Introduction to the Boundary Element Method

Over recent decades, the boundary element method (BEM) has received much attention from researchers and has become an important technique in the computational solution of a number of physical problems. In common with the better-known finite element method (FEM)[1] and finite difference method (FDM)[2], the boundary element method is a method for solving partial differential equations (PDEs)[3] and can only be employed when the physical problem can be expressed as such. Hence the BEM development is a topic in applied mathematics[4].

As with the other methods mentioned, the boundary element method is a numerical method[5] and hence it is an important subject of research amongst the numerical analysis community. However, the potential advantages of the BEM have seemed so considerable that the strongest impetus behind its development has come from the engineering community, in its enthusiasm to obtain flexible and efficient computer-based solutions to a range of engineering problems.

The boundary element method is derived through the discretisation of an integral equation that is mathematically equivalent to the original partial differential equation. The essential re-formulation of the PDE that underlies the BEM consists of an integral equation that is defined on the boundary of the domain and an integral that relates the boundary solution to the solution at points in the domain. The former is termed a boundary integral equation (BIE) and the BEM is often referred to as the boundary integral equation method or boundary integral method. Over the last twenty years the term boundary element method has become more popular. The other terms are still used in the literature however, particularly when authors wish to refer to the overall derivation and analysis of the methods, rather than their implementation or application.

An integral equation re-formulation can only be derived for certain classes of PDE . Hence the BEM is not widely applicable when compared to the near-universal adaptability of the finite element and finite difference method. However, in the cases in which the boundary element method is applicable, it often results in a numerical method that is easier to use and more computationally efficient than the competing methods. The advantages in the boundary element method arises from the fact that only the boundary (or boundaries) of the domain of the PDE requires sub-division to produce a surface or boundary mesh[6]. (In the finite element method or finite difference method the whole domain of the PDE requires discretisation.) Thus the dimension of the problem is effectively reduced by one, for example an equation governing a three-dimensional region is transformed into one over its surface. In cases where the domain is exterior to the boundary - for example the acoustic field surrounding a loudspeaker - the extent of the domain is infinite and hence the advantages of the BEM are even more striking; the equation governing the infinite domain is reduced to an equation over the (finite) boundary.

The mathematics underlying the boundary element method is far from simple. However, in the associated website www.boundary-element-method.com the mathematics is embedded in the codes which can be downloaded. All that is needed is an appreciation of the domains and physical quantities that are being modelled.

[1] www.finite-element-method.info
[2] www.numerical-methods.com/fdm.htm
[3] www.appliedmathematics.info/pde.htm
[4] www.applied-mathematics.info
[5] www.numerical-methods.com
[6] Boundary Representation in the Boundary Element Method