Numerical Methods

Linear systems of equation

Consider the following system:

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 4 & 12 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 1.2 \\ -0.2 \\ 0.8 \\ 1.5 \end{Bmatrix}$$

a. Use the Gauss elimination method to obtain the solution of x_i.
b. Calculate the determinant for the left-hand side matrix.

Non-Linear systems of equation

The relationship for friction factor f for a flow in a damping element with the Reynolds number R_e is given by:

$$\frac{1}{\sqrt{f}} = \frac{1}{k}\ln\left(R_e\sqrt{f}\right) + \left(14 - \frac{5.6}{k}\right)$$

where k is a constant for internal wall roughness for the damping element and is equal to 0.28. Calculate the value of f if $R_e = 3{,}750$.

Interpolation

The fuel consumption of an engine has been recorded as shown in the following table.

Time, hour	Fuel, liter
1.2	0.33201
1.7	0.54739
1.8	0.60496
2.0	0.73891

If a user runs the engine for 1.55 hours, determine the estimated fuel consumption using the Newton and Lagrange interpolation methods.

Curve fitting

The following data shows the height function of a hill a a distance x from a reference. Form a cubic polynomial via regression.

x_i	0	1	2	3	4	5	6	7	8
h_i	4	5	10	17	21	16	11	3	1

Also, calculate the corresponding standard deviation.

Optimization

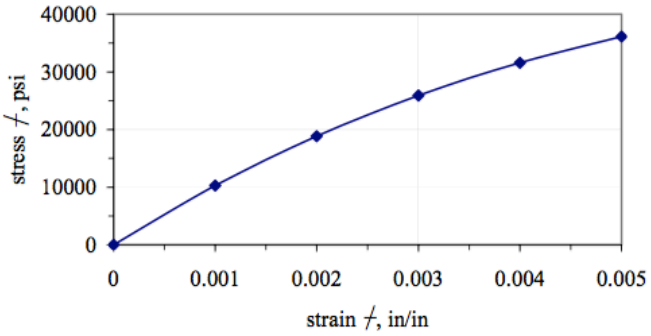
Obtain the minimum value of the following function at $x \geq 0$ using the quadratic interpolation function using the initial values of 0.1, 0.5 and 5.0, and the Newton method using the initial value of 0.5:

$$f(x) = x + \frac{1}{x}$$

Integrals

A stress-strain test has been conducted on an aircraft component and the result is tabulated as followed:

ε, in/in	σ, psi
0.000	0
0.001	10298
0.002	18852
0.003	25882
0.004	31586
0.005	36137



In this test, it is found that the component fails at the strain of 0.005 in/in. Use the trapezoidal rule and the Gauss quadrature to estimate the strain energy of the component which is required to assess the reability of the aircraft wing system. As a guidance, the curve for the test is given by:

$$\sigma = 11.2514 \times 10^6 \varepsilon e^{-88.52\varepsilon}$$

Ordinary Differential Equations (ODE)

Partial Differential Equations (PDE)