysschaert & Bergman

2011). Unaware prediabetic individuals P_U lack awareness of diabetes and more to engage in unhealthy behaviours such as sedentary lifestyles, increasing their risk of progression to diabetes due to the absence of preventive action. In contrast, aware prediabetes individuals have awareness about diabetes and their condition, which encourages behavioural changes and adopting healthy lifestyles to prevent or delay diabetes onset. Diabetes without complications, D refers to individuals who meet the diagnostic criteria for diabetes (Jones *et al.* 2006) and are not associated with end-organ damage (Johnson *et al.* 2023). This class includes both diagnosed and undiagnosed. Diabetes with complications, C includes individuals who show detectable signs of diabetes-related complications affecting organs such as eyes, feet, kidneys or other organs. This class also includes both diagnosed and undiagnosed. Complicated diabetes is defined as 'end organ damage' (Johnson *et al* 2023). The cumulative number of awareness campaigns driven by media, M. These campaigns are disseminated through print media, social media and electronic media.

Table 2: Description of parameters of the model

Parameters	Description	Unit
μ	Natural death rate	year^{-1}
Γ	Incidence of healthy individuals	year^{-1}
κ	Rate of unaware healthy individuals developing prediabetes	year^{-1}
ρ	Rate of unaware prediabetes individuals developing diabetes	year^{-1}
ρ_0	Rate of unaware prediabetes individuals developing complications	year^{-1}
γ	Rate of aware prediabetes reverting to a healthy state	year^{-1}
τ_1	Rate of diabetes individuals developing complications	year^{-1}
σ	Rate of aware healthy individuals losing awareness	year^{-1}
σ_0	Rate of unaware prediabetes individuals losing awareness	year^{-1}
δ	Death due to diabetes	year^{-1}
δ_1	Death due to complications of diabetes	year^{-1}
ϕ	Rate of unaware healthy individuals becoming aware	$\text{campaigns}^{-1}\ \text{year}^{-1}$
ϕ_0	Rate of unaware prediabetes becoming aware	$\text{campaigns}^{-1}\ \text{year}^{-1}$
ω_1	Growth rate of awareness campaigns	$\text{campaigns individual}^{-1}\ \text{year}^{-1}$
ω_2	Depletion rate of awareness campaigns	year^{-1}

New individuals are recruited into unaware healthy subpopulations at Γ. Diabetes is a non-infectious disease and does not spread between individuals (Mollah & Biswah 2021). Therefore, we assume some unaware healthy individuals develop prediabetes at a rate κH_U, transitioning into P_U class. Unaware healthy individuals become aware when exposed to media-driven awareness campaigns at a rate of $\phi H_U M$ moving into the H_A class. This indicates that increasing the dissemination of awareness campaigns through media directly raises the number of unaware healthy individuals being exposed to the information and adopting healthier lifestyles. Similarly, unaware prediabetes individuals may become aware at a rate of $\phi_0 P_U M$, transitioning into the P_A class. This nonlinear term illustrates that the interaction between the unaware individual and the awareness program is directly proportional. Unaware prediabetes individuals may either develop diabetes without complications ρP_U or diabetes with complications $\rho_0 P_U$ entering D or C class, respectively. Diabetes without complications can develop complications at a rate of $\tau_1 D$ and eventually move to C class. Individuals may be left D and C subpopulations due to diabetes-related deaths at rates δD and $\delta_1 C$.

Awareness can fade over time due to some psychological or social factors (Misra & Kumari 2023). As a result, aware healthy individuals and aware prediabetes individuals may lose and revert to an unaware state at rates σH_A and $\sigma_0 P_A$. Importantly, some aware prediabetes individuals may reverse their condition at the rate of γP_A and become healthy again by joining the H_A class. Some individuals will leave all subpopulations due to natural death. Diabetes can be found in almost every population in the world and epidemiology evidence suggests that, without effective prevention and control programmes, its prevalence will continue to rise (Alberti *et al.* 2007). This highlights the need for awareness campaigns. Therefore, with the study by Agarwal and Pathak (2014) and Do I Have Prediabetes? campaign (Centers for Disease Control and Prevention 2024) was introduced in response to the increase in prediabetes prevalence. Hence, we assume recruitment into the cumulative awareness programs by media depends on the number of affected individuals who are unaware prediabetes, and diabetes without and with complications ($\omega_1(P_U + D + C)$). As prediabetes and diabetes prevalence rise, more healthy individuals are at risk, highlighting the urgency of awareness programs to address these health issues. However, as the number of aware people increases, the cumulative awareness programs decrease. The depletion of awareness programs about the disease due to some psychological or social factors, $\omega_2 M$. A flow diagram illustrating the model structure is presented in Figure 1.

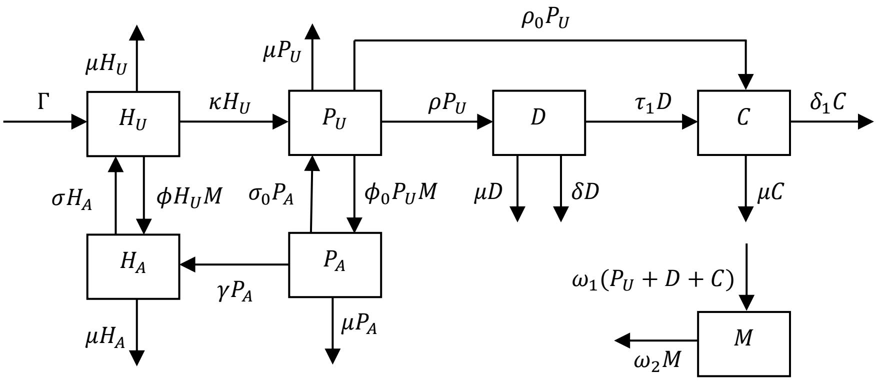

Figure 1: Flow diagram of diabetes

The system of ordinary differential equations (1)-(7) is formulated based on Figure 1, and the initial conditions of the model are ${H_U}_0 \geq 0,\ {P_U}_0 \geq 0,\ C_0 \geq 0,\ D_0 \geq 0,\ {H_A}_0 \geq 0,\ {P_A}_0 \geq 0$ and $M_0 \geq 0$

$$\frac{dH_U}{dt} = \Gamma + \sigma H_A - \kappa H_U - \phi H_U M - \mu H_U \tag{1}$$

$$\frac{dP_U}{dt} = \kappa H_U + \sigma_0 P_A - \rho P_U - \rho_0 P_U - \phi_0 P_U M - \mu P_U \tag{2}$$

$$\frac{dD}{dt} = \rho P_U - \tau_1 D - \delta D - \mu D \tag{3}$$

$$\frac{dC}{dt} = \tau_1 D + \rho_0 P_U - \mu C - \delta_1 C \tag{4}$$

$$\frac{dH_A}{dt} = \phi H_U M + \gamma P_A - \sigma H_A - \mu H_A \tag{5}$$

$$\frac{dP_A}{dt} = \phi_0 P_U M - \sigma_0 P_A - \gamma P_A - \mu P_A \tag{6}$$

$$\frac{dM}{dt} = \omega_1 (P_U + D + C) - \omega_2 M \tag{7}$$

3. Basic Properties of the Model

This section studies the basic properties of the system (1)-(7), including the existence and uniqueness of solution, nonnegativity of solution, and boundedness of solution. These properties are important to ensure the mathematical model of diabetes closely reflects the real-world scenario and allows for further analysis. For the diabetes model to be useful, it must have at least a solution and ensure that this solution is unique. A unique solution reflects the deterministic nature of the model, which produces only one outcome (Fred & Nohel 1989). This aligns with the idea that the model always yields the same outcome for a given set of initial conditions. Since the model involves population dynamics, it needs to satisfy the biological meaning which means the population can never be negative. Additionally, the bounded solution ensures that neither the population nor media-driven awareness campaigns grow indefinitely, reflecting real-world constraints such as limited resources and space (Wirkus & Swift 2014).

3.1. *Existence and uniqueness solution*

Theorem 3.1. *The system of equations (1)-(7) that satisfies a given initial condition exists and has a unique solution.*

Proof. All the expressions on the right-hand side of equations (1)-(7) consist of continuous functions and their partial derivatives with respect to state variables exist and are continuous. Thus, the Existence and Uniqueness Theorem guarantees the existence of a unique solution for the system (1)-(7) for any given initial condition. □

3.2. *Non-negativity of solution*

Theorem 3.2. *The solution $H_U(t), P_U(t), D(t), C(t), H_A(t), P_A(t), M(t)$ of the model system (1)-(7) with the non-negative initial condition will remain non-negative for all time $t > 0$.*

Proof. Let $H_U(t), P_U(t), D(t), C(t), H_A(t), P_A(t), M(t)$ be the solution of the system with non-negative initial condition. The continuity of the solution dictates that the functions $H_U(t), P_U(t), D(t), C(t), H_A(t), P_A(t), M(t)$ cannot go negative without crossing the axes $H_U = 0, P_U = 0, D = 0, C = 0, H_A = 0, P_A = 0, M = 0$. Let $t^* = min\{t_1, t_2, \dots, t_7\}$ where $H_U(t_1) = 0, P_U(t_2) = 0, D(t_3) = 0, C(t_4) = 0, H_A(t_5) = 0, P_A(t_6) = 0, M(t_7) = 0$ and $t_1, t_2, \dots, t_7 \geq 0$.

When $t^* = t_1$, we have $H_U(t_1) = 0$ and the functions $P_U(t_1)$, $D(t_1), C(t_1), H_A(t_1)$, $P_A(t_1), M(t_1) > 0$ at $t = t_1$, gives $H_U{}'(t^* = t_1) = \Gamma + \sigma H_A \geq 0$. This guarantees that whenever the function $H_U(t)$ intersects the axis $H_U = 0$, its derivative is non-negative, ensuring the function $H_U(t)$ does not decrease and will not enter the negative region.

Using a similar argument, we have $P_U{}'(t^* = t_2) = \kappa H_U + \sigma_0 P_A \geq 0$, $D(t^* = t_3) = \rho P_U \geq 0$, $C(t^* = t_4) = \tau_1 D + \rho_0 P_U \geq 0$, $H_A(t^* = t_5) = \phi H_U M + \gamma P_A \geq 0, P_A(t^* = t_6) = \phi_0 P_U M \geq 0$, and $M(t^* = t_7) = \omega_1(P_U + D + C) \geq 0$. This shows that if any state variable reaches zero first, its derivative always be non-negative at that time and thus state variable remains non-negative. If the case for all the state variables is zero simultaneously at time t^*, the derivatives will also be non-negative except for H_U, ensuring that the state variables P_U, D, C, H_A, P_A, M remain non-negative.

If follows that when $H_U(t) = 0, P_U(t) = 0, D(t) = 0, C(t) = 0, H_A(t) = 0, P_A(t) = 0$, $M(t) = 0$, we have derivatives $H_U{}' \geq 0, P_U{}' \geq 0, D' \geq 0, C' \geq 0, H_A{}' \geq 0, P_A{}' \geq 0, M' \geq 0$. Thus, the solution $H_U(t), P_U(t), D(t), C(t), H_A(t), P_A(t), M(t)$ will not cross the axes $H_U = 0, P_U = 0, D = 0, C = 0, H_A = 0, P_A(t) = 0, M = 0$. This guarantees that all solutions of the model system (1)-(7) are non-negative for any nonnegative initial condition and it satisfies the biological principle. This concludes the solution's nonnegativity proof. □

3.3. *Boundedness of solution*

Lemma 3.3. *The set* $\Omega_L = \left\{(H_U, P_U, D, C, H_A, P_A, M)\epsilon\mathbb{R}^7 : 0 \leq N \leq \frac{\Gamma}{\mu}, 0 \leq M \leq \frac{\omega_1\Gamma}{\omega_2\mu}\right\}$ *is positively invariant and attracts the system of nonlinear ordinary differential equations (1)-(7).*

Proof. Since the total population $N(t) = H_U(t) + P_U(t) + D(t) + C(t) + H_A(t) + P_A(t)$, we know that

$$\frac{dN}{dt} = \Gamma - \mu N - \delta D - \delta_1 C \tag{8}$$

Since δD and $\delta_1 C$ is positive (by Theorem 2.1.2 as $D \geq 0$ and $C \geq 0$),

$$\frac{dN}{dt} + \mu N \leq \Gamma.$$

Using the method of integrating factor, we obtain

$$N \leq \frac{\Gamma}{\mu}(1 - e^{-\mu t}) + N(0)e^{-\mu t}$$

As $t \to \infty, 0 < N \leq \frac{\Gamma}{\mu}$. So, if $N(0) \leq \frac{\Gamma}{\mu}$, then $\lim_{t\to\infty} N(t) \leq \frac{\Gamma}{\mu}$. On the other hand, if $N(0) > \frac{\Gamma}{\mu}$, then $N(t)$ will decrease to $\frac{\Gamma}{\mu}$.

Next, we have to prove the boundedness of M. From Eq. (7) and by using the upper bound of the population N, $\frac{\Gamma}{\mu}$ we get

$$\frac{dM}{dt} + \omega_2 M \leq \omega_1\left(\frac{\Gamma}{\mu}\right)$$

Again, using the method of integrating factor, we obtain

$$M \leq \frac{\omega_1\Gamma}{\mu\omega_2}(1 - e^{-\omega_2 t}) + M(0)e^{-\omega_2 t}$$

As $t \to \infty, 0 \leq M \leq \frac{\omega_1\Gamma}{\mu\omega_2}$. So, if $M(0) \leq \frac{\omega_1\Gamma}{\mu\omega_2}$, then $\lim_{t\to\infty} M(t) \leq \frac{\omega_1\Gamma}{\mu\omega_2}$. In addition, if $M(0) > \frac{\omega_1\Gamma}{\mu\omega_2}$, then $M(t)$ will decrease to $\frac{\omega_1\Gamma}{\mu\omega_2}$.

Thus, N and M are bounded and all sets of solutions for the model (1)-(7) approach, enter or remain in $\Omega_L = \left\{(H_U, P_U, D, C, H_A, P_A, M)\epsilon\mathbb{R}^7 : 0 \le N \le \frac{\Gamma}{\mu}, 0 \le M \le \frac{\omega_1\Gamma}{\omega_2\mu}\right\}$. □

4. Equilibrium and stability analysis

4.1. *Equilibrium point*

In this section, we examine the equilibrium point. The equilibrium point of the model (1)-(7) are obtained by setting $\frac{dH_U}{dt} = 0, \frac{dP_U}{dt} = 0, \frac{dD}{dt} = 0, \frac{dC}{dt} = 0, \frac{dH_A}{dt} = 0, \frac{dP_A}{dt} = 0$ and $\frac{dM}{dt} = 0$ and solve the equations simultaneously.

$$\Gamma + \sigma H_A{}^* - \phi H_U{}^* M^* - \psi_1 H_U{}^* = 0 \tag{9}$$

$$\kappa H_U{}^* + \sigma_0 P_A{}^* - \phi_0 P_U{}^* M^* - \psi_2 P_U{}^* = 0 \tag{10}$$

$$\rho P_U{}^* - \psi_3 D^* = 0 \tag{11}$$

$$\tau_1 D^* + \rho_0 P_U{}^* - \psi_4 C^* = 0 \tag{12}$$

$$\phi H_U{}^* M^* + \gamma P_A{}^* - \psi_5 H_A{}^* = 0 \tag{13}$$

$$\phi_0 P_U{}^* M^* - \psi_6 P_A{}^* = 0 \tag{14}$$

$$\omega_1 (P_U{}^* + D^* + C^*) - \omega_2 M^* = 0 \tag{15}$$

where $\psi_1 = \kappa + \mu$, $\psi_2 = \rho + \rho_0 + \mu$, $\psi_3 = \tau_1 + \delta + \mu$, $\psi_4 = \delta_1 + \mu$, $\psi_5 = \sigma + \mu$, and $\psi_6 = \sigma_0 + \gamma + \mu$.

From Eq. (12), we obtain

$$D^* = \frac{\rho}{\psi_3} P_U{}^* \tag{16}$$

To obtain the following Eq. (17), we substitute Eq. (16) into Eq. (12)

$$C^* = \frac{\tau_1 \rho P_U{}^* + \psi_3 \rho_0 P_U{}^*}{\psi_3 \psi_4} \tag{17}$$

Using the Eq. (16) and Eq. (17) in Eq. (15), we obtain

$$M^* = \psi_7 \omega_1 P_U{}^* \tag{18}$$

where $\psi_7 = \frac{1}{\omega_2} + \frac{\rho}{\psi_3 \omega_2} + \frac{\tau_1 \rho + \psi_3 \rho_0}{\psi_3 \psi_4 \omega_2}$.

Substitute equation Eq. (18) into Eq. (14), we get the following Eq. (19)

$$P_A{}^* = \frac{\psi_7\omega_1\phi_0 P_U{}^{*2}}{\psi_6} \tag{19}$$

We obtain value $H_A{}^*$ by Eq. (18) and Eq. (19) into Eq. (13)

$$H_A{}^* = \frac{\psi_7\omega_1}{\psi_5}\left(\phi H_U{}^* P_U{}^* + \frac{\gamma\phi_0 P_U{}^{*2}}{\psi_6}\right) \tag{20}$$

Substituting Eq. (18) and Eq. (20) into Eq. (9), we obtain

$$H_U{}^* = \frac{\psi_5\psi_6\Gamma + \psi_7\sigma\omega_1\gamma\phi_0 P_U{}^{*2}}{\psi_6\psi_7\mu\phi\omega_1 P_U{}^* + \psi_1\psi_5\psi_6} \tag{21}$$

Using the value of and $H_U{}^*$, $P_A{}^*$, and M^* Eq. (18), Eq. (19) and Eq. (21) into Eq. (10). We derive

$$A_3 P_U{}^{*3} + A_2 P_U{}^{*2} + A_1 P_U{}^* + A_0 = 0 \tag{22}$$

where,

$$A_3 = \omega_1{}^2\mu\psi_7{}^2(\gamma\phi\phi_0 + \mu\phi\phi_0)$$

$$A_2 = \omega_1\mu\psi_7(\kappa\sigma\phi_0 + \phi_0(\gamma+\mu)(\kappa+\psi_5) + \psi_2\psi_6\phi)$$

$$A_1 = \psi_1\psi_2\psi_5\psi_6$$

$$A_0 = -\psi_5\psi_6\kappa\Gamma$$

Clearly, the coefficient A_3, A_2 and A_1 are positive, whereas A_0 exhibits a negative value. To prove that Eq. (22) could have positive or negative roots, we will apply Descartes's rule sign. By using Descartes's rule sign, it shows that Eq. (22) may have one positive real root and possibilities of two or zero negative real roots. If there are one positive and zero negative real roots, thus the remaining roots could be a pair of complex roots. We conclude the roots obtained in Table 3.

Table 3: Number of roots for Eq. (22)

A_3	A_2	A_1	A_0	Number of real positive root	Number of real negative root	Number of complex root
> 0	> 0	> 0	< 0	1	2	0
				1	0	2

From Table 3, Eq. (22) has one real positive root and it can be considered as an equilibrium state for P_U since it satisfies the biological meaning. Hence, there is only one equilibrium point $(H_U{}^*, P_U{}^*, D^*, C^*, H_A{}^*, P_A{}^*, M^*)$ that exist.

4.2. *Local stability analysis*

In this section, we study the local stability of the equilibrium point using the linearisation method.

Theorem 4.2. *The equilibrium point for the system (1)-(7) is locally asymptotically stable*.

Proof. The Jacobian matrix at the equilibrium point is given below:

$$J^* = \begin{bmatrix} -a & 0 & 0 & 0 & b & 0 & -c \\ d & -e & 0 & 0 & 0 & f & -g \\ 0 & h & -k & 0 & 0 & 0 & 0 \\ 0 & l & m & -n & 0 & 0 & 0 \\ p & 0 & 0 & 0 & -q & r & c \\ 0 & s & 0 & 0 & 0 & -u & g \\ 0 & v & v & v & 0 & 0 & -w \end{bmatrix}$$

where $a = \psi_1 + \phi M^*, b = \sigma, c = \phi {H_U}^*, d = \kappa,\ e = \psi_2 + \phi_0 M^*,\ \ f = \sigma_0,\ g = \phi_0 {P_U}^*,\ h = \rho, k = \psi_3,\ l = \rho_0,\ m = \tau_1, n = \psi_4, p = \phi M^*, q = \psi_5, r = \gamma, s = \phi_0 M^*, u = \psi_6, v = \omega_1, w = \omega_2$.

To analyze the stability, we determine the roots of characteristic polynomial by solving $|J^* - \lambda I| = 0$. These roots represent the eigenvalues of the system. The local asymptotic stability of the equilibrium point $({H_U}^*, {P_U}^*, D^*, C^*, {H_A}^*, {P_A}^*, M^*)$ can be determined by checking whether all the eigenvalues are negative or have negative real parts. Thus, the characteristic polynomial is obtained by

$$|J^* - \lambda I| = \lambda^7 + B_1\lambda^6 + B_2\lambda^5 + B_3\lambda^4 + B_4\lambda^3 + B_5\lambda^2 + B_6\lambda + B_7$$

where,

$$B_1 = a + e + k + n + q + u + w$$

$$B_2 = a(E_1 + E_2 + E_3) + e(k + E_1 + E_2) + k(E_1 + E_2) + gv + n(q + E_2) + qE_2 + uw - bp - fs$$

$$B_3 = a\big(e(k + E_1 + E_2)\big) + gv(E_4 + l + E_1 + u + a) + uw(a + E_1 + E_3) + k(E_1 + E_2)(a + e) + n(q + E_2)(a + E_3) + qE_2(a + E_3 + n) + cdv - bp(E_3 + E_2 + n) - f(s(a + k + E_1 + w) + gv)$$

$$B_4 = nquw + aek(E_1 + E_2) + (agv + cdv)(E_4 + l + E_1 + u) + auw(E_3 + E_1) + bfps + (ekn + anE_3)(q + E_2) + (E_2)(eq(k + n) + aq(E_3 + n) + kqn) + euw(k + E_1) + gv[h(m + E_1 + u) + k(l + E_1 + u) + l(q + u) + n(q + u) + qu] + kuwE_1 - af[gv + s(k + E_1 + w)] - b[cdv + ep(k + n + E_2) + drs + kp(n + E_2) + gpv + np(E_2) + puw] - f[gv(E_4 + l + E_1) + ks(E_1 + w) + ns(q + s) + qsw]$$

$$
\begin{aligned}
B_5 = {} & a[ek(nq + nE_2) + e(n + k)(qE_2 + uw) + gv(E_4(E_1 + u) + kl + E_5(q + u) \\
& + qu + hm) + k(nqE_2 + uwE_1) + quw(n + e)] + bfp[gv + s(k + n + w)] \\
& + cdv[h(E_1 + m + u) + kE_5 + (E_5 + k)(q + u) + qu] + ek[nqE_2 + uwE_1] \\
& + gv[(hm + kl)(q + u) + E_4(nq + uE_1) + quE_5] + nquwE_3 - af[gv(E_4 + l \\
& + E_1) + ks(E_1 + w) + ns(q + w) + qsw] - b[(cdv + gpv)(E_4 + E_5 + u) \\
& + dr\big(gv + s(k + n + w)\big) + (ep + kp)(nE_2 + uw) + ep(kE_2 + kn)] \\
& - f[gv(hm + E_1E_4 + kl + qE_5) + ks(nq + wE_1)] - nw(bpu + fqs)
\end{aligned}
$$

$$
\begin{aligned}
B_6 = {} & a\big[gv\big(h(mq + mu + uE_1 + nq)\big) + ekn(qE_2 + uw)\big] + bf[gpv(E_4 + E_5) \\
& + ps(kn + kw + nw)] + cdv[h(mq + nq + mu + uE_1)] + gvqu[h(m + n) \\
& + kE_5] + quw(aek + aen + akn + ekn) + (agv + cdv)(k(qE_5 + lu + uE_1) \\
& + quE_5) - af[gv(hm + hE_1 + kl + kE_1 + qE_5) + ks(nq + wE_1) + nqsw] \\
& - b\big[cdv\ (hn + hm + kE_5 + uE_4 + uE_5) + dr(gv(E_4 + E_5) + ks(n + w) \\
& + nsw) + ep\big(k(nE_2 + uw)\big) + gpv(h(m + n + u) + k(E_5 + u) + uE_5) \\
& + npuwE_3\big] - fq[gv(hm + hn + kE_5) + knsw]
\end{aligned}
$$

$$
\begin{aligned}
B_7 = {} & aqu(gv(hm + hn + kE_5) + eknw) + bfp(gv(hm + hn + kE_5) + knsw) \\
& + cdvqu(hm + hn + kE_5) - afq(gv(hm + hn + kE_5) + knsw) \\
& - \ b[cdvu(hm + hn + kE_5) + drgv(hm + hn + kE_5) + gvpu(hm + hn \\
& + kE_5) + nw(dkrs + ekpu)]
\end{aligned}
$$

where $E_1 = n + q, E_2 = u + w, E_3 = k + e, E_4 = h + k, E_5 = l + n$

However, solving for eigenvalues directly is difficult and complex as it involves the seventh-degree polynomial. Instead, we use Lienard-Chipart criterion as an alternative approach for analyzing the eigenvalue. According to this criterion, all the roots of characteristic polynomial are negative and have negative real parts if any of four sets holds (Mat Daud 2021b). We selected one of these sets, in which the odd coefficient and the even order Hurwitz determinant are positive. We took this advantage to ensure the roots of polynomial have negative real part when these conditions are satisfied. The necessary and sufficient conditions for seventh-degree polynomial are (Mat Daud 2021b);

$$B_i > 0, i = 1,3,5,7$$

$$|H_2| = B_1B_2 - B_3 > 0$$

$$|H_4| = (B_1B_2 - B_3)\big((B_1B_6 - B_7) + (B_3B_4 - B_2B_5)\big) - (B_1B_4 - B_5)^2 > 0$$

$$
\begin{aligned}
|H_6| = {} & (B_1B_2 - B_3)(2(B_1B_6 - B_7)(B_5B_6 - B_4B_7) + (B_3B_4 - B_2B_5)(B_5B_6 - B_4B_7) \\
& - (B_3B_6 - B_2B_7)^2) \\
& + (B_1B_4 - B_5)\big(-(B_1B_4 - B_5)(B_5B_6 - B_4B_7) + (B_1B_6 - B_7)(B_3B_6 - B_2B_7)\big) \\
& - (B_1B_6 - B_7)^2 > 0
\end{aligned}
$$

Therefore, the equilibrium point $(H_U{}^*, P_U{}^*, D^*, C^*, H_A{}^*, P_A{}^*, M^*)$ is locally asymptotically stable if the conditions $B_1 > 0$, $B_3 > 0$, $B_5 > 0$, $B_7 > 0$, $|H_2| > 0$, $|H_4| > 0$ and $|H_6| > 0$ are satisfied. □

4.3. *Global stability analysis*

Theorem 4.3. *The model is globally asymptotically stable around the equilibrium point inside the region Ω_L.*

Proof. Consider the form of Lyapunov function as

$$V = \frac{1}{2}K_1(H_U - H_U{}^*)^2 + \frac{1}{2}K_2(P_U - P_U{}^*)^2 + \frac{1}{2}K_3(D - D^*)^2 + \frac{1}{2}K_4(C - C^*)^2 + \frac{1}{2}K_5(H_A - H_A{}^*)^2 + \frac{1}{2}K_6(P_A - P_A{}^*)^2 + \frac{1}{2}K_7(M - M^*)^2 \tag{23}$$

where $K_1, K_2, \ldots, K_7$ are positive constants and can be chosen. Differentiating Eq. (23) with respect to time obtain

$$\dot{V} = K_1(H_U - H_U{}^*)\dot{H_U} + K_2(P_U - P_U{}^*)\dot{P_U} + K_3(D - D^*)\dot{D} + K_4(C - C^*)\dot{C} + K_5(H_A - H_A{}^*)\dot{H_A} + K_6(P_A - P_A{}^*)\dot{P_A} + K_7(M - M^*)\dot{M}$$

Then, we obtain

$$\begin{aligned}\dot{V} = &-K_1(\psi_1 + \phi M)(H_U - H_U{}^*)^2 - K_2(\psi_2 + \phi_0 M)(P_U - P_U{}^*)^2 - K_3\psi_3(D - D^*)^2 \\ &- K_4\psi_4(C - C^*)^2 - K_5\psi_5(H_A - H_A{}^*)^2 - K_6\psi_6(P_A - P_A{}^*)^2 - K_7\omega_2(M - M^*)^2 \\ &+ (K_1\sigma + K_5\phi M)(H_U - H_U{}^*)(H_A - H_A{}^*) - K_1\phi H_U{}^*(H_U - H_U{}^*)(M - M^*) \\ &+ K_2\kappa(P_U - P_U{}^*)(H_U - H_U{}^*) + (K_2\sigma_0 + K_6\phi_2 M)(P_U - P_U{}^*)(P_A - P_A{}^*) \\ &- (K_2\phi_0 P_U{}^* - K_7\omega_1)(P_U - P_U{}^*)(M - M^*) + K_3\rho(D - D^*)(P_U - P_U{}^*) \\ &+ K_4\tau(D - D^*)(C - C^*) + K_4\rho_0(C - C^*)(P_U - P_U{}^*) \\ &+ K_5\gamma(H_A - H_A{}^*)(P_A - P_A{}^*) + K_5\phi H_U{}^*(H_A - H_A{}^*)(M - M^*) \\ &+ K_6\phi_0 P_U{}^*(P_A - P_A{}^*)(M - M^*) + K_7\omega_1(M - M^*)(D - D^*) \\ &+ K_7\omega_1(M - M^*)(C - C^*)\end{aligned}$$

The sufficient conditions for the $\dot{V}$ to be negative definite are:

$$K_1(\sigma)^2 < \frac{1}{4}K_5(\psi_1 + \phi M)(\psi_5)$$

$$K_1(\phi H_U{}^*)^2 < \frac{1}{7}K_7(\psi_1 + \phi M)(\omega_2)$$

$$K_2(\kappa)^2 < \frac{1}{7}K_1(\psi_2 + \phi_0 M)(\psi_1 + \phi M)$$

$$K_1(\sigma_0)^2 < \frac{1}{7}K_6(\psi_2 + \phi_0 M)(\psi_6)$$

$$K_2(\phi_0 P_U{}^*)^2 < \frac{4}{49}K_7(\psi_2 + \phi_0 M)(\omega_2)$$

$$K_3(\rho)^2 < \frac{1}{6}K_2(\psi_3)(\psi_2 + \phi_0 M)$$

$$K_4(\tau)^2 < \frac{4}{9}K_3(\psi_3)(\psi_4)$$

$$K_4(\rho_0)^2 < \frac{1}{6}K_2(\psi_4)(\psi_2 + \phi_0 M)$$

$$K_5(\phi M)^2 < \frac{1}{4}K_6(\psi_1 + \phi M)(\psi_5)$$

$$K_5(\gamma)^2 < \frac{4}{9}K_6(\psi_5)(\psi_6)$$
$$K_5(\phi H_U{}^*)^2 < \frac{1}{7}K_7(\psi_5)(\omega_2)$$
$$K_6(\phi_0 M)^2 < \frac{1}{7}K_2(\psi_2 + \phi_0 M)(\psi_6)$$
$$K_6(\phi_0 P_U{}^*)^2 < \frac{1}{7}K_7(\psi_6)(\omega_2)$$
$$K_7(\omega_1)^2 < \frac{4}{49}K_2(\psi_2 + \phi_0 M)(\omega_2)$$
$$K_7(\omega_1)^2 < \frac{1}{6}K_3(\psi_3)(\omega_2)$$
$$K_7(\omega_1)^2 < \frac{1}{6}K_4(\psi_4)(\omega_2). \square$$

5. Numerical simulation

In this section, numerical simulations are conducted to observe the population dynamics of diabetics. The values of parameters in Table 4 are used to plot the graph in Figures 2-11. The parameter values given in Table 4 are obtained based on assumptions except μ. We use the estimated value of 5.9 per 1000 people by the Department of Statistics Malaysia (2024). The numerical simulations were carried out using the ode45 command in MATLAB software.

By substituting the value of parameters into the system of equations (9)-(15), we obtained the equilibrium point $H_U{}^* = 461438.9$, $P_U{}^* = 7107.85$, $D^* = 5482.34$, $C^* = 16580.3$, $H_A{}^* = 29122829.7$, $P_A{}^* = 880956.5$ and $M^* = 408 .39$ The eigenvalues obtained are -13.0263, -4.8896, -0.0052, $-0.0236 + 0.0146i$, $-0.0236 - 0.0146i$, -0.0324 and -0.0506. All the eigenvalues obtained have negative real parts, which ensures that the equilibrium is locally asymptotically stable.

Furthermore, the global stability of the equilibrium is illustrated graphically. The outputs of numerical simulation displayed in Figures 2-8, show the behaviour of the state variables under different initial conditions. These variables include: the aware healthy individuals H_U, unaware prediabetes individuals P_U, diabetes without complications D, diabetes with complication C, aware healthy individuals H_A and aware prediabetes individuals P_A. The figures show that all populations approach their equilibrium point. Hence, the numerical simulation confirms that the equilibrium point is globally stable.

Table 4: Assumption value of parameters

Parameter	Value (year^{-1})	Source
μ	0.0059	Department of Statistics Malaysia (2024)
Γ	180,000	Assume
κ	0.05	Assume
ρ	0.02	Assume
ρ_0	0.01	Assume
γ	0.02	Assume
τ_1	0.02	Assume
σ	0.04	Assume
σ_0	0.04	Assume
δ	0.00003	Assume
δ_1	0.005	Assume
ϕ	0.007	Assume
ϕ_0	0.02	Assume
ω_1	0.0007	Assume
ω_2	0.05	Assume

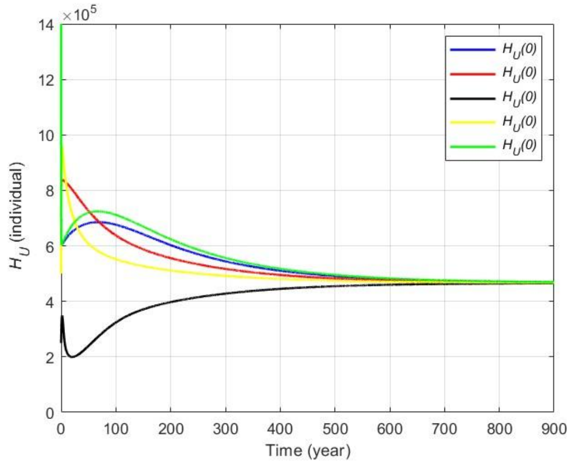

Figure 2: The number of unaware healthy individuals $H_U(t)$ with any initial condition.

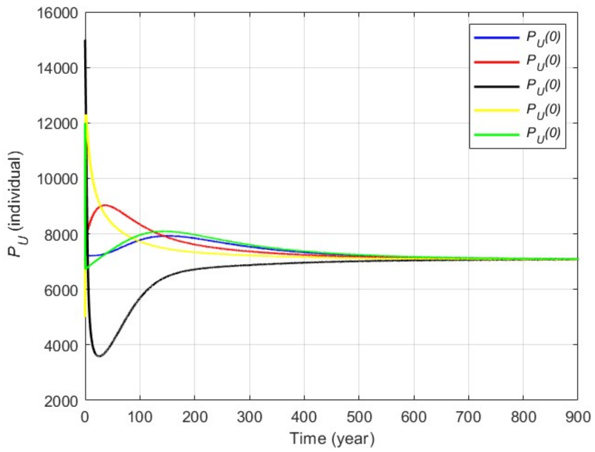

Figure 3: The number of unaware prediabetes individuals $P_U(t)$ with any initial condition.

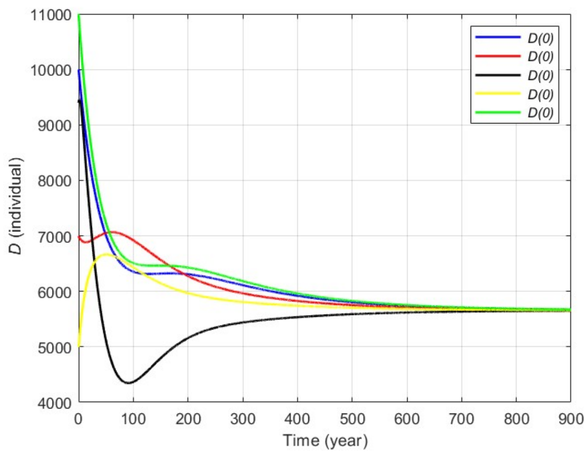

Figure 4: The number of diabetes individuals without complications $D(t)$ with any initial condition.

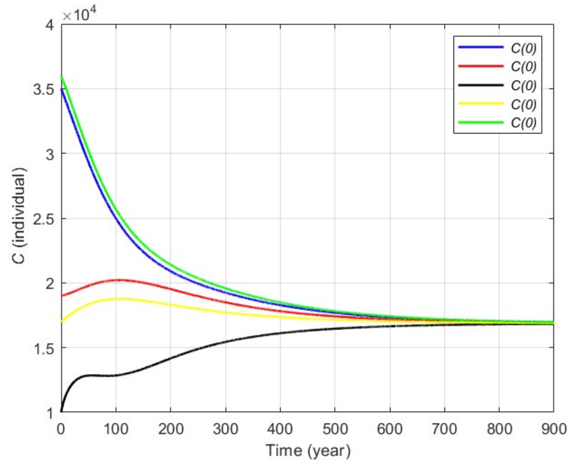

Figure 5: The number of diabetes individuals with complications $C(t)$ with any initial condition

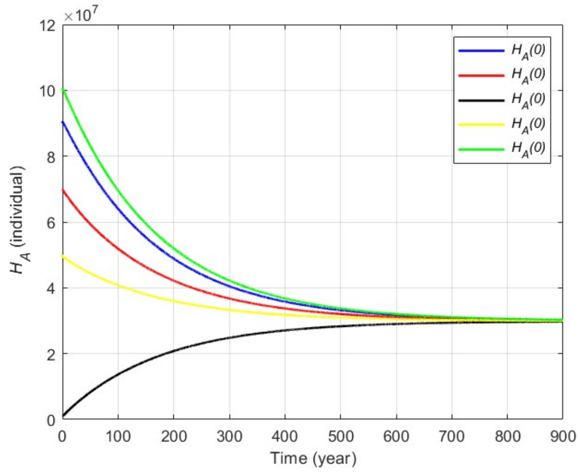

Figure 6: The number of aware healthy individuals $H_A(t)$ with any initial condition

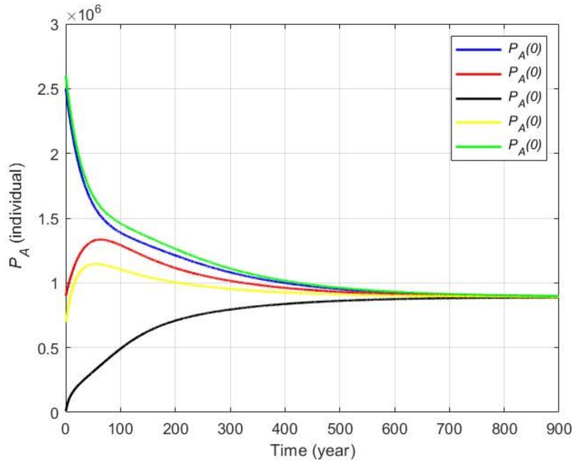

Figure 7: The number of aware prediabetes individuals $P_A(t)$ with any initial condition

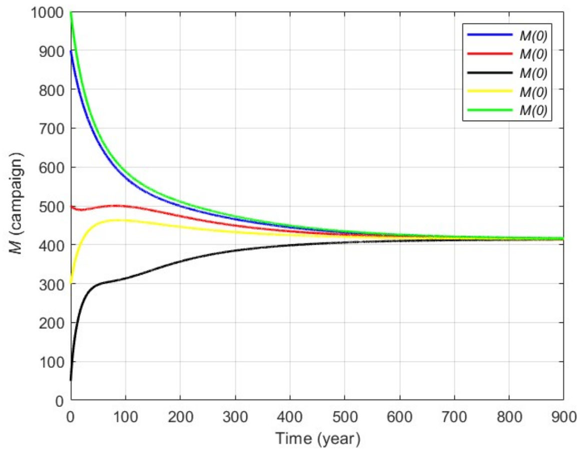

Figure 8: The cumulative number of awareness campaigns $M(t)$ with any initial condition

To show the impact of media-driven awareness campaigns on population dynamics, we explore the impact of parameters related to awareness campaigns such as the growth rate of awareness campaigns, ω_1, the rate at which unaware healthy individuals gain awareness, ϕ and the rate at which unaware prediabetes gain awareness, ϕ_0. We conducted numerical simulations using the following initial conditions $H_U(0) = 250{,}000$; $P_U(0) = 15{,}000$; $D(0) = 9{,}400$; $C(0) = 10000$; $H_A(0) = 1{,}000{,}000$, $P_A(0) = 10{,}000$ and $M(0) = 50$. However, these initial conditions were chosen arbitrarily to explore the dynamics of the model rather than achieving precise predictions. To visualize the effect of parameter variation, we plotted the Figure 9-14, which illustrate changes in $H_U(t)$, $P_U(t)$, $D(t)$, $C(t)$, $H_A(t)$, $P_A(t)$ and $M(t)$. These figures demonstrate the impact of varying different values of ϕ, ω_1 and ϕ_0. While sensitivity analysis could offer valuable insights into which parameters are dominant, the current study does not examine this aspect.

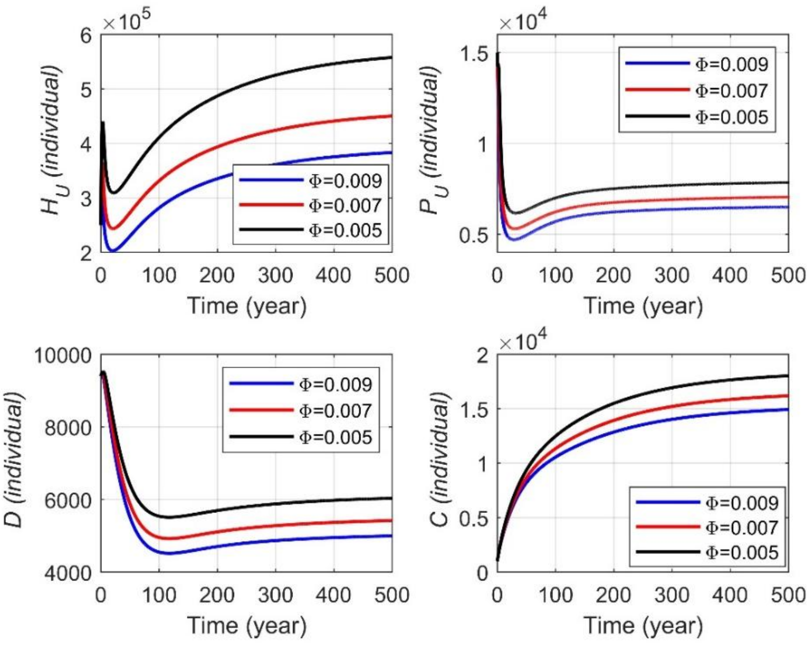

Figure 9: Simulation for $H_U(t)$, $P_U(t)$, $D(t)$, and $C(t)$, with different values of ϕ

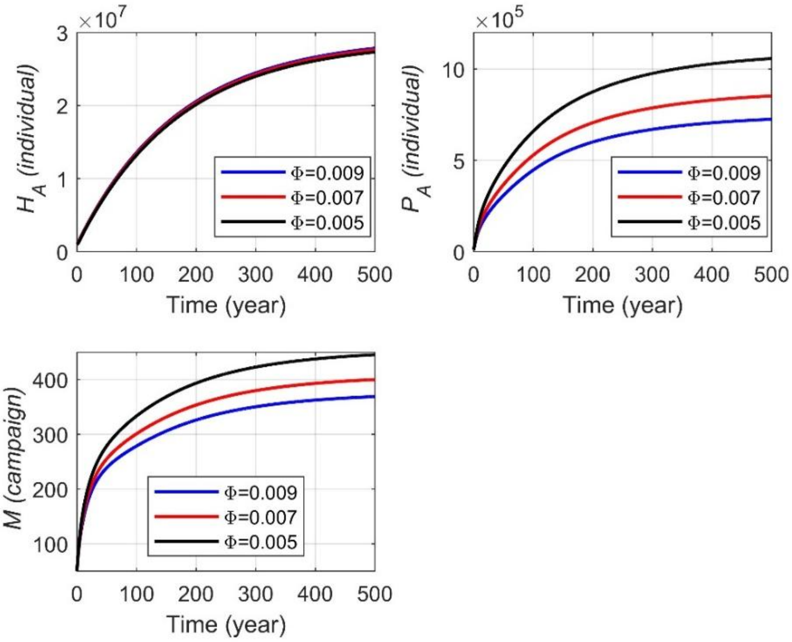

Figure 10: Simulation for $H_A(t)$, $P_A(t)$ and $M(t)$ with different values of ϕ

The numerical simulations in Figure 9 and Figure 10 illustrate the changes in the dissemination rate of awareness ϕ affect different stages of populations and the cumulative awareness program. The simulation considers three different values: $\phi = 0.005$, $\phi = 0.007$ and $\phi = 0.009$. From these figures and Table 5, we observe that as ϕ increases, the number of unaware prediabetes individuals and diabetes individuals (D and C) decreases. This suggests that a higher dissemination rate of awareness effectively reduces the number unaware prediabetes and diabetes individuals. At the same time, aware healthy individuals increases with increasing ϕ indicating a positive impact on awareness levels.

Table 5: Populations after 500 years with $\phi = 0.005$, $\phi = 0.007$ and $\phi = 0.009$

Population	$H_U(t)$	$P_U(t)$	$D(t)$	$C(t)$	$H_A(t)$	$P_A(t)$	$M(t)$
Population after 500 years with $\phi = 0.005$	5.57×10^5	7.84×10^3	6.03×10^3	1.8×10^4	2.73×10^7	1.06×10^6	4.45×10^2
Population after 500 years with $\phi = 0.007$	4.5×10^5	7×10^3	5.42×10^3	1.62×10^4	2.76×10^7	8.52×10^5	4×10^2
Population after 500 years with $\phi = 0.009$	3.83×10^5	6.49×10^3	4.99×10^3	1.49×10^4	2.78×10^7	7.25×10^5	3.68×10^2

Similarly, Figure 11 and Figure 12 illustrate the effect of the growth rate of awareness campaigns on the variation of state variables with respect to time for different values of $\omega_1 = 0.005, 0.007, 0.009$. From these figures and Table 6, we observed that as the growth rate of media-driven awareness programs increases, the number of unaware prediabetes and diabetes decreases (D and C). This concludes that an increase in media-driven awareness campaigns plays a significant role in reducing diabetes prevalence. The results suggest that increasing media-driven awareness campaigns can be an effective strategy for controlling the development of prediabetes and diabetes within the population.

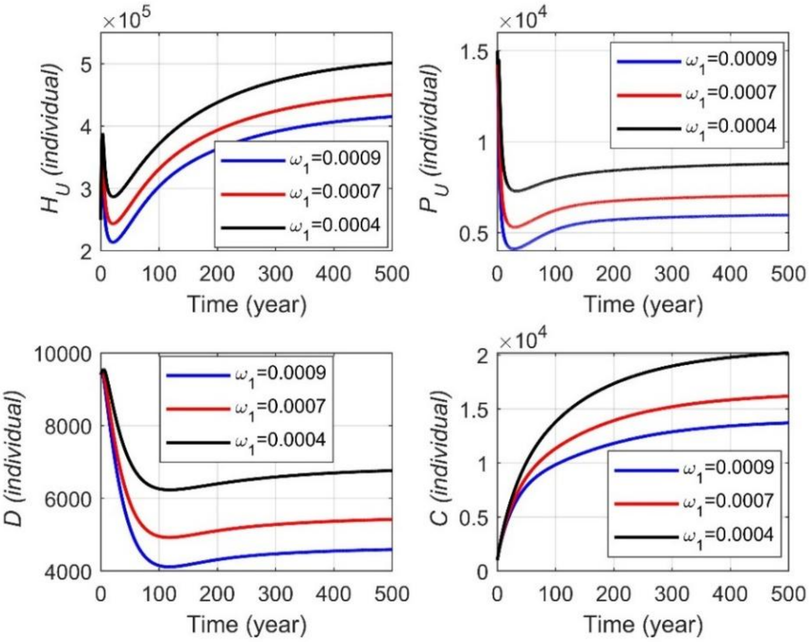

Figure 11: Simulation for $H_U(t)$, $P_U(t)$, $D(t)$ and $C(t)$ with different values of ω_1

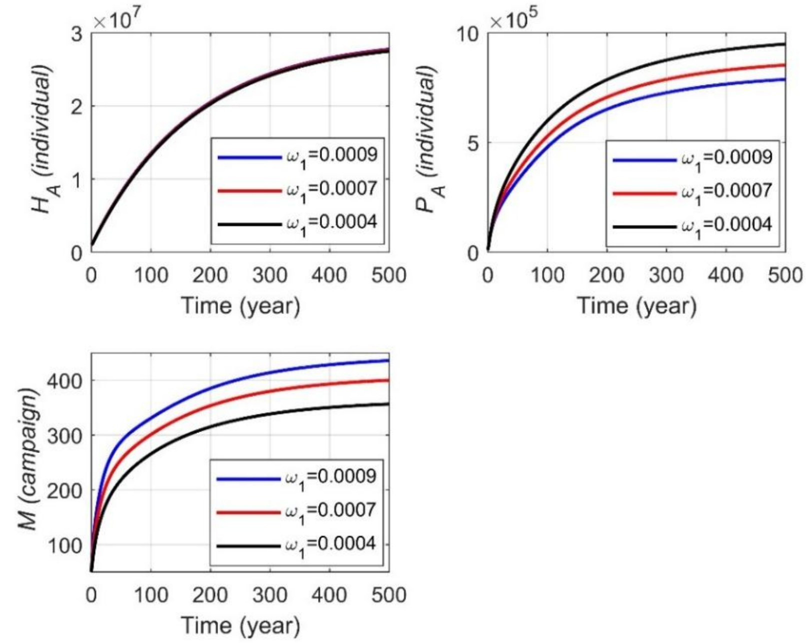

Figure 12: Simulation for $H_A(t)$, $P_A(t)$ and $M(t)$ with different values of ω_1

Table 6: Populations after 500 years with $\omega_1 = 0.0004$, $\omega_1 = 0.0007$ and $\omega_1 = 0.0009$

Population	$H_U(t)$	$P_U(t)$	$D(t)$	$C(t)$	$H_A(t)$	$P_A(t)$	$M(t)$
Population after 500 years with $\omega_1 = 0.0004$	5.01×10^5	8.79×10^3	6.76×10^3	2×10^4	2.74×10^7	9.48×10^5	3.56×10^2
Population after 500 years with $\omega_1 = 0.0007$	4.5×10^5	7×10^3	5.42×10^3	1.62×10^4	2.76×10^7	8.52×10^5	4×10^2
Population after 500 years with $\omega_1 = 0.0009$	4.1×10^5	5.96×10^3	4.59×10^3	1.37×10^4	2.77×10^7	7.87×10^5	4.35×10^2

The numerical simulation in Figure 13 and Figure 14 suggests that the transition unaware prediabetes individuals gain awareness significantly impacts the behaviour of different stages of populations and the cumulative awareness program. Using different values of $\phi_0 = 0.04$, $\phi_0 = 0.02$ and $\phi_0 = 0.009$, observe from figures and Table 7 that as ϕ_0 increases, the number unaware prediabetes individuals and diabetes individuals (D and C) decrease. This indicates that a higher rate of unaware prediabetes individuals gains awareness shows the effectiveness of awareness campaigns.

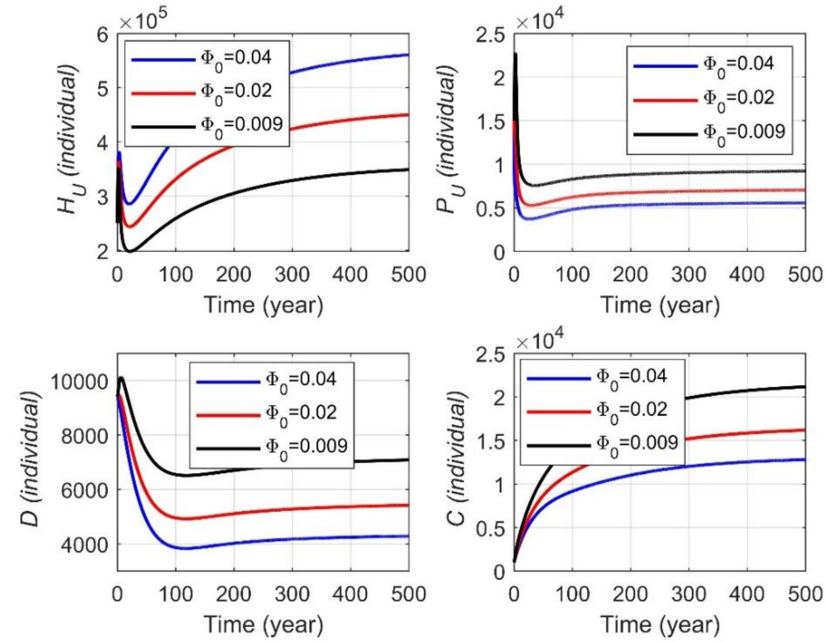

Figure 13: Simulation for $H_U(t)$, $P_U(t)$, $D(t)$ and $C(t)$ with different values of ϕ_0

We can conclude that the growth rate of awareness campaigns, ω_1, the rate at which unaware healthy individuals gain awareness, ϕ and the rate at which unaware prediabetes gain awareness, ϕ_0 have a significant influence on the prevalence of prediabetes and diabetes. The awareness campaign enhances people's knowledge about the risks, symptoms and complications and changes their behaviour toward a healthy lifestyle

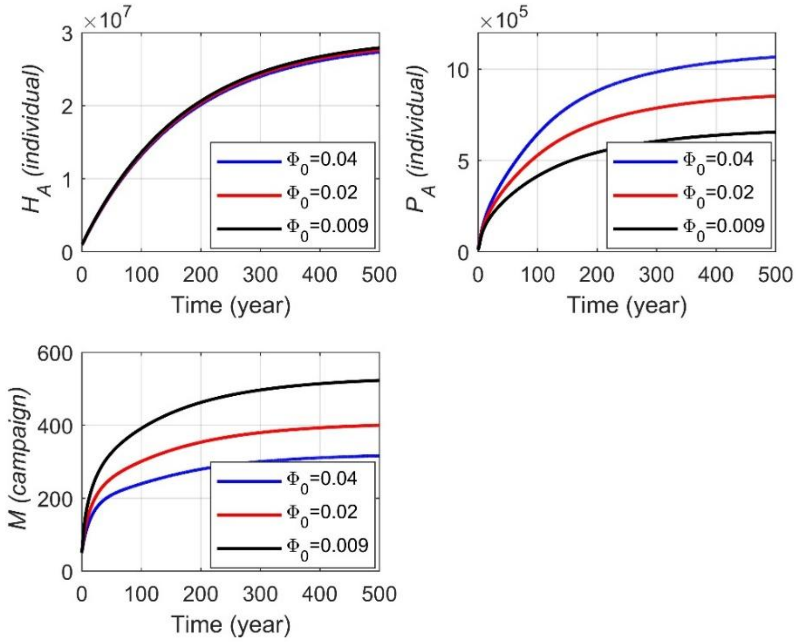

Figure 14: Simulation for $H_A(t)$, $P_A(t)$ and $M(t)$ with different values of ϕ_0

Table 7: Populations after 500 years with $\phi_0 = 0.04$, $\phi_0 = 0.02$ and $\phi_0 = 0.009$

Population	$H_U(t)$	$P_U(t)$	$D(t)$	$C(t)$	$H_A(t)$	$P_A(t)$	$M(t)$
Population after 500 years with $\phi_0 = 0.04$	5.6×10^5	5.57×10^3	4.28×10^3	1.27×10^4	2.73×10^7	1.06×10^6	3.16×10^2
Population after 500 years with $\phi_0 = 0.02$	4.5×10^5	7×10^3	5.42×10^3	1.62×10^4	2.76×10^7	8.52×10^5	4×10^2
Population after 500 years with $\phi_0 = 0.009$	3.48×10^5	9.21×10^3	7.08×10^3	2.11×10^4	2.79×10^7	6.55×10^5	5.22×10^2

This information may be helpful for government officials or policymakers to manage the rising incidence of diabetes each year. The increasing prevalence causes a financial burden on the government to cover the expenses associated with necessary treatments for affected individuals. The findings suggest that implementing further educational campaigns for the public through various media outlets - including online media, printed media, radio, and television - could be beneficial in addressing the diabetes epidemic. Hence, this information, along with other factors such as available resources, may be incorporated into public health decision-making. While the beneficial effects of health campaigns have long been recognized, the present findings provide further justification for their positive influence.

The outcomes from analysing the equilibrium point and stability may assist policymakers or government agencies in knowing the number of stable populations for each state variable in the future by inputting the data into the equilibrium point. Inputting the data does not necessarily involve the whole of Malaysia; it may involve the preferred state. As a result, government agencies may pinpoint which states are likely to have high prevalence in the future. With this knowledge, government agencies may allocate resources more effectively, focusing on states that require interventions such as health campaigns, increased healthcare facilities or adequate supply of medication. Also, the government may assess the effectiveness of interventions over time. By observing changes in prevalence before and after specific actions are implemented, the government may evaluate the impact of these measures and make adjustments as needed.

6. Conclusion

Diabetes is an incurable disease that contributes to premature death primarily due to its associated complications, as indicated in the introduction. These complications not only increase the economic burden due to treatment costs but also lead to disabilities and reduce the quality of life. However, this severe scenario can be alleviated through pragmatic measures targeting modifiable risk factors. Increasing awareness and educating individuals to encourage behaviour changes toward a healthy lifestyle are some of the key strategies to combat these risks. In this study, we developed a mathematical model to analyse diabetes population dynamics, comprising seven ordinary differential equations. By considering different stages of diabetes progression, our model provides deeper insights into how awareness as a prevention effort influences various population groups. Our model also includes the potential for prediabetes reversal through lifestyle modification among aware prediabetic individuals. The model treats the cumulative number of awareness campaigns as a state variable and assumes that the growth of media-driven awareness campaigns depends on unaware prediabetes, diabetes with complications and diabetes without complications. This approach offers a realistic representation of how awareness influences the transition between unaware and aware subpopulations, enhancing the model's applicability to real-world prevention strategies. From the nonlinear model, only one equilibrium point is considered biologically meaningful. The study employs local stability analysis using the Jacobian matrix and the Lienard-Chipart criteria, while global stability analysis is conducted using a Lyapunov function. When the solution satisfies the given conditions, it guarantees that all the state variables are stable at the equilibrium point, maintaining a constant value over time. Numerical simulations demonstrate the existence and stability of all the subpopulations, thus verifying both local and global stability. These results ensure that population dynamics remain manageable in the long term, allowing lifestyle intervention strategies can be planned with confidence. Furthermore, simulations were performed to explore the impact of parameters that related to awareness campaigns such as the growth rate of media-driven awareness campaigns, ω_1, the rate at which unaware healthy individuals gain awareness, ϕ and the rate at which unaware prediabetes gain awareness, ϕ_0. The resulting figures (Figure 9-14) clearly illustrate the influence of different parameter values in order to convince health decision-makers that investing in primary health care yields certain benefits. This study underscores the need to adopt awareness strategies targeting unaware healthy individuals and unaware prediabetes individuals for diabetes prevention and management, rather than focusing solely on treatment. These results indicate that the progression of prediabetes and diabetes may be controlled or reduced with the help of media-driven awareness campaigns. This study suggests that investment in awareness is a strategy to reduce healthcare costs and improve the quality of life for the population. Moreover, this study has certain limitations due to assumptions made. Specifically, we consider the interaction between unaware healthy individuals and awareness campaigns to be proportional. Thus, the rate of healthy individuals being aware increases when awareness campaigns increase. However, in reality, people will not respond in the same proportion. This paper can be extended by considering the implementation of awareness programs by media has limited impact on the unaware individuals that may give better insight into the population dynamics of diabetics.

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