Halo Structure Investigation of Nucleus $^8$He via Core Deformation Parameter
(Penyelidikan Struktur Halo Nukleus $^8$He melalui Parameter Kecanggaan Teras)

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ABSTRACT

Helium-8 ($^8$He) is a four neutrons halo nucleus and is the halo nucleus that attract attention to observe because it belongs to the group of light dripline nuclei. We studied the two neutrons halo nucleus of $^8$He. This halo nucleus lies on the neutron rich dripline in the nuclear landscape. Nucleus $^8$He was investigated in the Three-body Model with two different configurations (T and Y). To describe the Three-body Model, Jacobi coordinate systems for few bodies was used for this hyper-nucleus ($^8$He). In this study, core ($^6$He) + two valence neutrons ($n + n$) were considered as the Three-body. Based on the finding, theoretical data on configuration Y met a good agreement with the experimental data of binding energy and rms matter radius. While for configuration T, the data slightly differ from the experimental data. The results showed that the core deformation parameters, $\beta_2$ of $^8$He were 0.12, 0.19, 0.35, 0.40, and 0.47. The binding energies -3.112 MeV, -3.416 MeV, and -3.625 MeV were calculated for the deformation parameters 0.35, 0.40, and 0.47 and rms radii 2.032 fm, 2.305 fm, and 2.39 fm were found for the deformation parameters 0.19, 0.35, and 0.4, respectively. In conclusion, $^8$He exhibits in prolate due to positive values of core deformation parameters.

Keywords: Core deformation parameter; halo nucleus; Three-body Model

INTRODUCTION

Nuclear structure beyond the stability line N/Z is highly attractive for many researchers. Most interesting among these was the nuclear structure of halo nuclei, which opened a new era of the physics world. There are many experiments and theoretical studies have been done on halo nuclei. Halo is a composites of a core plus with either one, two or four valence nucleons. The valence nucleons are orbiting far from the core with low binding energy (unbound systems) and lead a sizeable abnormal root mean square (rms) (Ershov et al. 2010; Frederico et al. 2012; Hansen et al. 1995; Vaagen & Gridnev 2000).

Generally, the halo phenomenon only occurred on nuclei that are situated near to neutron or proton drip lines. This phenomenon happened in which particles are held in short-range potential wells (Jensen et al. 2004). Nucleus halo consists of a core with loosely bound nucleon(s), which cause an extension to spatial distance (Khan & Das 2001; Liang et al. 2008). Moreover, halo phenomenon was related to Borromean system where it only exists when both sub-system n-n and core-n were loosely bound with low energy (Bacca et al. 2012; Mueller et al. 2007; Nielsen 2001).

Helium -8 ($^8$He) is a four neutrons halo nucleus (Moon 2014). $^6$He and $^8$He are the two-halo nuclei that attract attention to observe because these two nuclei belong to the group of light dripline nuclei. In addition, $^8$He was the first nucleus that was observed in its structure. $^4$He is a composite of core ($^4$He) with two neutrons and $^6$He is a composite of a core ($^4$He) with four neutrons. These two nuclei are characterized as a Borromean halo system.
and had been investigated in experimental and theoretical studies (Chulkov et al. 2005; Kanada-En'yo 2007; Keeley et al. 2007; Hagino et al. 2008; Ye 2012). The ground state of $^8$He is $^0+$, and energies for the excited state at $^2+$ are in between 3.6 and 3.9 MeV, while at state $^1+$, the energies are 5.3 to 5.5 MeV (Golovkov et al. 2009; Korsheninnikov et al. 2003). $^8$He has $(1p_{3/2})^4$ structure and other configurations such as $(1p_{3/2})^2(1p_{1/2})^2$ based on probability in the ground state wave function (Hagino et al. 2008; Keeley et al. 2007).

In previous studies, properties of $^8$He were determined in electromagnetic fragmentation reactions, quasi-free scattering and knockout reaction. $^8$He was also studied using a cluster shell model, microscopic cluster model (resonating group model) and large-basis no-core shell model. $^8$He has been observed in a bound state with 2.140 MeV was needed to remove a neutron pair and 3.112 MeV was need to remove two neutron pairs (Audi et al. 2003; Pfeiffer 2012). This 3.112 MeV is much closer to the binding energy, 3.1 MeV for four neutron halo nuclei $^4$He (Lemasson et al. 2010). In addition, Jacobi coordinates Three-body model is applied to a systematic study for the energy and hyper-nucleus structure (Hiyama et al. 1999). In this present work, Jacobi coordinate as well as Hamiltonian of three-body of the system are used to produce the theoretical values of the binding energy of the three-body and the rms matter radii. Because the binding energies for the deformation parameters 0.12, 0.19, and 0.35 and the rms radii for the core deformation parameters 0.12, and 0.47 for the $^8$He nucleus were obtained experimentally but the rest of them were waiting for experiment. This work will predict the binding energy for core deformation parameters 0.4, 0.47, and rms radii for the core deformation parameters 0.19, 0.35, 0.40 which are still waiting to obtain experimentally. This paper has been described as the following structure: Literature review is described in the introduction, followed by the theoretical framework (methods). After that there will be results, discussion and conclusion have been discussed sequentially in this paper.

**THEORETICAL FRAMEWORK**

The system of three-body is used in two different configurations which are configuration-T and configuration-Y as shown in Figure 1. These two configurations are explained in Jacobi coordinates. In this section, to discuss the $^8$He halo nucleus is considered as the three-body system. To explain the system, Jacobi coordinate has been used. There are three sets of Jacobi coordinates of this system. In this work, core ($^4$He) + n + n model was considered for the $^8$He hyper-nucleus. In this model, we employed the realistic nn interaction of the Bonn-A type and a one range Gaussian potential for the nn correlations between the valence nucleons and core ($^4$He) + n interactions, respectively.

However, coupled-rearrangement channel Gaussian basis variational method was employed for the three-body wave equation for the above interactions. For these interactions, the total wave function of these three bodies was described as a sum of the amplitude using three rearrangement channels $c = 1$-$3$, where $c = 1$-$2$ are used for the Y-configuration and $c = 3$ is for the T-configuration. Figure 1 shows the Y- and T-configurations. However, Y-configuration shows the rearrangement channel is $c = 1$, not for $c = 2$ (Hiyama & Kamimura 2018; Hiyama et al. 1999).

**FIGURE 1. Three-body system of T-configuration and Y-configuration**
Set of eigenstates \( \phi_{\text{core}} \) and eigenvalues \( \varepsilon_{\text{core}} \) are used in the intrinsic Hamiltonian operation of the core with:

\[
\hat{H}_{\text{core}}(\xi_{\text{core}}) \phi_{\text{core}}(\xi_{\text{core}}) = \varepsilon_{\text{core}} \phi_{\text{core}}(\xi_{\text{core}}) \quad (1)
\]

The Jacobi coordinates are included in the total wave function of the three-body system:

\[
\psi^{JM}(x, y, \xi) = \phi_{\text{core}}(\xi_{\text{core}}) \psi(x, y) \quad (2)
\]

The wave function \( \psi(x, y) \) contains information on radius, angle, and spin associated with the two particles relative to the core. The hypersphere coordinate is used to describe the relative motion in the system of three-body containing a mass of \( A_i \), where \( i = 1, 2, 3 \). The distances between each pair of particles, \( r_{ij} \) and the distance between the centre mass of the particle pairs and the third particles can be based on the Jacobi coordinates \( (x, y) \) (Brida et al. 2006; Nunes et al. 1996; Raynal & Revai 1970):

\[
x = \sqrt{A_{jk}r_{jk}} = \frac{A_{jk}r_{jk}}{\sqrt{A_{jk}A_{k}r_{jk}}} \quad (3)
\]

Hyper-radius \( \rho \) and hyper-angular \( \theta \) are defined as:

\[
\rho^2 = x^2 + y^2 \quad \text{and} \quad \theta = \arctan \left( \frac{y}{x} \right) \quad (4)
\]

The expansion of hypersphere coordinates that are converted from Jacobi coordinates \( (x, y) \). The angular and radial part of wave function of three-body system is (Brida et al. 2006; Nunes et al. 1996; Raynal & Revai 1970):

\[
R_n(\rho) = \frac{s}{\rho^5} \int_{(a+b)}^{\infty} L^2_{nl\text{max}}(z) e^{-\frac{z}{\rho}} \quad (5)
\]

\[
\phi^{1\ell y}_{k}(\theta) = N_{k}^{1\ell y}(\sin\theta)^{\ell}(\cos\theta)^{y}p_{n}^{\ell + \frac{3}{2} + \frac{1}{2}}(\cos2\theta) \quad (6)
\]

where \( \rho = \sqrt{J(J+1)} \) and \( \rho_n = \sqrt{m(m+1)} \), while, \( J = l+1 \) and \( m = -j, j+1, \ldots, j \), where \( J \) is the total angular momentum of the valence nucleon; \( z = \rho/\rho_0 \) where \( z \) is dependence on \( \rho \) and \( \rho_0_L_{\text{lang}} \) is associated Laguerre polynomials of order \( n_{\text{lang}} = 0, 1, 2, 3, \ldots \). \( N_k^{1\ell y} \) is normalization coefficient and \( k \) is the hyper-angular momentum quantum number with \( k = 2n+1 \) and \( \ell = l+1 \) and \( P_{n}^{\ell+\frac{3}{2}+\frac{1}{2}}(\cos2\theta) \) is the Jacobi polynomial, where \( n = l+1 \). The wave function of valence nucleon (neutron) is (Brida et al. 2006; Nunes et al. 1996; Raynal & Revai 1970):

\[
\psi^{1\ell y}_{nK}(\rho, \theta) = R_n(\rho)\phi^{1\ell y}_{k}(\theta) \quad (7)
\]

Hamiltonian \( \hat{H} \) in the three-body system is defined as (Hwash et al. 2014, 2012; Salih et al. 2018, 2017):

\[
\hat{H} = H + h_c(\xi) + V_{c-n_1}(r_{c-n_1}) + V_{c-n_2}(r_{c-n_2}) + V_{n-n}(r_{n-n}) \quad (8)
\]

where \( T \) is referred to kinetic energy and \( h_c(\xi) \) is the Hamiltonian of the core which depends on the variable \( (\xi) \). The potential energies for two interacting bodies which are \( V_{c,n_1} \) and \( V_{c,n_2} \) as follows:

\[
V_{c-n_1}(r_{c-n_1}) = -\frac{1}{4\pi}\frac{\gamma_{c-n_1}(\theta, \phi)}{r_{c-n_1}} \quad (9)
\]

\[
V_{n-n}(r_{n-n}) = -\frac{1}{4\pi}\frac{\gamma_{n-n}(\theta, \phi)}{r_{n-n}} \quad (10)
\]

with: \( R(\theta, \phi) = R_0[1 + \beta_2 Y_{20}(\theta, \phi)] \) for \(^{4}\text{He} \quad (11)\)

The potential energies for (9) and (10) use deformed Wood-Saxon potential and also included with (11) where the radius \( R(\theta, \phi) \) of the deformed core is expanded in spherical harmonics with the quadrupole term, \( \beta \). We used \( R_0 = 1.25A_{1}^{1/3} \), \( R_{r_0} = R \) and \( l \) is the operator of orbital momentum, \( S \) is the operator of the nucleon’s spin, \( m \) is the mass of pion and \( \beta \) is the core’s quadrupole deformation.

The operators of squared distance two nucleons for an A-body system from the position of the total centre of mass are (Hwash et al. 2014, 2012; Salih et al. 2018, 2017):

\[
\tilde{r}_{CM} = \frac{1}{A} \sum_{i=1}^{A} \tilde{r}_i \quad (12)
\]

\[
\tilde{r}_{CM}^2 = \frac{1}{A} \sum_{i=1}^{A} (\tilde{r}_i - \tilde{r}_{CM})^2 \quad (13)
\]

\[
(r_{CM}^2)^{1/2} = \frac{1}{A} [A_{c}(r_{CM}^2(\text{core}) + \langle \rho^2 \rangle)] \quad (14)
\]

where \( \tilde{r}_{CM} \) is the centre of mass; \( \tilde{r}_i \) is the position of i nucleon; and \( (r_{CM}^2)^{1/2} \) is the root mean square of the radius. The total quadrupole moment for halo nucleus can be written as \( Q = Q_c + Q_r \), where \( Q \) is the amount of quadrupole moment comprising of \( Q_c \) caused by a loose nucleon, while \( Q_r \) is due to the core. Generally, \( Q_r \gg Q_c \) (Homyak 1975):

\[
Q_c = Q_j \left[ \left( \frac{3\Omega^2}{2J^2} \right) - 1 \right] \quad (15)
\]

Equation (15) can be rewritten in the form:

\[
Q_c = Q_j \frac{J}{2J+3} \left[ \left( \frac{3\Omega^2}{2J+1} \right) - 1 \right] \quad (16)
\]

where \( J \) is the total angular momentum or nuclear spin; \( \Omega \) is the projection angular momentum of nuclear; \( j \) and \( Q \) (intrinsic quadrupole moments) can be taken as:

\[
Q = \frac{4}{5} 8ZR^2
\]
Z is the parameter of proton number; R is the radius of the nucleus, and parameter δ is related to the parameter of deformation $\beta_2$ where $\beta_2=2/(3(4\pi/5)^{1/2})\delta$ (Homyak 1975). Equation (7) is a wave function for the three-body system. The wave function is the result of multiplication of internal wave functions of the cluster (between core with neutron valence). However, for the core like $\alpha$-core in nucleus $^4$He, where the core configuration is $(1s_{1/2})^2$ for the proton and neutron, the wave function can be ignored from the three-body system. The total wave function for the entire system is similar to the wave function of valence neutron, which includes correlation of valence neutron and core movements.

Variable in Jacobi polynomial in (6) was used to link up with the principal quantum number of the polynomials so that input values of the quantum numbers into the calculation of computational methods can produce different results. Table 1 shows the parameter values used in the computational process by using MATLAB.

### Table 1. The parameter values used for calculation of computational in present work

<table>
<thead>
<tr>
<th>$l_x$</th>
<th>$l_y$</th>
<th>n</th>
<th>K</th>
<th>$\rho_p$</th>
<th>$\rho_o$</th>
<th>$r_o$ (fm)</th>
<th>a (fm)</th>
<th>$a_{so}$ (fm)</th>
<th>$V_o$ (MeV)</th>
<th>$V_{so}$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>0.866</td>
<td>0.866</td>
<td>1.25</td>
<td>0.65</td>
<td>0.65</td>
<td>-74</td>
<td>-7.5</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>0.866</td>
<td>1.936</td>
<td>1.25</td>
<td>0.65</td>
<td>0.65</td>
<td>-74</td>
<td>-7.5</td>
</tr>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.936</td>
<td>-</td>
<td>1.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td>0.866</td>
<td>2.958</td>
<td>1.25</td>
<td>0.65</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td>1.936</td>
<td>-</td>
<td>1.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>


Based on shell model, the ground state of $^8$He was $(p_{3/2})^4$ for valence neutron configuration and recent finding suggested that $^8$He ground state was $(p_{3/2})^2(p_{1/2})^2$. Probability of wave function for $(p_{3/2})^4$ was 0.349, while for configuration $(p_{3/2})^2(p_{1/2})^2$ was 0.237. In this study, theoretical calculation was performed only for the values of $l_x$ and $l_y$ from 0 to 2 based on the previous study on neutron halo (Chulkov et al. 2005; Hagino et al. 2008; Itagaki et al. 2001; Keeley et al. 2007; Korsheninnikov et al. 2003; Makenroth 2001; Markenroth et al. 2001; Skaza et al. 2006).

### RESULTS AND DISCUSSION

In this study, $^4$He ground states are considered to have a $^4$He + 4n ($\alpha$ + 4n) structure and $^4$He + 2n structure. These two structures showed exciting behaviour. This behaviour is related to the deformation parameter and binding energy. The binding energy and the matter radius of $^4$He were calculated using the experimental deformation value which is showed in Figures 2 and 4, respectively, through the normalization. For the Y-configuration and T-configuration, the binding energy was found -3.416 MeV and -3.625 MeV, respectively, for a given deformation parameter $\beta_2$. To calculate the deformation of the core ($^4$He) and the binding energy, the experimental matter radius of $^4$He was used. To obtain the Figure 2, we solved the Hamiltonian for Y-and T-configurations to collect the theoretical binding energy for a particular deformation value. Then, the theoretical binding energies were plotted with the deformation parameters. After this, the experimental binding energies were drawn in the Figure 2. These binding energies intersect the curves in Figure 2. From these intersect points, the theoretical deformation parameter values were determined. Moreover, theoretical binding energies for the given deformation parameters were used to calculate rms matter radius. These rms matter radii and yielded core deformations are plotted in Figure 4. These yielded core deformations intersect the curve in the figure which shows the theoretical rms matter radius.

The experimental binding energy for $\alpha$+4n was 3.112 ± 0.0285 MeV (Audi et al. 2003; Pfeiffer 2012), while for $^4$He + 2n were -2.125 ± 0.0001MeV and -2.140 ± 0.0070 MeV (Audi et al. 2003; Pfeiffer 2012). Based on Lemasson et al. (2010) study, binding energy for one, two, three and four were 2.6, 2.1, 4.0, and 3.1 MeV (Ye 2012). The theoretical core deformation parameter, $\beta_2$, based on experimental binding energies were 0.12, 0.19, 0.35, and 0.40. These values will be used to determine the rms matter radius of $^8$He to produced theoretical values that correspond to the experimental value of binding energies. Core deformation parameter, $\beta_2=0.47$, was produced through normalization based on rms matter.
radius graph. Theoretical binding energy were -3.416 and -3.625 MeV based on $\beta_2=0.47$. These values differ by 10% to 16% from $3.112 \pm 0.0285$ MeV (Audi et al. 2003; Pfeiffer 2012). All the experimental and theoretical values of the binding energy of two and four valence nucleon are tabulated in Table 2.

![Figure 2](image.png)

**FIGURE 2.** Binding energies of the bound states in $^8$He in two different configurations as functions of the core deformation parameter

Based on Golovkov et al. (2009), the highest resonance peak for $^8$He was $3.57 \pm 0.12$ MeV for the first excitation from the ground state. From this work, the binding energy range is -3.416 MeV to -3.625 MeV at 0.47 (core deformation parameter). However, Figure 3 shows the energy level of $^4$He plus with two and four-neutron valences. Core deformation parameter, $\beta_2=0.47$ came from the normalization of optical limit approximation values from Figure 3. Based on the Y-configuration and T-configuration, we could assume the $^8$He nucleus is either $^4$He + n + n or $^4$He + 2n + 2n with the three-body system. But the $^4$He + n + n configuration is not possible due to the weak binding energy of $^4$He. On the other hand, $^4$He + 4n configuration with three-body system exists for the binding energy 3.16 MeV (Wurzer & Hofmann 1997). The binding energy showed the neutron skin (4 neutrons) surrounded in the $^4$He was bound around 3.112 to 3.625 MeV. Figure 3 shows the energy level of the valence neutrons bound with the $^4$He.

<table>
<thead>
<tr>
<th>Core deformation parameter, $\beta_2$</th>
<th>Binding energy of three-body (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>-2.125 $\pm$ 0.0001 (Audi et al. 2003)</td>
</tr>
<tr>
<td>0.19</td>
<td>-2.140 $\pm$ 0.0070 (Pfeiffer 2012)</td>
</tr>
<tr>
<td>0.35</td>
<td>-3.112 $\pm$ 0.0285 (Audi et al. 2003; Pfeiffer 2012)</td>
</tr>
<tr>
<td>0.40</td>
<td>-3.416</td>
</tr>
<tr>
<td>0.47</td>
<td>-3.625</td>
</tr>
</tbody>
</table>
Core deformation parameter values, $\beta_2$, obtained through normalization of the experimental binding energy of three-body. Figure 4 is used to determine the rms matter radii of $^4$He. The rms matter radii of $^4$He are 2.032, 2.305, and 2.395 fm. All the experimental and theoretical values of rms matter radii are tabulated in Table 3.

Based on Tanihata et al. (1992), rms matter radius of $^8$He was $2.49 \pm 0.04$ fm. This value was closed to rms matter radius at $\beta_2 = 0.35$ and 0.40, which gave 2.305 and 2.395 fm, respectively. While experimental data, $1.92 \pm 0.03$ fm was close to core deformation parameter $\beta_2 = 0.19$ giving matter radius 2.032 fm. Both experimental values of the binding energy of three-body and rms matter radius had the same core deformation parameter $\beta_2$, which was 0.12. This showed that the core of $^4$He was deformed. We assumed that $^4$He exhibits as $^4$He + 4$n$ structure. This $^4$He + 4$n$ structure is clustering in the four configurations ($\alpha$-$n$-$n$-$n$, $\alpha$-$n$-$nn$, $\alpha$-$nn$-$n$, and $\alpha$-$nn$-$nn$) in the $^8$He nucleus (Wurzer & Hofmann 1997). Among of these clusterings, $\alpha$-$nn$-$nn$ structure is the only for three-body system in $^4$He nucleus. Therefore, structure of $^8$He is $^4$He + $2n$ + $2n$ due to the core deformation parameter $\beta_2 = 0.35$, 0.40, and 0.47 was closed with four-neutron separation energy. This was supported by rms matter radius calculation from the relativistic mean field model and optical limit approximation (Ozawa et al. 2011; Tanihata et al. 1992).
The different theoretical rms matter radius with experimental was 6 to 25% and 5 to 19% differ with optical limit approximation (Angeli & Marinova 2013; Ozawa et al. 2001). Nuclear radii based on liquid drop model with $r_0 A^{1/3}$ law, cannot predict nuclear radii of halo nucleus well. In this work, we used (14) to calculate the rms matter radius of $^8$He halo nucleus due to the successful achievement on previous works (Hwash et al. 2014, 2012; Salih et al. 2018, 2017; Wang et al. 2001). Based on the theoretical data, expansion in nuclear radius for core + 2n and core + 4n was different. This showed that the density distribution of nucleon for core + 4n was larger than core + 2n. Therefore, adding four neutrons into $^4$He nucleus caused the large extension of the density distribution compared to adding two neutrons. Based on the core deformation parameter, $^4$He exhibit in prolate shape due to the positive value of $\beta_2$. In this respect, the theoretical result of $^4$He meet an agreement with experimental data, meaning a nearly closed shell for neutrons more strongly bounded compared to $^6$He (Audi et al. 2003; Pfeiffer 2012). Thus, $^4$He need more energy to remove two or four neutron valence and the radius become a bit smaller compared to $^6$He due to tightly bounded system.

**CONCLUSION**

Based on the discussion, the theoretical calculation was performed in two configurations based on Jacobi coordinates. Experimental data were used through normalization for this study especially, to calculate rms matter radius and core deformation of $^8$He. Experimental data met an agreement with the theoretical data for configuration-Y. This model showed that the core of the $^8$He is deformed and exhibited as a prolate shape (positive). Deformation has an effect on the bound state of the three-body system. The low binding energy of three-body and large rms matter radius of $^8$He nucleus have been confirmed throughout this work.

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