techniques like rezoning. Mesh rezoning, however, is tedious and time consuming, and may introduce additional inaccuracy into the solution.

The difficulties and limitations of the grid-based methods are especially evident when simulating hydrodynamic phenomena such as *explosion* and *high velocity impact* (HVI). In the whole process of an explosion, there exist special features such as large deformations, large inhomogeneities, moving material interfaces, deformable boundaries, and free surfaces. These special features pose great challenges to numerical simulations using the grid-based methods. High velocity impact problems involve shock waves propagating through the colliding or impacting bodies that behave like fluids. Analytically, the equations of motion and a high-pressure equation of state are the key descriptors of material behavior. In HVI phenomena, there exist large deformations, moving material interfaces, deformable boundaries, and free surfaces, which are, again, very difficult for grid-based numerical methods. As can be seen from the next chapters, simulation of hydrodynamic phenomena such as explosion and HVI by methods without using a mesh is a very promising alternative.

The grid-based numerical methods are also not suitable for situations where the main concern of the object is a set of discrete physical particles rather than a continuum, e.g., the interaction of stars in astrophysics, movement of millions of atoms in an equilibrium or non-equilibrium state, dynamic behavior of protein molecules, and etc. Simulation of such discrete systems using the continuum grid-based methods may not always be a good choice.

1.3 Meshfree methods

A recent strong interest is focused on the development of the next generation of computational methods — meshfree methods, which are expected to be superior to the conventional grid-based FDM and FEM for many applications. The key idea of the meshfree methods is to provide accurate and stable numerical solutions for integral equations or PDEs with all kinds of possible boundary conditions with a set of arbitrarily distributed nodes (or particles) without using any mesh that provides the connectivity of these nodes or particles. Details on many existing meshfree methods can be found in a recent monograph by Liu (2002). One important goal of the initial research is to modify the internal structure of the grid-based FDM and FEM to become more adaptive, versatile and robust. Much effort is concentrated on problems to which the conventional FDM and FEM are difficult to apply, such as problems with free surface, deformable boundary, moving interface (for FDM), large deformation (for FEM), complex mesh generation, mesh adaptivity, and multi-scale resolution (for both FDM and FEM). Recently, a number of meshfree methods have been proposed for analyzing solids and structures as well as fluid flows. These meshfree methods share some common features, but are different in the means of function approximation and the implementation process.

Smoothed particle hydrodynamics (SPH) (Lucy, 1977; Gingold and Monaghan, 1977), as a meshfree and particle method, was originally invented for modeling astrophysical phenomena, and later widely extended for applications to problems of continuum solid and fluid mechanics. The SPH method and its different variants are the major type of particle methods, and have been incorporated into many commercial codes.

Liszka and Orkisz (1980) proposed a generalized finite difference method that can deal with arbitrary irregular grids. Nayroles et al. (1992) are the first to use moving least square approximations in a Galerkin method to formulate the so-called diffuse element method (DEM). Based on the DEM, Belytschko et al. (1994) advanced remarkably the element free Galerkin (EFG) method. The EFG is currently one of the most popular meshfree methods, and applied to many solid mechanics problems (Krysl and Belytschko, 1996a; b; Noguchi, et al. 2000; etc.) with the help of a background mesh for integration.

Atluri and Zhu (1998) have originated the Meshless Local Petrov-Galerkin (MLPG) method that requires only local background cells for the integration. Because the MLPG does not need a global background mesh for integration, it has been applied to the analysis of beam and plate structures (Atluri, et al., 1999; Gu and Liu, 2001e, Long and Atluri, 2002), fluid flows (Lin and Atluri, 2001; Wu and Liu, 2003a,b; etc.), and other mechanics problems. Detailed descriptions of the MLPG and its applications can be found in the monograph by Atluri and Shen (2000).

W. K. Liu and his co-workers (Chen and Liu, 1995, Liu et al., 1995a, b; 1996a, b, c; 1997), through revisiting the consistency and reproducing conditions in SPH, proposed a reproducing kernel particle method (RKPM) which improves the accuracy of the SPH approximation especially around the boundary. There are comprehensive literatures available on RKPM and its applications (see, W. K. Liu et al., 1996b; Li and Liu, 2002).

G. R. Liu and his colleagues in a series of papers developed the point interpolation method (PIM) and some variants (Liu 2002; Gu and Liu, 2001a, c; 2002; Liu and Gu, 1999; 2001a-d). Their struggle has been on the singularity issue in the polynomial PIMs, and different ways to solve the problem have been attempted (Liu, 2002). The use of radial basis function (or together with the polynomials) has well resolved the problem for both the local Petrov-Galerkin weak-form (Liu and Gu, 2001b) and the global Galerkin weak-form (Wang and Liu, 2000; 2001a; 2002). Recently, a meshfree weak-strong (MWS) form formulation based on a combined weak and strong forms (Liu and Gu, 2002; 2003c; 2003; Wu and Liu 2003b) has been proposed. The MWS method uses both MLS and the radial PIM shape functions, and needs only a local background mesh for nodes that is near the natural boundaries of the problem domain.

Table 1.3 Some typical meshfree methods in chronological order

Methods	**References**	**Methods of approximation**
Smoothed particle hydrodynamics (SPH)	Lucy, 1977; Gingold and Monaghan, 1977, etc.	Integral representation
Finite point method	Liszka and Orkisz, 1980; Onate et al., 1996, etc.	Finite difference representation
Diffuse element method (DEM)	Nayroles et al., 1992	Moving least square (MLS) approximation Galerkin method
Element free Galerkin (EFG) method	Belytschko et al., 1994, 1996; 1998; etc.	MLS approximation Galerkin method
Reproduced kernel particle method (RKPM)	Liu et al., 1995; 1996, etc.	Integral representation Galerkin method
HP-cloud method	Duarte and Oden, 1996, etc.	MLS approximation, Partition of unity
Free mesh method	Yagawa and Yamada, 1996; 1998, etc.	Galerkin method
Meshless local Petrov-Galerkin (MLPG) method	Atluri and Zhu, 1998; 1999; Atluri and Shen, 2002; etc.	MLS approximation Petrov-Galerkin method
Point interpolation method (PIM)	Liu and Gu, 1999; 2001a-d; Gu and Liu, 2001a,c; Liu,2002; Wang and Liu, 2000; 2001; 2002	Point interpolation, (Radial and Polynomial basis), Galerkin method, Petrov-Galerkin method
Meshfree weak-strong form (MWS)	Liu and Gu, 2002; 2003c; 2003; etc.	MLS, PIM, radial PIM (RPIM), Collocation plus Petrov-Galerkin

Other notable representatives of meshfree methods include the HP-cloud method by Duarte and Oden (1996), free mesh method (FMM) by Yagawa and Yamada (1996; 1998), the point assembly method by Liu (1999), etc. Some typical meshfree methods either in strong or weak form are listed in Table 1.3.

Comprehensive investigations on meshfree methods are closely related to the applications to complex computational solid and fluid mechanics problems. Since the computational frame in the meshfree methods is a set of arbitrarily distributed nodes rather than a system of pre-defined mesh/grid, the meshfree methods are attractive in dealing with problems that are difficult for traditional grid-based methods. The interesting applications of meshfree methods include large deformation analyses in solids (Chen et al., 1996; 1997; 1998; Jun et al., 1998; Li et al., 2000a, b; 2002; etc.), vibration analyses especially for plates and shells (Gu and Liu, 2001d,e; Liu and Gu, 2000a; Liu and Chen, 2001; Liu and Tan, 2002; L. Liu et al., 2002; Dai et al., 2002; etc.), structure buckling problems (Liu and Chen, 2002), piezoelectric structure simulations (Liu, Dai et al., 2002a, b), non-linear foundation consolidation problems (Wang et al. 2000; 2001a,b; 2002), incompressible flows (Lin and Atluri, 2001; Wu and Liu, 2003a,b; etc.), singular boundary-value problems (X. Liu et al., 2002), and impact and explosion simulation that will be discussed in the following chapters.

Meshfree methods have also been developed for boundary integral equations to develop boundary meshfree methods, in which only the boundary of the problem domain needs to be represented with nodes. The formulation developed by Mukherjee, et al. (1997a; b) was based on the formulation of EFG using the MLS approximation. Boundary point interpolation methods (BPIM) were developed by Gu and Liu using polynomial PIM and radial PIM interpolations (Gu and Liu, 2001; 2002; Liu and Gu, 2003b), which give much a set of much smaller discretized system equations due to the delta function property of the PIM shape functions.

In practical applications, a meshfree method can be coupled with other meshfree methods or a conventional numerical method to take the full advantages of each method. Examples include SPH coupling with FEM (Attaway et al., 1994; Johnson, 1994; Century Dynamics, 1997), EFG coupling with boundary element method (BEM) (Gu and Liu, 2001b; Liu and Gu, 2000b), and MPLG coupling with BEM or FEM (Liu and Gu, 2000d). A meshfree method can also be coupled with another meshfree method for particular applications (Liu and Gu, 2000c). An adaptive stress analysis package based on the meshfree technology, MFree2D©, has been developed (Liu and Tu, 2001; 2002; Liu, 2002).

There are basically three types of meshfree methods: methods based on strong form formulations, methods based on weak form formulations, and particle methods. The *strong form method* such as the *collocation method* has attractive advantages of being simple to implement, computationally efficient and "truly" meshfree, because no integration is required in establishing the discrete system equations. However, they are often unstable and less accurate, especially when irregularly distributed nodes are used for problems governed by partial differential equations with Neumann (derivative) boundary conditions, such as solid mechanics problems with stress (natural) boundary conditions. On the other hand, *weak form method* such as the EFG, MLPG and PIM has the advantages of excellent stability, accuracy. The Neumann boundary conditions can be naturally satisfied due to the use of the weak form that involves smoothing (integral) operators[3]. However, the weak form method is said not to be "truly" meshfree, as a background mesh (local or global) is required for the integration of the weak forms.

Recently, a novel meshfree weak-strong (MWS) form method is proposed by Liu and Gu (2002; 2003c; 2003) based on a combined formulation of both the strong form and the local weak form. In the MWS method, the strong form formulation is used for all the internal nodes and the nodes on the essential boundaries. The local weak form (Petrov-Galerkin weak form) is used only for nodes near the natural boundaries. Hence, there is no need for numerical integrations for all the internal nodes and the nodes on the essential boundaries. The numerical integration is performed locally only for the nodes on the natural boundaries and thus only local background cells for the nodes near the natural boundaries are required. The locally supported radial point interpolation and the moving least squares (MLS) approximation have been used to construct the meshfree shape functions for the MWS. The final system matrices were sparse and banded for computational efficiency. The MWS method is, so far, the meshfree method that uses least meshes in the entire computation and produce stable solutions even for solid mechanics problems using irregularly distributed nodes. It is one more step close to realize the dream of the "truly" meshfree method that is capable of producing stable and accurate solutions for solid mechanics problems using irregularly distributed nodes.

Some excellent reviews on the meshfree and particle methods can be found in papers by Belytschko et al. (1996), Li and Liu (2002), etc. In the monograph on meshfree methods, Liu (2002) has comprehensively addressed the history, development, theory and applications of the major existing meshfree methods. According to Liu (2002), the theories of function approximations used in the meshfree methods can be classified into three major categories: integral representation methods, series representation methods, and differential representation methods.

This book focuses on meshfree particle methods that in fact are the earliest class of meshfree methods, particularly on the smoothed particle hydrodynamics (SPH) method that uses the integral representation method for field function approximation. As mentioned by Liu (2002, page 54), the SPH method is

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[3] The integral operator is know as a smoothing operator that can smear out the errors contained in the function been operated on. (c.f., e.g., Chapter 2 of a monograph by Liu and Han (2003)).

actually very much similar to the weak form method. The difference is that the weak form operation (integral operation) is implemented in the stage of function approximation in the SPH, rather than in the stage of creating the discrete system equation as in the usual weak form method (EFG, MLPG, RKPIM, PIM, etc.). The use of the integral representation of field functions passes the differentiation operations[4] on the field function to the smoothing (weight) function. It reduces the requirement on the order of continuity of the approximated field function. Therefore, the SPH has been proven stable for arbitrarily distributed nodes for many problems with extremely large deformations. The accuracy of the solution in the SPH method depends, naturally, very much on the choice of the smoothing function (see Chapter 3).

1.4 Meshfree particle methods (MPMs)

A meshfree particle method (MPM) in general refers to the class of meshfree methods that employ a set of finite number of discrete particles to represent the state of a system and to record the movement of the system. Each particle can either be directly associated with one discrete physical object, or be generated to represent a part of the continuum problem domain. The particles can range from very small (nano or micro) scale, to meso scale, to macro scale, and even to astronomical scale. For CFD problems, each particle possesses a set of field variables such as mass, momentum, energy, position etc, and other variables (e.g., charge, vorticity, etc.) related to the specific problem. The evolution of the physical system is determined by the conservation of mass, momentum and energy. Some typical particle methods or particle-like methods are listed in Table 1.4.

Based on the *length scale*, the meshfree particle methods can be roughly divided into three classes: atomistic/microscopic scale meshfree particle methods, mesoscopic meshfree particle methods, and macroscopic meshfree particle methods.

A typical atomistic MPM is the molecular dynamics (MD) method, either *ab inito* or classic that uses force potential functions. Mesoscopic MPMs include dissipative particle dynamics (DPD) (Hoogerbrugge and Koelman, 1992; Español, 1998), lattice gas Cellular Automata (CA) (Wolfram, 1983; Kandanoff et al. 1989) etc. Macroscopic MPMs includes SPH (Lucy, 1977; Gingold and Monaghan, 1977), Particle-in-Cell (PIC) (Harlow, 1963; 1964), Marker-and-Cell (MAC) (Harlow, 1964), Fluid-in-Cell (FLIC) (Gentry et al., 1966), MPS

[4] The differential operator is know as a harshening operator that can magnify the errors contained in the function been operated on. (see, e.g., Chapter 2 of a monograph by Liu and Han (2003)).