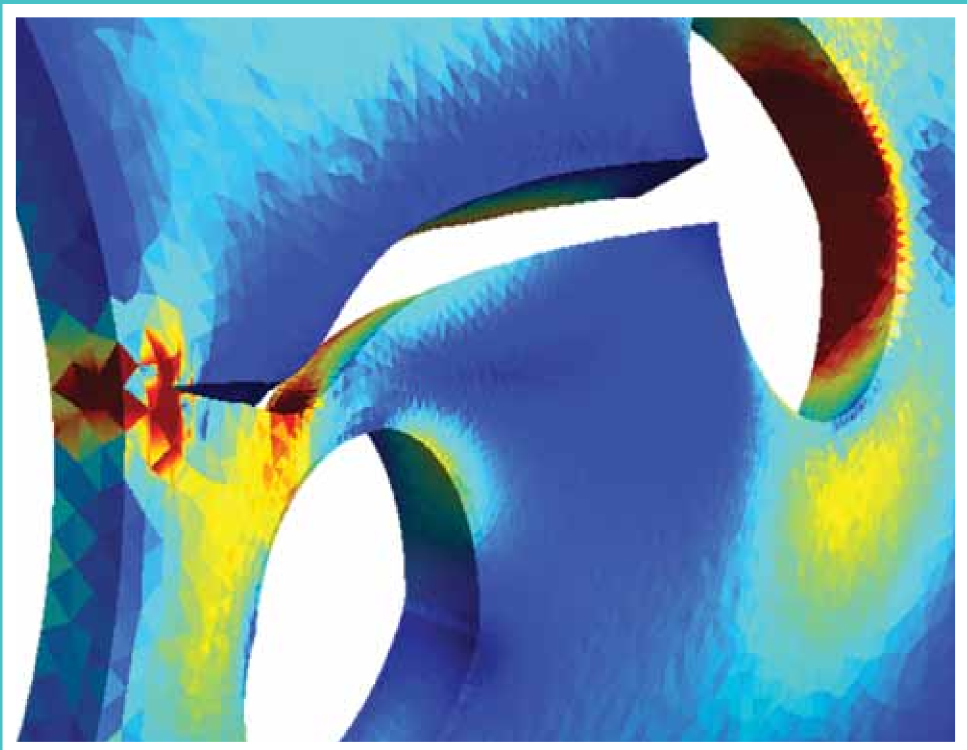
XFEM 2011

29th June - 1st July 2011, Cardiff University, UK

The 2nd International Conference on the EXtended Finite Element Method

Partition of Unity Enrichment: Recent developments and applications

ECCOMAS / IACM Special Interest Conference



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Chairmen

Institute of Mechanics and Advanced Materials
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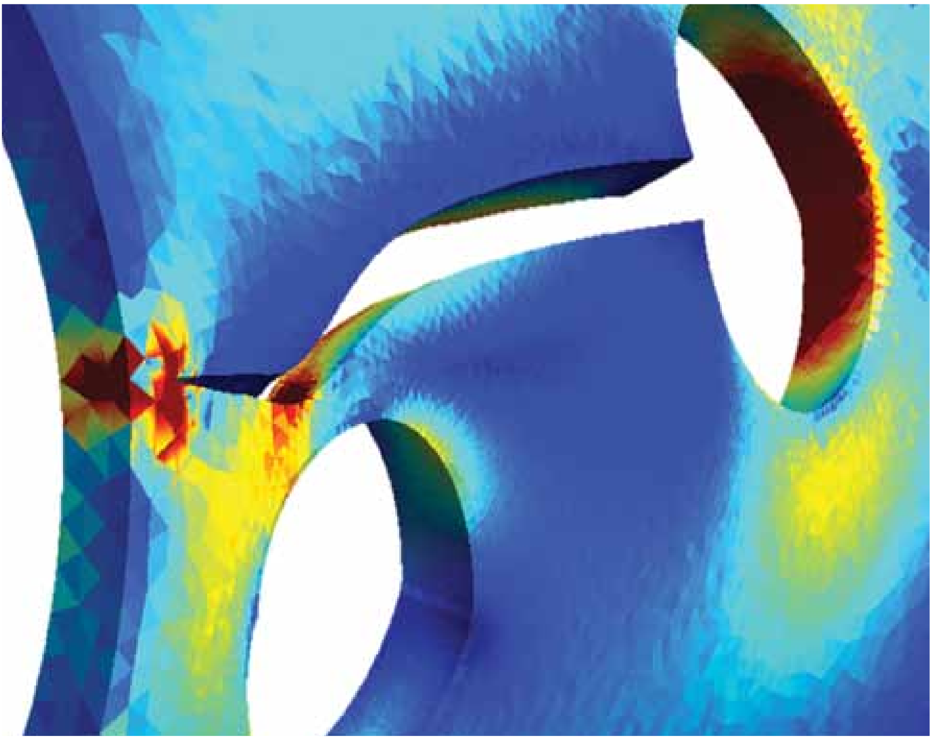

Objectives

The eXtended Finite Element Method (XFEM) along with the Generalized Finite Element Method (GFEM), hp clouds and enriched meshfree methods belong to the class of Partition of Unity Methods (PUM).This class of methods has played an increasingly important role to simulate various phenomena in Engineering and Science.

The principal feature of these methods is the ability to add, locally, a priori knowledge about the solution to the approximation space. This enrichment allows to capture particular features such as discontinuities and singularities present in the solution exactly. XFEM in particular has been used successfully to solve crack initiation and propagation problems, multi-material systems, fluid flow with boundary layers, combustion problems, fluid structure interaction, growth of hydrogels and biofilms among others, with minimal meshing and remeshing of the moving boundaries involved.

After the very successful XFEM2009 conference organized in Aachen, Germany by Prof. Thomas-Peter Fries and Prof. Andreas Zilian, it is our pleasure to have been offered to organize the second event in Cardiff and we do hope that these first two conferences are only the start of a fruitful series.

XFEM 2011 is one of the successful series of ECCOMAS thematic events. XFEM 2011 is a medium-size conference with a balanced participation covering both the theoretical aspects of the subject and engineering applications.

Conference Topics

Over the years, research has refined the initial version of the method and one can identify the following salient topics under particularly intensive study:

- Solid and Fracture Mechanics
- Structural Optimization
- Fluid-structure interaction, multi-fluid, free-surface flow
- Multi-field problems
- Mathematical aspects: preconditioners, convergence, numerical integration, application of boundary conditions on moving interfaces, blending
- A priori/posteriori error estimation, adaptive schemes, hierarchical methods
- Coupling with advanced mesh generation methods
- Multiscale problems
- Parallelization of XFEM and level set algorithms, real-time simulations
- Enriched meshfree methods
- Identification: extended digital image correlation

We would like to thank you for your participation and sincerely hope that your stay in Cardiff will be enjoyable both scientifically and personally.

Stéphane P.A. BORDAS
Pierre KERFRIDEN
Bhushan L. KARIHALOO

June 2011

Organizers and Committees

Conference chairmen

Prof. Stéphane P.A. BORDAS
Dr. Pierre KERFRIDEN
Prof. Bhushan L. KARIHALOO

Institute of Mechanics & Advanced Materials (IMAM)
School of Engineering
Cardiff University, Queen's Buildings
The Parade, CARDIFF CF24 3AA, Wales, UK.

Scientific Committee

- Prof. Olivier Allix, LMT Cachan, France
- Prof. Ted Belytschko, Northwestern University, USA (TBC)
- Prof. Dr.-Ir. Rene de Borst, Eindhoven University of Technology, The Netherlands
- Prof. Alain Combescure, INSA Lyon, France
- Prof. Armando Duarte, University of Illinois at Urbana-Champaign, USA
- Dr. Marc Duflot, CENAERO, Belgium
- Dr.-Ing. Thomas P. Fries, RWTH Aachen University, Germany
- Dr. Juan José Ródenas García, Universidad Politécnica de Valencia, Spain
- Prof. Peter Hansbo, Chalmers University of Technology, Sweden
- Prof. Patrick Laborde, Institut de Mathématiques de Toulouse, France
- Prof. Nicolas Moës, Ecole Centrale de Nantes, France (TBC)
- Prof. Brian Moran, KAUST, Saudi Arabia
- Dr Angelo Simone, TU Delft, The Netherlands
- Prof. Natarajan Sukumar, UC Davis, USA
- Prof. Jon Trevelyan, Durham University, UK
- Prof. Dr.-Ing. Andreas Zilian, Technical University of Braunschweig, Germany

The conference committee would like to dedicate this conference to the improvement of Professor Ted Belytschko's health.

Local Scientific Organizing Team

Institute of Mechanics and Advanced Materials, Theoretical and Computational Mechanics group, Cardiff University:

- Prof Stéphane P. A. Bordas, Professor, Director of Institute
- Dr Pierre Kerfriden, Lecturer
- Prof Bhushan L. Karihaloo, Professor
- Dr Sivakumar Kulasegaram, Lecturer
- Dr Octavio Andrés González Estrada, Post-doctoral research fellow
- Dr Robert Simpson, Lecturer
- PhD Students:
  - Mr Sundararajan Natarajan
  - Mr Ahmad Akbari Rahimabadi
  - Mr Olivier Goury
  - Mrs Rola Deeb
  - Mr Akbar Ghanbari
  - Mr Sultanul Md Islam
  - Mr Lian Haojie
  - Mr Chang-Kye Lee

Conference secretariat

LSM Consultancy is pleased to work with the Institute of Mechanics and Advanced Materials,Theoretical and Computational Mechanics at Cardiff University to provide conference administration services and delegate management for XFEM 2011. We would like to welcome our international guests to Wales and will be available at the registration desk throughout the event to provide any help or assistance required.

Louise Morris
Rebecca Milton
www.lsmconsultancy.com

Sponsors and Partners

Sponsors

Cenaero is an applied research center providing to any company, involved in a technology innovation process, high fidelity numerical simulation methods and tools to invent and design more competitive products. The major expertise field of the center consists in both composite manufacturing modelling and technical/economical optimization of multi-materials structures design, i.e. simulation technologies.

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Partners

- European Community on Computational Mechods in Applied Sciences (ECCOMAS)
- International Association for Computational Mechanics (IACM)
- Engineering and Physical Science Research Council
- Royal Academy of Engineering
- The Leverhulme Trust
- British Council
- European Research Council
- Framework Programme 7

Conference banquet

Thursday 30th June 2011

Cardiff Castle, Castle Street, Cardiff, CF10 3RB

www.cardiffcastle.com

(+44) 029 2087 8100

Drinks reception 19.00 - 20.00

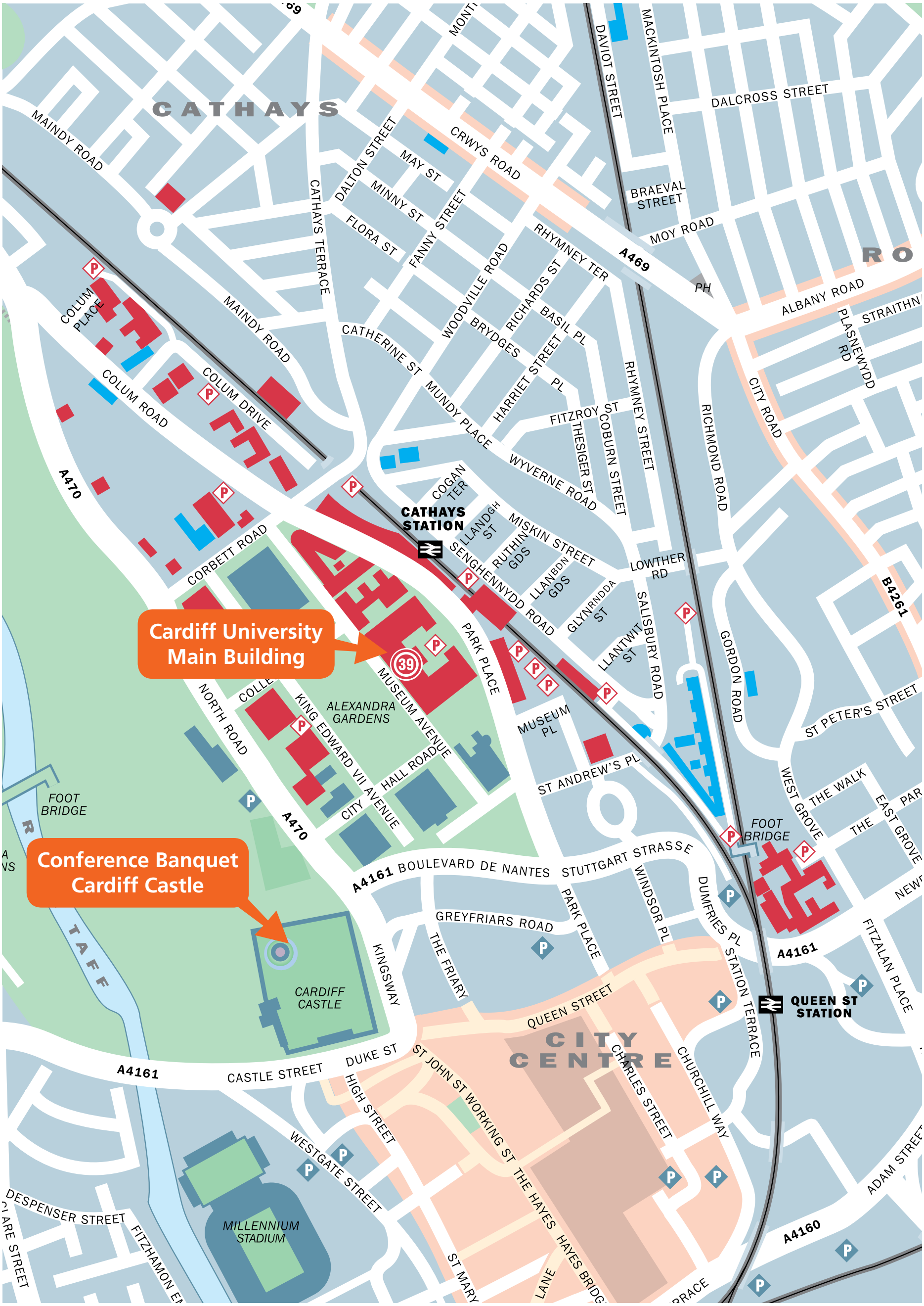
Banquet 20.00 - 22.00

Those registered for the banquet will be welcomed to a drinks reception in the Grand Library and will enjoy a sumptuous 4 course Welsh inspired menu in the magnificent Banqueting Hall at Cardiff Castle.

- Tickets will be distributed on arrival to those who have requested to attend.
- Please ensure you bring your ticket to the banquet.
- Cardiff Castle can be located on the map on page 7. Guests should enter the castle from the main entrance on Castle Street.

Directions to Main Building

Cardiff University Main Building is number 39.



Maps and Guidance

Conference venue Cardiff University,
Main Building, Museum Avenue, Cardiff, CF10 3AT

Main Building - North Wing

- Large chemistry lecture theatre (1.123) (Plenary lecture theatre)
- Large Shandon lecture theatre (-1.64)
- Small chemistry lecture theatre (1.122)

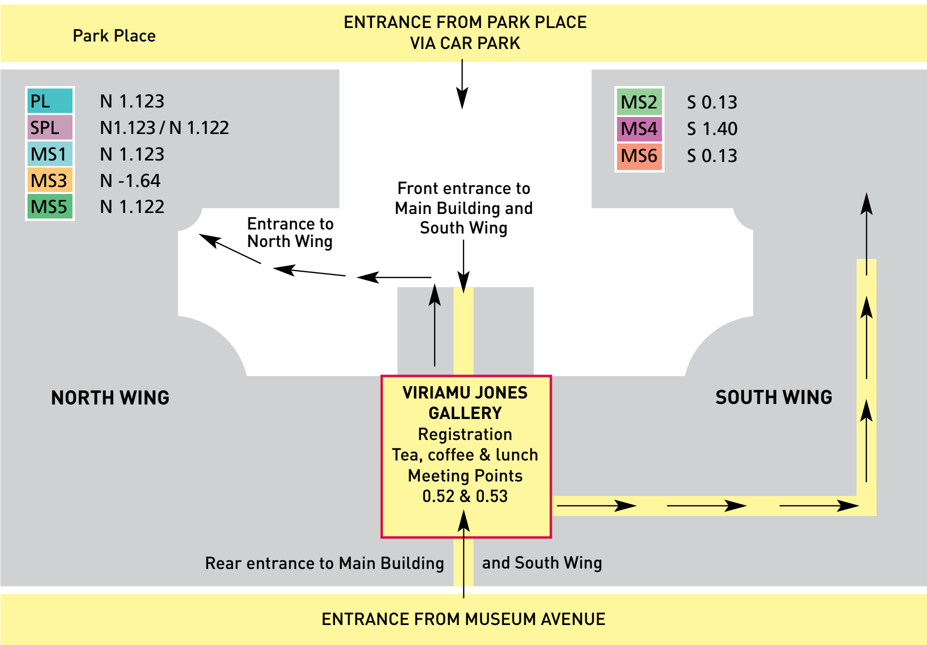
Main Building - South Wing

- Wallace lecture theatre (0.13)
- Beverton lecture theatre (1.40)

Main Building - Viriamu Jones Gallery

- Registration desk
- Tea & coffee breaks
- Lunch
- Meeting point (0.52)
- Meeting point (0.53)

Plan of Main Building



XFEM 2011 Program - Event overview

	Wednesday 29th June 2011	Thursday 30th June 2011	Friday 1st July 2011
9:00 – 9:45	Allix (PL)	Duarte (PL)	Sukumar (PL)
9:45 – 10:30	Banerjee (PL)	Moës (PL)	Gravouil (SPL) / Wells (SPL)
10:30 – 11:00	Break	Break	Break
11:00 – 11:45	Combescure (PL)	Gracie (SPL) / Rabczuk (SPL)	Khoei (SPL) / Mohammadi (SPL)
11:45 – 12:30	Prof. H Thomas Pro-Vice-Chancellor, Cardiff University	Sluys, Simone (SPL) / Löhnert (SPL)	Lunch
12:30 – 13:00	Lunch	Lunch	Lunch
13:00 – 13:30	Lunch	Lunch	Session 5
13:30 – 15:40	Session 1 MS1 MS2 MS5 MS3	Session 3 MS1 MS2 MS5 MS3	MS1 MS4 MS3 MS6 PhD Prize Competition Result
15:40 – 16:00	Break	Break	End
16:00 – 17:30	Session 2 MS1 MS4 MS5 MS6	Session 4 MS1 MS4 MS5 MS6	End

Legend:

Room #

PL	N 1.123	Plenary lecture
SPL	N 1.123 / N 1.122	Semi-Plenary lecture
MS1	N 1.123	Fracture
MS2	S 0.13	Multi Field Problems
MS3	N -1.64	Error estimation and mathematical aspects
MS4	S 1.40	3D and Large scale applications
MS5	N 1.122	PUFEM and Beyond
MS6	S 0.13	Contact, solid mechanics and other applications

Room

N 1.123	Large chemistry lecture theatre – North wing
N 1.122	Small chemistry lecture theatre – North wing
N -1.64	Large Shandon lecture theatre – North wing
S 0.13	Wallace lecture theatre – South wing
S 1.40	Beverton lecture theatre – South wing

XFEM 2011 Program

Plenary

Registration 8:30 – 9am

Wednesday, June 29, 2011
1.123 Large Chemistry Theatre - Chair: S. P. A. Bordas

Time	Speaker	Title
9:00 – 9:45	O. Allix	Multiscale parallel strategies and PUM: the two sides of the same question?
9:45 – 10:30	U. Banerjee	The Stable Generalized Finite Element Method
11:00 – 11:45	A. Combescure, T. Elguedj, D. Haboussa, F. Cazes and A. Simatos	Modeling damage brittle ductile fracture transition with combined XFEM cohesive zone
12:00 – 12:30	H. Thomas	TBC

Thursday, June 30, 2011
1.123 Large Chemistry Theatre - Chair: A. Combescure

Time	Speaker	Title
9:00 – 9:45	C. A. Duarte	Bridging scales with a generalized FEM
9:45 – 10:30	N. Moës	The thick level method (TLS) to model damage growth and transition to fracture
10:30 – 11:00		**Break**

Friday, July 1, 2011
1.123 Large Chemistry Theatre - Chair: N. Moës

Time	Speaker	Title
9:00 – 9:45	N. Sukumar	Partition-of-unity finite elements for large, accurate quantum-mechanical materials calculation

XFEM 2011 Program

Semi-plenary

Wednesday, June 29, 2011. No semi-plenary on the first day.

Thursday, June 30, 2011

Session 1: 1.123 Large chemistry lecture theatre - Chair: N. Sukumar

Time	Speaker	Title
11:00 – 11:45	R. Gracie	Adaptivity in Multiscale Simulations using XFEM
11:45 – 12:30	F. K. F. Radtke, A. Simone and L. J. Sluys	A partition of unity finite element method for simulating non-linear debonding and matrix failure in thin fibre composites and reinforced concrete

Session 2: 1.122 Small chemistry lecture theatre - Chair: C. A. Duarte

Time	Speaker	Title
11:00 – 11:45	T. Rabczuk and S. P. A. Bordas	Partition of unity enriched methods for multiscale fracture and fluid structure interaction driven brittle and ductile failure
11:45 – 12:30	S. Löhnert and M. Holl	Crack propagation and coalescence in an adaptive multiscale framework

Friday, July 1, 2011

Session 1: 1.123 Large chemistry lecture theatre - Chair: R. Gracie

Time	Speaker	Title
9:45 – 10:30	A. Gravouil, E. Pierres and M. C. Baietto	Global-local XFEM for 3D non-planar frictional crack
11:00 – 11:45	A. R. Khoei, T. Mohammadnejad and E. Haghighat	Modeling of crack propagation and fluid flow in multi-phase porous media using a modified XFEM technique

Session 2: 1.122 Small chemistry lecture theatre - Chair: S. Löhnert

Time	Speaker	Title
9:45 – 10:30	G. N. Wells and M. Nikbakht	Domain specific languages and automated code generation: application to the extended finite element method
11:00 – 11:45	S. Mohammadi	Anisotropic/orthotropic XFEM for fracture analysis of structures

XFEM 2011 Program

Wednesday, June 29, 2011

Session 1 MS1: Fracture

N 1.123 Large chemistry lecture theatre - Chair: J. Réthoré

Time	Speaker	Title
13:30 – 14:00	H. Waisman (Keynote)	An extended stiffness derivative technique to extract the strain energy release rates based on the extended finite element method
14:00 – 14:20	V. F. González-Albuixech, E. Giner, J. E. Tarancón and F. J. Fuenmayor	Efficiency of different domain energy integrals with XFEM and level sets
14:20 – 14:40	A. Moradi, S. A. Hosseini Kordkheili and S. Adibnazari	Development of extended finite element method to solve arbitrary multiple curved cracks
14:40 – 15:00	E. Benvenuti	Mesh-size objective XFEM for the simulation of a finite-width process-zone in concrete like materials
15:00 – 15:20	P. Broumand and A. R. Khoei	Modeling ductile fracture with damage plasticity using XFEM technique
15:20 – 15:40	M. R. R. Seabra and J. M. A. César de Sá	Ductile fracture approach using the XFEM

Session 1 MS2: Multi Field Problems

S 0.13 Wallace lecture theatre - Chair: T.-P. Fries

Time	Speaker	Title
13:30 – 14:00	A. Zilian (Keynote)	Modeling and enriched approximations for multi-field problems in engineering applications
14:00 – 14:20	A. Legay	Modal reduction of a vibroacoustic problem for a parametric study using XFEM
14:20 – 14:40	F. Pasenow, A. Zilian and D. Dinkler	XFEM coupling techniques for landslide-fluid interaction
14:40 – 15:00	O. Laghrouche and A. El-Kacimi	Pressure and shear plane wave decomposition in finite elements for elastic wave problems in inhomogeneous media
15:00 – 15:20	M. Kästner, C. Lux, J. Goldmann and V. Ulbricht	XFEM-analysis of stationary magnetic fields and magneto-structural coupling
15:20 – 15:40	A. Ifis, F. Bilteryst and M. Nouari	Development of a new hybrid model for thermomechanical modeling of thin layers

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Wednesday, June 29, 2011

Session 1 MS5: PUM and beyond

N 1.122 Small chemistry lecture theatre - Chair: P. M. Baiz

Time	Speaker	Title
13:30 – 14:00	Y. Shen and A. J. Lew (Keynote)	A robust discontinuous-Galerkin-based extended finite element method for fracture problems with nearly incompressible elasticity
14:00 – 14:20	H. Pathak, A. Singh and I. V. Singh	Numerical comparison of XFEM and EFGM solutions for fracture mechanics problems
14:20 – 14:40	R. Emre Erkmen	Locking-free analysis of shear deformable beams by coupling finite element and meshfree methods
14:40 – 15:00	S. Natarajan, P. Kerfriden, S. P. A. Bordas, D. Roy Mahapatra and T. Rabczuk	Enriched element free Galerkin method for gradient elasticity with application to fracture mechanics
15:00 – 15:30	H. -S. Oh, C. Davis and J. W. Jeong (Keynote)	Meshfree particle methods for thin plates
15:40 – Break		

Session 1 MS3: Error estimation and mathematical aspects

N -1.64 Large Shandon lecture theatre - Chair: U. Banerjee

Time	Speaker	Title
13:30 – 14:00	J. J. Ródenas, O. A. González-Estrada, F. J. Fuenmayor, E. Nadal and S. P. A. Bordas (Keynote)	On the use of recovery techniques for accurate error estimation and error bounding in XFEM
14:00 – 14:20	L. Berger-Vergiat, H. Waisman, B. Hiriyur, R. Tuminaro and D. Keyes	Inexact Schwarz-AMG preconditioners for crack problems modeled by XFEM
14:20 – 14:40	A. El-Kacimi, O. Laghrouche and M. S. Mohamed	PUFEM combined to wavelet based ILU preconditioners for solving short wave problems in the frequency domain
14:40 – 15:00	C. Hoppe, S. Löhnert and P. Wriggers	Recovery based error estimation for the discretization error of twoscale crack calculations
15:00 – 15:20	O. A. González-Estrada, J.J. Ródenas, E. Nadal, S. P. A. Bordas, P. Kerfriden and T. Rabczuk	Accurate evaluation of K in XFEM using error estimation in quantities of interest based on equilibrated recovery
15:40 – Break		

Wednesday, June 29, 2011

Session 2 MS1: Fracture
N 1.123 Large chemistry lecture theatre - Chair: E. Chatzi

Time	Speaker	Title
16:00 – 16:30	J. -C. Passieux, A. Gravouil, J. Réthoré and M.-C. Baietto (Keynote)	Direct estimation of generalized stress intensity factors using a multigrid XFEM
16:30 – 16:50	A. R. Daneshyar and S. Mohammadi	Simulation of strong tangential discontinuity for XFEM shear band evolution
16:50 – 17:10	Z. Shabir, L. Ponson and A. Simone	Self-affine scaling of simulated intergranular cracks in brittle polycrystals
17:10 – 17:30	S. Chakraborty, D. Roy Mahapatra, S. Natarajan and S. P. A. Bordas	Polygonal XFEM for modelling deformation of polycrystalline microstructures

Session 2 MS4: 3D, Large scale
N 1.122 Small chemistry lecture theatre - Chair: T. Nagashima

Time	Speaker	Title
16:00 – 16:30	M. Duflot (Keynote)	Industrial applications of XFEM for 3D crack propagation with MORFEO/Crack and ABAQUS
16:30 – 16:50	C. Henrard, M. Bruyneel, G. Janssen, K. Saade and A.-P. Gonze	Computational fracture mechanics using XFEM in SAMCEF: validation on a industrial test case
16:50 – 17:10	H. Minnebo and E. Wyart	Conform XFEM for automatic three dimensional crack propagation
17:10 – 17:30	A. Ghouali and A. Pyre	A numerical estimation of a fully plastic J-integral using XFEM based MORFEO software

Session 2 MS5: PUM and beyond
S 0.13 Wallace lecture theatre - Chair: S. Mohammadi

Time	Speaker	Title
16:00 – 16:30	R. N. Simpson, S. P. A. Bordas, P. Kerfriden and J. Trevelyan	Suppressing mesh generation for linear elastic fracture simulation: isogeometric analysis with boundary element methods
16:30 – 16:50	S. Sh. Ghorashi, N. Valizadeh and S. Mohammadi	Analysis of cracked orthotropic media using the extended isogeometric analysis (XIGA)
16:50 – 17:10	N. Valizadeh, S. Sh. Ghorashi, S. Mohammadi, S. Shojaee and H. Ghasemzadeh	An improved isogeometric analysis using the Lagrange multiplier method

Wednesday, June 29, 2011

Session 2 MS6: Contact, solid mechanics and applications

S 0.13 Wallace lecture theatre - Chair: S. Mohammadi

Time	Speaker	Title
16:00 – 16:30	D. Givoli, D. Rabinovich and S. Vigdergauz **(Keynote)**	XFEM applied to inverse wave problems
16:30 – 16:50	M. -C. Baietto, E. Pierres and A. Gravouil	3D fretting crack propagation analysis through a combined experimental and XFEM simulation
16:50 – 17:10	F. Lefèvre, S. Lohrengel and S. Nicaise	An extended finite element method for Maxwell's equations
17:10 – 17:30	G. Ferte and P. Massin	Cohesive zone models and path-following methods in the extended finite element formulation (XFEM)

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Thursday, June 30, 2011

Session 1 MS1: Fracture

N 1.123 Large chemistry lecture theatre - Chair: J. J. Ródenas

Time	Speaker	Title
13:30 – 14:00	Q. Z. Zhu, J. F. Shao, J. Yvonnet and Q. C. He (Keynote)	Modelling of imperfect interface effects via XFEM with developments of enrichment functions
14:00 – 14:20	S. Esna Ashari and S. Mohammadi	Debonding propagation analysis of FRP reinforced beams by the extended finite element method
14:20 – 14:40	I. V. Singh, B. K. Mishra and S. Bhattacharya	Numerical simulation of fatigue crack propagation in FGM using XFEM
14:40 – 15:00	B. Vandoren and K. De Proft	Mesoscopic modelling of masonry using embedded weak discontinuities based on partitions of unity
15:00 – 15:20	M. Silani, S. Ziaei-Rad and V. B. C. Tan	Modeling of matrix damage initiation and propagation in clay-nanocomposites using extended finite element method (XFEM)

Session 1 MS2: Multi Field Problems

S 0.13 Wallace lecture theatre - Chair: A. Zilian

Time	Speaker	Title
13:30 – 14:00	G. Meschke and D. Leonhart (Keynote)	Recent advances in XFEM based crack modeling in the context of poromechanics
14:00 – 14:20	R. Keshavarzi and S. Mohammadi	Fully coupled modeling of interaction between hydraulic and natural fractures in naturally fractured reservoirs by XFEM: The effect of pore pressure change
14:20 – 14:40	N. Watanabe, O. Kolditz, W. Wang, J. Taron and J. N. Görke	Mechanically enriched fracture elements for a hydromechanical problem in fractured rocks
14:40 – 15:00	I. Skarydova and M. Hokr	Examples of groundwater problems with discrete features and singularities
15:00 – 15:20	H. Bian, Y. Jia and J. Shao	Hydro-mechanical modelling of rock features under normal stress and fluid pressure based on XFEM
15:20 – 15:40	J. Březina	Numerical couplings for mixed-hybrid discretization of fracture flow

XFEM 2011 Program

Thursday, June 30, 2011

Session 1 MS5: PUM and beyond

N 1.122 Small chemistry lecture theatre - Chair: H. -S. Oh

Time	Speaker	Title
13:30 – 14:00	P. M. Baiz (Keynote)	Application of partition of unity concepts in the boundary element method
14:00 – 14:20	H. Lian, R. N. Simpson, S. P. A. Bordas, P. Kerfriden, J. J. Ródenas and E. Nadal	Isogeometric analysis with boundary element methods: comparison with quad-tree finite element method with implicit domain definition
14:20 – 14:40	Kh. Kunter, T. Heubrandtner, G. Trattnig, B. Mlekusch, B. Fellner and R. Pippan	Simulation of crack propagation in high strength automotive steel sheets using hybrid Trefftz method
14:40 – 15:00	A. Ahmed, F. P. van der Meer and L. J. Sluys	A geometrically non-linear, discontinuous solid-like shell element
15:00 – 15:20	D. A. F. Torres, P. T. R. Mendonça and C. S. de Barcellos	A framework for fracture modeling using implicitly defined enrichments over Ck partitions of unity simultaneously based on finite elements and mesh-free nodes
15:20 – 15:40	A. Akbari R., S. P. A. Bordas, P. Kerfriden and T. Rabczuk	Enriched Meshless Local Petrov-Galerkin (MLPG) Method to model strong discontinuities and singularities with application to cracks in functionally graded materials

Session 1 MS3: Error estimation and mathematical aspects

N -1.64 Large Shandon lecture theatre - Chair: L. Chamoin

Time	Speaker	Title
13:30 – 14:00	E. Chahine, P. Laborde and Y. Renard (Keynote)	On the mathematical analysis of XFEM and some of its variants
14:00 – 14:20	S. M. Quraishi, K. Sandeep, P. Kerfriden, S. Kulasegaram, S. P. A. Bordas and T. Rabczuk	B-spline XFEM for problems with discontinuities
14:20 – 14:40		
14:40 – 15:00	K. Shibanuma, T. Utsunomiya and S. Aihara	Correction of incompleteness of blending approximation in XFEM
15:00 – 15:20	S. Müller, M. Kästner and V. Ulbricht R.	Higher order XFEM-modelling of material interfaces and cohesive cracks
15:20 – 15:40	Chakir and P. Zunino	An analysis of extended finite elements for the approximation of large contrast problems

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Thursday, June 30, 2011

Session 2 MS1: Fracture

N 1.123 Large chemistry lecture theatre - Chair: S. Löhnert

Time	Speaker	Title
16:00 – 16:30	J. Réthoré and R. Estevez (Keynote)	Identification of cohesive zone models using XFEM and digital images
16:30 – 16:50	C. Jin, J. Shao, Q. Zhu and Q. He	XFEM modeling of interface damage in heterogeneous geomaterials
16:50 – 17:10	V. X. Tran, S. Geniaut and E. Galenne	Development and applications of XFEM axisymmetric models in fracture mechanics
17:10 – 17:30	E. Feulvarch, M. Fontaine, M. Geuffrard and J. -M. Bergheau	XFEM investigation of a crack propagation influenced by quenching residual stresses

Session 2 MS4: 3D, Large scale

S 1.40 Beverton lecture theatre - Chair: M. Duflot

Time	Speaker	Title
16:00 – 16:30	T. -P. Fries and K. -W. Cheng (Keynote)	Three-dimensional h-adaptive XFEM for two-phase incompressible flow
16:30 – 16:50	B. Prabel, A. Simatos, T. Yuritzinn and T. Charras	3D crack propagation in inelastic material
16:50 – 17:10	S. Gross	An extended velocity-pressure finite element pair for 3D incompressible liquid-liquid/liquid-gas flows
17:10 – 17:30	S. Kanaun	Numerical calculation of physical fields in homogeneous media with isolated heterogeneous inclusions

Session 2 MS5: PUM and beyond

N 1.122 Small chemistry lecture theatre - Chair: T. Rabczuk

Time	Speaker	Title
16:00 – 16:30	V. Chiaruttini, F. Feyel, J. Rannou and J. Guilie (Keynote)	An hybrid optimal approach for crack propagation: mixing conform remeshing and crack front enrichment
16:30 – 16:50	C. Davis and H. -S. Oh	Reproducing polynomial boundary particle methods for numerical solutions of boundary integral equations
16:50 – 17:10	J. S. Hale and P. M. Baiz Villafranca	Simulation of shear deformable plates using meshless maximum entropy basis functions
17:10 – 17:30	O. Goury, P. Kerfriden, L. Margetts and S. P. A. Bordas	Rationalised computational time in fracture simulation: adaptive model reduction and domain decomposition

Thursday, June 30, 2011

Session 2 MS6: Contact, solid mechanics and applications

S 0.13 Wallace lecture theatre - Chair: A. Simone

Time	Speaker	Title
16:00 – 16:20	A. Caron, P. Massin and N. Moës	Large sliding contact along discontinuities with quadratic XFEM elements
16:20 – 16:40	J. Zhang, W. H. Zhang, J. H. Zhu and L. Xia	Integrated layout design of multi-component system using XFEM
16:40 – 17:00	S. Amdouni, P. Hild, V. Lleras, M. Moakher and Y. Renard	A stabilized Lagrange multiplier method for the enriched finite-element approximation of cracked elastostatic contact problems
17:00 – 17:20	M. Parchei, S. Mohammadi and H. Zafarani	Two-dimensional dynamic extended finite element method for simulation of seismic fault rupture

XFEM 2011 Program

Friday, July 1, 2011

Session 1 MS1: Fracture

N 1.123 Large chemistry lecture theatre - Chair: J. -C. Passieux

Time	Speaker	Title
13:00 – 13:30	E. N. Chatzi, B. Hiriyur, H. Waisman and A. W. Smyth (Keynote)	Detection of structural flaws using the extended finite element method and a novel genetic algorithm
13:30 – 13:50	V. P. Nguyen, O. Lloberas-Valls, M. Stroeven and L. J. Sluys	Computational homogenization for multiscale crack modelling
13:50 – 14:10	T. Elguedj, A. Gravouil and H. Maigre	An explicit dynamics extended finite element method with standard critical time step
14:10 – 14:30	H. Bayesteh and S. Mohammadi	Crack propagation in a pressurized stiffened cylindrical shells by XFEM
14:30 – 15:00	T. Nagashima, Y. Kawamoto and H. Suemasu (Keynote)	Stress analysis of a CFRP stiffened panel with a delamination by XFEM

Session 1 MS4: 3D, Large scale

S 1.40 Beverton lecture theatre - Chair: H. Minnebo

Time	Speaker	Title
13:00 – 13:30	G. Legrain, N. Chevaugeon and K. Dréau (Keynote)	High order extended finite element method and level sets: uncoupling geometry and approximation
13:30 – 13:50	M. Moumnassi, S. Belouettar, S. P. A. Bordas, É. Béchet, D. Quoirin and M. Potier-Ferry	Accurate and optimally convergent implicit geometry definition and finite element analysis preserving sharp edges and corners from parametric primitives
13:50 – 14:10	Y. Zhang, W. Zhang and M. Shi	GPU accelerated XFEM and its application in the simulation of underground excavation
14:10 – 14:40	D. Colombo and P. Massin	A robust level set formulation for XFEM 3D mixed mode crack propagation simulation at high kink angles

Friday, July 1, 2011

Session 1 MS3: Error estimation and mathematical aspects

N -1.64 Large Shandon lecture theatre - Chair: O. A. González-Estrada

Time	Speaker	Title
13:00 – 13:30	R. Cottereau and P. Díez (Keynote)	How bad XFEM behaves at estimating fluxes close to the interface and how to improve that behavior
13:30 – 14:00	L. Chamoin (Keynote)	Robust goal-oriented error estimation in the XFEM framework
14:00 – 14:20	K. Mansouri, Y. Renard and H. Hassis	Two XFEM methods and two constitutive laws for convergence analysis within a nonlinear incompressible elasticity framework
14:20 – 14:40	A. Byfut and A. Schröder	Adaptive h- and hp- XFEM with multi-level hanging nodes via constraint approximation
14:40 – 15:00	H. Waisman	Quasi-algebraic multigrid preconditioners for fracture problems modeled by extended finite element methods

Session 1 MS6: Contact, Solid mechanics and applications

S 0.13 Wallace lecture theatre - Chair: A. Khoei

Time	Speaker	Title
13:00 – 13:20	J. J. Ródenas, E. Nadal, J. Albelda and M. Tur	A fast and accurate solver based on Cartesian grids for shape optimization problems
13:20 – 13:40	M. Siavelis, M. L. E. Guiton, P. Massin and N. Moës	XFEM with large sliding contact-friction along branching surfaces
13:40 – 14:00	C. Yongen	Study on the dynamic process of tsunami earthquake by FEM with liquid-solid coupling
14:00 – 14:20	M. Purcar, V. Topa, C. Munteanu and A . Avram	Electrode shape change simulations based on the XFEM

Guidelines for oral presentations

- All presentations are to be made in English.
- Each plenary and semi-plenary session lasts 45 minutes.
- Each keynote lecture lasts 30 minutes including time for discussion.
- Each presentation in a session lasts 20 minutes including time for discussion.

All rooms will be equiped with a PC and a projector. Authors are welcome to bring their own laptops but they should arrange to check compatibility and set up well in advance of their talks to avoid any delay to the programme.

In case Authors do not use their own equipment, it is advised that they should verify the compatibility between their presentation material and the computer provided before their talk.

Members of the local organizing team will be present in each room 15 minutes before a session starts to help the speakers of the following session set up their presentations. Members of the organizing team will be in the room at all times during the session.

PhD student competition

Papers by PhD students will be considered within the Best PhD Paper Competition organized within the conference. These will be judged on the following criteria:

1. quality of the research presented;
2. quality of the presentation material;
3. quality of the oral presentation;
4. ability to answer questions during the discussion.

Judges will be selected by the session chairs.

The three prizes are an iPad, and iPod touch and an iPod nano for the first, second and third places, respectively. Good luck!

At the beginning of their presentation, all PhD students should state their PhD-student status.

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The registration desk will be situated in the Viriamu Jones Gallery for the duration of the event. Please visit the desk if you have any queries.

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Rooms 0.52 and 0.53 will be available throughout the event for delegates to meet. Booking is not required and space is available on a first some first serve basis.

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A message board will be available during the conference for delegates to post messages or requests. This will be situated at the information desk.

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FREE wifi will available throughout the duration of the conference. All delegates will be provided with an unique wifi password and access information which will be included in delegate bags.

Lunch

Delegates who have paid to take lunch will be provided with lunch vouchers in delegate bags. Lunch will be served in the Viriamu Jones Gallery in Main Building, near to the registration area.

For those who have not opted for lunch, the main Building has a dining room and snack bar which offers a range of hot and cold meals.

The Students Union has a number of food outlets, and there are also a range of coffee shops and restaurants in the city centre.

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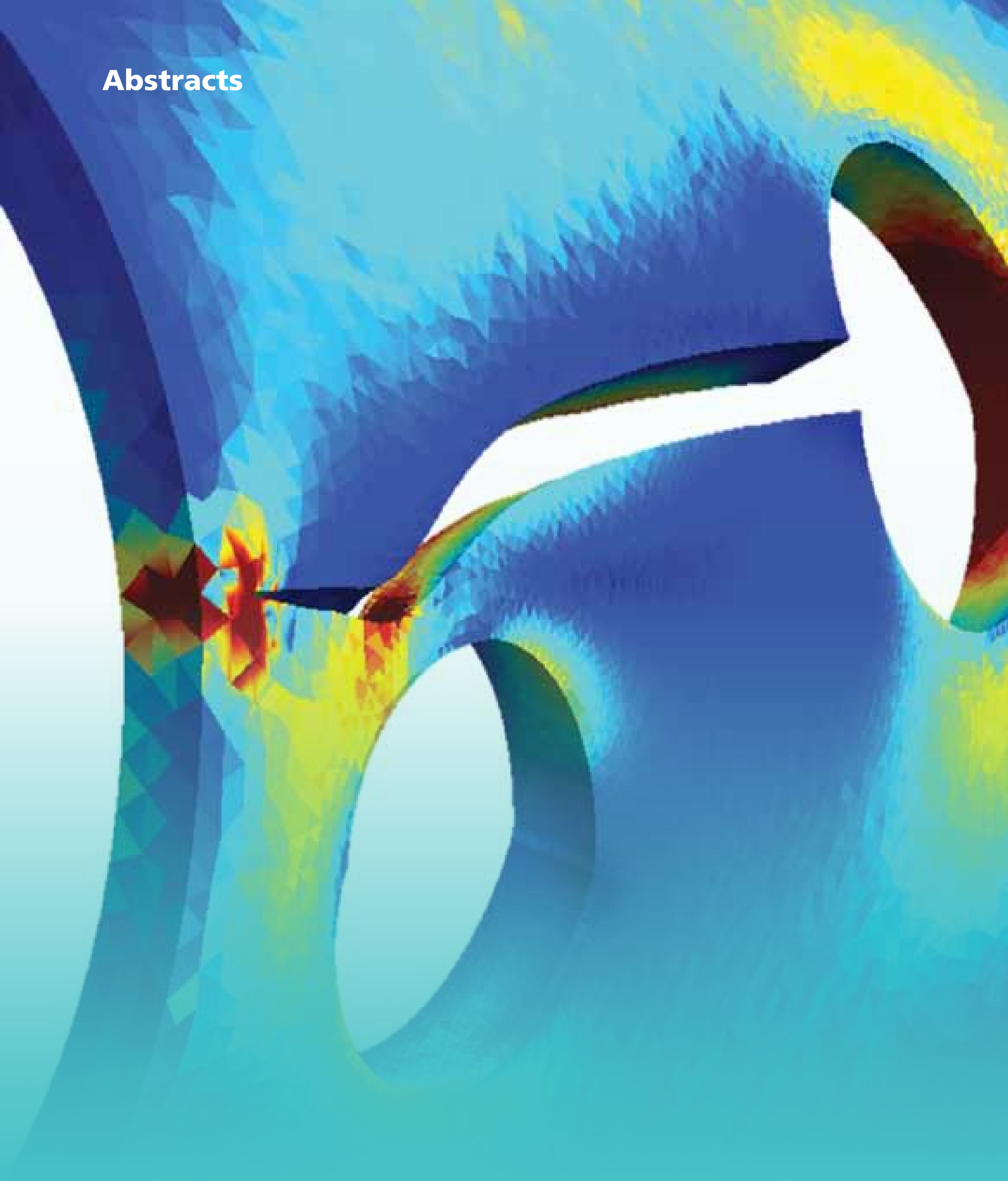
Abstracts



A Geometrically Non-linear, Discontinuous Solid-Like Shell Element

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Key Words: *Discontinuous solid-like shell element, cohesive cracking, geometrical instabilities.*

ABSTRACT

A new continuum based shell finite element for the simulation of propagating discontinuities is presented with application to geometrical instabilities. Shell structures like others are susceptible to various types of defects and damages such as initiation and propagation of cracks, global and local buckling etc. which may impair structural soundness.

The proposed discontinuous shell element is based on a solid-like shell theory [1]. The element has the same geometry as the solid/volume element but possesses shell kinematics. Finite strain kinematics is employed and the discontinuous shell behavior is achieved by incorporating discontinuities in the displacement field of the solid-like shell element. An alternative approach to XFEM, based on the Hansbo and Hansbo method [2], where an element crossed by a discontinuity is replaced by overlapping elements, is exploited.

The discontinuous shell element has only displacement degrees of freedom and provides a complete three-dimensional state of stress. The inelastic material behavior near the crack tip zone is assumed to be lumped on a single plane and is captured by a cohesive constitutive law which relates the interface tractions to the jump in the displacement field. The discontinuous solid-like shell element allows the analysis of propagating matrix cracking in thin structures independent of the finite element mesh. This avoids the need for predefining crack locations and adaptive meshing in finite element simulations as the crack grows. Additionally, the proposed model also addresses the issue of simultaneously capturing the effects of geometrical instabilities in combination with material in-elasticity.

Finally, the performance of the newly developed discontinuous solid-like shell element is elaborated by means of several numerical examples to demonstrate the potential for modeling geometrical instabilities and matrix cracking in thin laminates.

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Enriched Meshless Local Petrov-Galerkin (MLPG) Method to Model Strong Discontinuities and Singularities with Application to cracks in functionally graded materials

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Key Words: *MLPG method, enriched weight functions, fracture mechanics, functionally graded material (FGM).*

ABSTRACT

In this paper, enriched weight functions [1] are introduced in the MLPG1 method [3,5] to analyse the crack tip fields in a two dimensional functionally graded material (FGM). The mechanical properties of FGMs depend on the volume fraction of its constituent materials. The material is assumed to be graded from ceramic to metal in only one direction and the effective properties are determined by the Mori-Tanaka homogenization scheme. The diffraction criteria is used to introduce the discontinuities in the MLPG method [2,4]. A node with enriched weight function which was proposed for the Element Free Galerkin method [1] is added to each crack tip to capture the singularities. In the MLPG1, the test function is chosen as the weight function but with different size of support, so it does not require any special technique to integrate over the cracked sub-domains. The stress intensity factors are computed by the interaction integrals and compared with those available in the literature.

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MULTISCALE PARALLEL STRATEGIES AND PUM: THE TWO SIDES OF THE SAME QUESTION?

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Key Words: *enrichment, multiscale, scale separation*

ABSTRACT

Initially the PUM was introduced by Babuska and Melenk to recover the rate of convergence of FEM in the case of singularity. Since then the flexibility of the method has been used in many application such as XFEM [1], GFEM[2] and is used as a general framework to enrich locally the solution. In most of the case the underlying mesh has nevertheless to be refined where the enrichment is introduced. The reason is that no clear decoupling between the local effect that should be captured by the enrichment and the Classical Finite Element solution that is introduced. The question of scale separation / interaction is also a key aspect of multi-scale computation. In fact when reading the introduction of many papers on both subjects, it can be seen that the same type of motivations are argued. In this presentation we will try to discuss the connection and complementarity of those two class of approaches. Multiscale and parallel strategies allows to use a very fine mesh everywhere, local mesh that can be seen as an enrichment of the one that is associated with the domain decomposition that has been chosen. In all parallel strategies the efficiency, i.e, extensivity, can be obtained if a pertinent information is transmitted between the sub-domain, what we call here macro-information. This can be done like in BDD [3] or FETI [4] method by adding pertinent constraints in pre-conjugate Gradient or GMRES strategies. This can be done more explicitly in the micro-macro two-scale strategy proposed in [5-6]. Therefore PUM-based and Domain Decomposition based strategies can be seen as two complementary classes of strategies:

- the first one class aiming at enriching locally the solution in order to gain efficiency and flexibility - the second one aiming at enriching globally the solution in order to ensure extensivity, that is ensuring a number of iteration independent of the number of sub domain in the iterative procedure which has to be used to gain efficiency in parallel computation with respect to direct methods.

Therefore in our view an interesting path to follow is to combined the two class of approaches. That is what has motivated our recent work in the field of non-linear computational mechanics. Several examples will serve to discuss this idea: - The Multiscale Extended Finite Element Method (MSXFEM) [7] - The non-linear relocalization method used to deal with both geometrical [8]and material non-linearities as delamination [9] in a parallel framework - Non-Intrusive Computation [11-12]

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A stabilized Lagrange multiplier method for the enriched finite-element approximation of cracked elastostatic contact problems

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Key Words: *extended finite element method, frictionless contact, elastostatic problem, Barbosa-Hughes-Nitsche stabilisation.*

ABSTRACT

The purpose of this paper is to study the frictionless contact problem with X-FEM. An extension of the “Barbosa-Hughes-Nitsche” concept is used in order to formulate the approximation of the contact condition in linear elastostatics problem with X-FEM. Such method which provides stability of the multiplier by adding supplementary terms in the weak formulation have been originally introduced in [2].

In this work, similarly as in [1], we propose three approximations of the contact condition, we prove that the normal stress component is not singular on the contact zone for the frictionless problem and we achieve an error analysis of the stabilised formulation for each approximation of the contact problem. The Numerical results, performed with GETFEM++ on a cracked square such that we have both a contact zone and a non-contact zone, illustrate the theoretical results and show the capabilities of the method.

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S. Bordas, B. Karihaloo and P. Kerfriden

3D FRETTING CRACK PROPAGATION ANALYSIS TROUGH A COMBINED EXPERIMENTAL AND X-FEM SIMULATION.

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Key Words: *extended finite element method, crack, friction, contact, propagation, fretting*.

ABSTRACT

The numerical crack growth simulation is a large research topic which involves the understanding and the modelling of numerous local phenomena like confined plasticity or interfacial frictional contact. Contact with friction between the crack faces notably occurs in contact fatigue problems, such as fretting or rolling fatigue. These possible time-dependent, multi-axial, non proportional loadings may induce a very complex distribution of open, sliding and stick areas between the crack faces, which impact directly the propagation rate and path of the cracks. Moreover, the spatial scale of these local phenomena is usually much smaller than the scale of the crack and of the structure itself. A global-local strategy has been developed to capture accurately the key features at the different scales, namely those of the structure, the components, the two-body interface, the cracks, the interfacial non linearities. Ribeaucourt *et al.* [1] emphasized as an essential pre-requisite the need for a fine discretization of the crack interface to capture accurately the complex frictional contact conditions. The crack propagation was then adressed dealing with crack direction according to adapted criteria [2]. The crack interface discretization [3] being dependent on the underlying finite element mesh, refining the crack discretization implies refining the finite element mesh. This is in conflict with the concept of crack mesh independency inherent to X-FEM. To overcome this difficulty, a global-local strategy was proposed [4] to capture in a single framework the different scales involved. The interfacial discretization is refined independently of the mesh of the structure, by successive sub-division of the interface elements, up to matching the scale of the contact non linearities. The crack is further considered as an autonomous entity with its own discretization scheme, variables, and constitutive law. It is connected in a weak sense to the structure and a three-mixed field formulation is used. This model, called hereafter NLLA-XFEM model, is used to simulate a 3D ball/plate fretting test inducing crack initiation and propagation. Using the actual recorded experimental fretting loading cycles, the local friction coefficient is determined. The contact pressure and traction distribution at the ball/plate interface are then computed during the fretting cycle. Two cracks initiated symmetrically to the contact area center propagate whilst the others sel-arrest. They are considered for simulation according to NLLA-X-FEM modelling. Their 3D non planar crack shapes are reconstructed from metallographic observations performed on several cross sections. They are then described using the level set techniques. This is performed on several samples at different number of cycles, leading to data on crack profile evolution versus time. The sequences of frictional contact conditions at crack interfaces are determined during the fretting cycle and the stress intensity factors are computed at both crack fronts during the fretting cycle. Their evolution versus time allows a better understanding of the crack propagation.

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APPLICATION OF PARTITION OF UNITY CONCEPTS IN THE BOUNDARY ELEMENT METHOD

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Key Words: *Partition of Unity, Boundary Element Method, Drilling Rotations.*

ABSTRACT

This work presents novel applications of partition of unity (PU) enrichment within the boundary element method (BEM) [1]. Applications of partition of unity in BEM (also known as PUBEM) started within Trevelyan's group in 2002 with the work of Perrey-Debain et al [2] on short wave problems by BEM. As mentioned in their work, it is natural to ask whether special finite element shape functions which come under the generic group of PUFEM can be applied to BEM. A more recent work on PUBEM is that of Simpson and Trevelyan [3] for an enriched Dual BEM formulation for accurate evaluation of stress intensity factors in crack problems. In all reported cases, PUBEM has shown to be a powerful new method for a variety of engineering problems.

In the present work, an approach to include drilling rotations in the classical two-dimensional boundary element method will be considered. The approach is based on a simple partition of unity strategy that gives rise to a fictitious rotational degree of freedom (using the well known Allman's triangle strategy) [4]. A functional based on the rotational residual ties the average fictitious drilling rotation field to the true rotation field induced from the two-dimensional elasticity problem. The approach maintains the boundary only character of BEM and the partition of unity enrichment makes it general (totally kernel independent) and efficient (just certain areas of the boundary could be enriched). The accuracy of the proposed method is assessed with well known benchmark problems (e.g. Cook's membrane).

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The Stable Generalized Finite Element Method

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Key Words: *Stable generalized finite element method*

ABSTRACT

The Generalized Finite Element Method (GFEM) is a Partition of Unity Method (PUM), where the trial space of standard Finite Element Method (FEM) is augmented with non-polynomial shape functions with compact support. These shape functions mimic the local behavior of the unknown solution of the underlying variational problem. GFEM has been successfully used to solve a variety of problems with complicated features, e.g., crack-propagation problems, problems with microstructure etc. microstructure. However, the stiffness matrix of GFEM is badly conditioned (much worse compared to the standard FEM) and there could be a loss of accuracy in the computed solution of the associated linear system.

In this talk, we will address this issue and propose a modification of the GFEM, referred to as the Stable GFEM (SGFEM).

- We will show that the conditioning of the stiffness matrix of SGFEM is not worse than that of the standard FEM. Moreover, SGFEM retains the excellent approximation properties of the GFEM.
- We will present a general theoretical framework, where it is possible to analyze the loss of accuracy of the GFEM.
- We will also show that the "scaled condition number" of the associated stiffness matrix is the appropriate indicator of the loss of accuracy in GFEM; this idea will be validated using several computational results, obtained using different computational platforms.

These ideas will communicated through very simple examples.

Crack propagation in a pressurized stiffened cylindrical shells by XFEM

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Key Words: *CTOA, XFEM, shell crack tip enrichments, crack propagation.*

ABSTRACT

Shell and plate elements have been extensively used in modeling of tubular structures, such as natural gas transmission and pressurized pipes. The presence of cracks decreases the load-bearing capacity of a structure and may even lead to its collapse. Generally, occurrence of three types of cracks is more likely: longitudinal crack in pipes whose diameter is large, circumferential crack in pipes which bending is dominant and their combination. Since in a longitudinal crack, unstable crack can likely occur when the load reaches a critical value, lateral stiffeners are employed to prevent crack propagation in axial direction. Although the J integral is commonly used to calculate fracture parameters such as the stress intensity factor, but in pressurized thin-walled structures modeled by shell elements, this method is less efficient and problematic due to the presence of pressure on cracked elements. The alternative criterion to assess the crack propagation is the Crack Tip Opening Angle (CTOA). In addition to mesh dependency, the need for remeshing, and inability to estimate the singular stress field in fracture problems, the mesh dependency of CTOA criterion is an extra problem in the conventional FEM simulations. Alternatively, the extended finite element method (XFEM) which enriches the solution around the crack affected fields, avoids the mesh dependency and remeshing, while captures the displacement discontinuity across a crack and reduces the crack tip stress singularity near the crack tip. This also leads to accurate estimation of CTOA and substantial decrease of mesh dependency of this criterion.

In this paper, crack propagation in stiffened pressurized cylindrical shells are investigated by the CTOA criterion. For this purpose, XFEM has been extended to model crack in shells by considering crack tip enrichments in shear-deformable shell elements. Then, the effect of crack tip enrichments on estimation of CTOA and the number of required elements to model have been studied for several problems.

Mesh-size objective XFEM for the simulation of a finite-width process-zone in concrete-like materials

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Key Words: *regularized XFEM, softening, process zone.*

ABSTRACT

The present contribution shows that a proper variant of the standard eXtended Finite Element Method, called Regularized XFEM approach (Re-XFEM), makes it possible to achieve mesh-objective results in the case of structural elements made of softening materials, such as concrete, without having recourse to viscous regularization [1], or non-local internal variables [2,3]. The key ingredients are: a regularized kinematics, independent constitutive modelling of the associated emerging stress fields, and a computational strategy for the continuous/discontinuoius transition which is consistent with the possible occurrence of strain localization and crack inception and propagation. Consequently, a finite width process zone (PZ) is modeled. The width can be easily controlled and influences the ductility of the structural response. Note that the development of PZ-width-dependent cracking models is still an open issue in the current technical literature. For instance, contrarily to [2,3], Re-XFEM avoids sudden loss of stiffness at the continuous/discontinuous transition. The Re- XFEM-based approach and related elemental quadrature procedures were discussed for linear and non-linear interfaces in an elastic bulk in References [4,5], while a consistent continuous/discontinuous transition in an elasto-damaging bulk has been recently presented [6]. The implementation of Re-XFEM is straightforward within a standard XFEM code, and offers additional features with a reduced computational burden. Comparisons with commercial codes and experimental results will be presented for tensile tests, in order to show that Re-XFEM can be highly competitive with both classic continuum regularized models and existing continuous/discontinuous transition approaches[1-3].

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Inexact Schwarz-AMG preconditioners for crack problems modeled by XFEM

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Key Words: *XFEM, extended finite elements, domain decomposition, Schwarz preconditioner, fracture analysis, algebraic multigrid, smoothed aggregation multigrid.*

ABSTRACT

Traditional algebraic multigrid (AMG) preconditioners are not well suited for crack problems modeled by extended finite element methods (XFEM). This is mainly due to the unique XFEM formulations which embed discontinuous fields in the linear system by addition of special degrees of freedom. These degrees of freedom are not properly handled by the AMG coarsening process, which may lead to slow convergence. In this paper we propose a simple domain decomposition approach that retains the AMG advantages on well behaved domains by avoiding the coarsening of enriched degrees of freedom. The idea is to employ a multiplicative Schwarz preconditioner where the physical domain is partitioned into a healthy (or unfractured) and cracked subdomains. First, the healthy subdomain containing only standard degrees of freedom, is solved approximately by one AMG V-cycle, followed by concurrent direct solves of cracked subdomains. This strategy alleviates the need to redesign special AMG coarsening strategies that can handle XFEMdiscretizations. Numerical examples on various crack problems clearly illustrate the superior performance of this approach over a brute force AMG preconditioner applied to the linear system.

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Hydro-mechanical modelling of rock fractures under normal stress and fluid pressure based on XFEM

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Key Words: *XFEM, hydromechanical coupling, rock fracture.*

ABSTRACT

Generally rock masses contain various natural fractures (discontinuities) in different scales. The presence of such discontinuities constitutes the key weak point for the stability and safety of many engineering constructions, such as underground nuclear waste disposal. Indeed, in the natural state, these engineering constructions and so as the fractures undergo various coupling actions which involve the mechanical, hydraulic, chemical and eventually thermal solicitations. The permeability of fractured rock is one of the most important physical parameters in the safety and durability study of these engineering constructions. The overall permeability of the fracture rock is influenced not only by characteristic geometric parameters of fractures, but also the fluid pressure. Understanding the influence of stresses and fluid pressure on the permeability and flow pathways is very important for numerical modelling of the coupled hydro-mechanical process in fractured rocks.

Devoted to describe of the hydro-mechanical behavior of rock fractures under normal and shear stresses, many experimental studies have been carried out during the previous decades. Further investigations are still necessary for the numerical modelling of coupled hydro-mechanical behavior of fractured rocks, particularly for cases with high variation of fluid pressure. Based on the classic or modified cubic law, the classic hydro-mechanical models only take into account the fracture aperture variation due to applied stresses in the calculation of permeability. The effects of fluid pressure on fracture deformation is neglected or not properly considered. However, the experimental researches show that the fluid pressure has a significant influence on the mechanical behavior of rock fractures.

The XFEM originally proposed for solving discontinuity problems by relieving the remesh burden (Moës et al., 1999; Belystshko et al., 1999) is adopted. The purpose of this paper is to propose a new numerical model for coupled hydro-mechanical behavior of rock fracture based on the XFEM. The discussion in this paper is limited to rock fracture subjected to normal stress and pore pressure.

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Numerical couplings for mixed-hybrid discretization of fracture flow.

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Key Words: *Fracture flow, Mortar method, Mixed-hybrid finite elements*

ABSTRACT

The granite rock represents one of the suitable sites for a nuclear waste deposit. Water in the granite massive is conducted by the complex system of fractures of various sizes. In our approach the small fractures are modeled by an equivalent permeable continuum, while the preferential flow in the substantial geological dislocations and their intersections is considered as a 2D flow on corresponding manifolds and 1D flow on lines respectively.

In our contribution, we will consider two types of coupling between dimensions. The first type of 2D fracture allows jump in the 3D flow field, but keeps pressure continuous, while the second type permits a discontinuity as in the flow field as in the pressure. We will investigate possible ways how to reproduce these couplings on a mesh with a non-conforming discretization of individual dimensions. To this end we employ Mortar and XFEM finite elements. For some of the possible numerical couplings, we present numerical results and comparison to the analytical solution.

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Modeling ductile fracture with damage plasticity using X-FEM technique

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ABSTRACT

Fracture of ductile materials is the consequence of a progressive damaging process and considerable plastic deformation usually precedes the ultimate failure. The numerical prediction of damage evolution and crack propagation can be described by the means of continuum damage mechanics. In this paper, a new technique is presented to model the crack growth in ductile materials by using the damage-plasticity model in the framework of extended finite element method (X-FEM). The X-FEM has been extensively proposed in linear fracture mechanics of brittle materials and the cohesive crack problems of quasi-brittle materials [1–3]. However, there are a few research works performed on the ductile and elasto-plastic fracture problems [4]. Ductile fracture originates at a microscopic level as a result of voids initiated at inclusions in the material matrix. These microscopic degradation processes lead to degradation of the macroscopic mechanical properties which cause softening, strain localization and formation of macroscopic cracks. Thus, a continuum model which takes the softening and damage effects into account must be applied.

In this study, the X-FEM technique is proposed to model the crack propagation in ductile fracture using the damage-plasticity model. The crack is modeled as a discontinuity by the Heaviside enrichment function and the crack tip enrichment functions are employed based on the crack tip damage zone. An associated plasticity model is employed using the von-Mises yield criterion with isotropic hardening. The material degradation is modeled based on the Lemaitre damage model, which is suitable for ductile metals. The triaxiality and plastic strain are derived as the damage driving forces, and the damage-plasticity equations are solved in a decoupled manner. Special strategy is employed to alleviate the need for data transferring in crack propagation. Finally, the robustness and effectiveness of proposed computational model are verified by numerical examples.

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Adaptive h- and hp-XFEM with Multi-Level Hanging Nodes via Constraint Approximation

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Key Words: *adaptivity, higher-order, extended finite element method, multi-level hanging nodes, constraint approximation.*

ABSTRACT

In this presentation, the concept of constraint approximation for standard higher-order finite element methods is carried over to the extended finite element method (XFEM). As a consequence, meshes containing multi-level hanging nodes may be employed in the adaptive h- and hp-XFEM.

Considering the usual definition of the XFEM via the Heaviside and in particular the crack tip functions spanning the Westergaard field, one may conclude that adaptivity is generally unnecessary in the XFEM. While uniform mesh refinement is sufficient for the rather academic mode I / II benchmark problems to obtain optimal convergence rates, it is generally insufficient for a variety of other problems. This includes problems where properties of the domain such as re-entrant corners as well as properties of the boundary conditions may cause strong gradients and low regularity, which in turn may inhibit optimal convergence rates. In modern finite element schemes, h- and hp-adaptivity based on a posteriori error control is a standard tool to overcome these inhibitors. Using this adaptivity in the XFEM, the inability of the crack tip functions to model arbitrary crack tip singularities can be compensated as well, cf. [1].

For h- or hp-adaptivity, it is necessary to locally refine meshes. Whenever one mesh element is refined but at least one of its neighboring elements is not, then (multi-level) hanging nodes may occur, if they are not eleminated from the mesh using sophisticated refinement strategies. For the standard h- and hp-FEM, constraint approximation is the technique of choice to ensure the continuity of finite element shape functions associated to these hanging nodes, see also [2]. This presentation will show, how constraint approximation for standard FEMs based on Lagrange, intergrated Legendre and Gauss-Lobatto polynomials can be carried over to the XFEM, to allow for h- and hp-adaptivity with the appropriate convergence rates.

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LARGE SLIDING CONTACT ALONG DISCONTINUITIES WITH QUADRATIC X-FEM ELEMENTS

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Key Words : *X-FEM, quadratic elements, contact, large sliding*

ABSTRACT

The extended finite element method developped by Moës and al. [1] allows the modeling of crack discontinuities while avoiding the task of remeshing. The discontinuity, represented by level set functions is taken into account thanks to a local enrichment satisfying partition of unity conditions. For the modeling of a closed crack, contact friction unknows between the sides of the crack must be added to the displacement enrichment. Building on the previous work of Géniaut and al. [2] which deals with small perturbations, and of Nistor and al. [3] which deals with large displacements, we present a quadratic contact friction element based on the mixed displacement- pressure formulation for the contact friction problem derived from Ben Dhia and al. [4], including a geometrical update procedure and contact locus pairing. The interest of such a quadratic formulation [5][6] with its extension to contact friction relies on the increased accuracy of these elements with respect to linear ones as well as higher order representation of the contact surface.

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On the mathematical analysis of XFEM and some of its variants

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Key Words: *extended finite element method, error estimates, numerical analysis, integral matching.*

ABSTRACT

An increasing number of works are using XFEM methods and exploring their potential in many kind of problems. However, very few addressed their theoretical analysis. We present the mathematical tools and results required for the formulation and the analysis of some optimal XFEM variants. We show as well the theoretical analysis leading to the corresponding *a priori* error estimates.
Based on some new XFEM approaches introduced in [1], we show a first error estimate for the *cutoff XFEM* where the singular enrichment is localized by means of a cutoff function [2]. The estimate is slightly sub-optimal due to the assumed regularity of the solution. A more optimal estimate for this variant as well as the first optimal estimate for the surface enrichment XFEM are given in [3]. We introduce moreover the *integral matching XFEM* [4] where a *mortar* type bonding condition is prescribed between the singular enriched and the non-enriched areas. The mathematical formulation of this method and the main steps of its proof of convergence are presented. We show that this variant has an optimal error estimate and we describe how it improves the accuracy and the computational cost of former approaches. Other variants that use pre-computed finite element solutions as singular enrichments are also presented in [5] and [6] to deal with partially or totally unknown asymptotic behavior.

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AN ANALYSIS OF EXTENDED FINITE ELEMENTS FOR THE APPROXIMATION OF LARGE CONTRAST PROBLEMS

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Key Words: *large contrast problems, fictitious domain methods, extended finite elements, Nitsche's method, preconditioners*

ABSTRACT

We aim to approximate contrast problems by means of a numerical scheme which does not require that the computational mesh conforms with the interface of discontinuity between coefficients. We restrict to the approximation of diffusion-reaction equations in the framework of finite elements.

In order to improve the unsatisfactory behaviour of Lagrangian elements for this particular problem, we resort to an enriched approximation space, which involves sub-elements cut by the interface. For this reason, following the approach of [3], we first analyse the H^1-stability of the extended finite element space with respect to the position of the interface. This analysis, applied to the conditioning of the discrete system of equations, shows that the scheme may be ill posed for some configurations of the interface, as also recently discussed in [1]. Then, we propose a stabilisation strategy, based on a scaling technique, which restores the standard properties of a Lagrangian finite element space and results to be very easily implemented.

We finally apply the extended finite element space to the discretisation of large contrast problems with unfitted interface. As in [2], we exploit Nitsche's method for the enforcement of interface conditions at the discrete level and we discuss a choice of Nitsche's penalty terms such that the extended finite element scheme with penalty is robust for the worse case among small sub-elements and highly discontinuous coefficients.

The robustness and the simplicity of implementation of the proposed scheme motivates its extension to time dependent problems with moving interface. Perspectives and results along this direction will be discussed.

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Polygonal XFEM for modelling deformation of polycrystalline microstructures

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Key Words: *XFEM, polycrystalline material, microstructure modelling, grain boundary, graded material property, SC mapping.*

ABSTRACT

In this paper, we develop a polygonal eXtended Finite Element Method (XFEM) [1] based scheme to deal with the material property of polycrystalline microstructure including grain boundary effects and simulate deformation behaviour with minimial meshing. The presence of micro-cracks are treated using standard XFEM. A binary alloy microstructure is considered wherein the crystallographic orientation in each grain is treated using a polygonal element representating a single grain [2, 3]. The grain boundary or interface material properties are modelled by assuming graded material properties with the grain boundary density and width as parameters. The crystallographic axis for each grain are rotated according to the point group of symmetry and energy minima. The proposed scheme is expected to be well suited for investigating the propagation of cracks in polycrystalline microstructure.

For numerical integration on the polygonal domain we use an optimal scheme that involves a two-scale mapping: (i) physical frame to a regular polygon; and (ii) regular polygon to a unit disk using Schwarz-Christoffel conformal mapping [4]. The numerical integration points are optimized on the disk for minimum error in the stiffness matrix based on Frobenious norm and minimum error in infinity norm of the displacement. These optimum points for numerical integration on unenriched elements are used along with the standard triangulation scheme for enriched elements having cracks on the boundaries. Since the relative number of the enriched elements is less compared to the number of nodes that are enriched, the computational efficiency is increased without sacrificing the accuracy required to model irregular shaped polycrystalline microstructure. Numerical results will be presented on the influence of crystallographic orientation on the initiation of crack under various types of loadings.

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Robust goal-oriented error estimation in the XFEM framework

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Key Words: *error estimation, adjoint problem, strict bounds, XFEM, stress intensity factors.*

ABSTRACT

We present a method which leads to strict and high-quality local error bounds in the context of fracture mechanics. We investigate in particular the capability of this method to evaluate the discretization error for quantities of interest computed using the eXtended Finite Element Method (XFEM).

The goal-oriented error estimation method we are focusing on uses the concept of Constitutive Relation Error (CRE) along with classical extraction techniques [1]. The main innovation resides in the methodology employed to construct admissible fields in the XFEM framework, which involves enrichments with singular and level set basis functions [2]. We show that this construction can be performed through a generalization of the classical procedure used for the standard finite element method [3]. Another breakthrough is the enrichment of the adjoint solution, by means of handbook techniques. This enrichment, which depends on the output of interest, enables to optimize the quality of local error bounds in a non-intrusive manner [4].

Eventually, the resulting goal-oriented error estimation method we propose leads to relevant and very accurate information on quantities of interest which are specific to fracture mechanics, such as stress intensity factors. It also constitutes a robust tool in order to drive adaptive procedures if need be.

The technical aspects and the effectiveness of the method will be illustrated through several numerical examples.

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DETECTION OF STRUCTURAL FLAWS USING THE EXTENDED FINITE ELEMENT METHOD AND A NOVEL GENETIC ALGORITHM

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Key Words: *System Structural Health Monitoring, Extended Finite Element Method (XFEM), Genetic Algorithms (GA), Flaws, Non-destructive techniques, NDT, NDE, inverse problem*

ABSTRACT

Flaw detection in structural components is extremely important in the area of structural health monitoring. As a result, computational approaches which formulate the nondestructive inspection or evaluation (NDE) as an inverse problem, have recently gained much attention. In this approach, trial flaws are modeled using the Extended Finite Element Method (XFEM) as the forward problem solver, while Genetic Algorithms (GAs) are employed as the optimization method to converge to the true flaw location and size. Flaws are considered as straight cracks, circular/elliptical holes and non-regular shaped holes. Measurements are obtained from strain sensors which are attached to the surface of the structure at specific locations providing the target solution to the GA. The main advantage of the approach is that XFEM alleviates the need for re-meshing the domain at every new iteration of the inverse solution process and GAs have proven to be robust and efficient optimization techniques in particular for this type of problems.
Additionally, a sawtooth type GA enhanced with a new mutation scheme based on the weighted average estimate of each design parameter is suggested. The Weighted Average Mutation GA (WAM-GA) was employed to replace the conventional creep mutation in order to enhance the search process, speed up the convergence and alleviate entrapment in local optima. The performance of the proposed WAM-GA was assessed on a number of nonlinear benchmark functions with two design variables. Furthermore, numerical simulations exploring the effectiveness of the proposed methodology are presented and finally the validity of the scheme is verified trough its application on actual experimental data.

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An hybrid optimal approach for crack propagation: mixing conform remeshing and crack front enrichment

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Key Words: *hybrid X-FEM, conform remeshing, geometric enrichment, optimal convergence.*

ABSTRACT

In order to address a wide class of industrial fracture problems, robust and efficient strategies are particularly welcomed specially when uncracked structure computations are already highly time consuming. In order to tackle such problems, for the last ten years a lot of energy has been focused in the development of computational methods based on the PUM such as the X-FEM [1]. On the one hand, such an approach hexibits many advantages: no need for conform representation of the crack surface, optimal convergence using geometric enrichment, efficient propagation using level set, etc. On the other hand, X-FEM faces some drawbacks: need for a specific and expensive integration rule, required remeshing process in the crack vicinity for long path propagations, difficulties to take into account crack lips contact efficiently, etc.

Improvements in robustness of 3D remeshing tools [2] and techniques to generate 3D cracked structure meshes [3] have recently been made. Thus conform crack description approach now appear to be a mature alternative method for 3D crack growth simulations. In order, to insure a better converge rate in such a context, an hybrid formulation that mixes crack tip functions enrichment with conform representation of the discontinuity has been developed. Some numerical assessments have been carried out and will be presented for 2D and 3D cracked structures.

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A ROBUST LEVEL SET FORMULATION FOR X-FEM 3D MIXED MODE CRACK PROPAGATION SIMULATION AT HIGH KINK ANGLES

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Key Words: *extended finite element method, level set, upwind scheme, domain localisation, kink.*

ABSTRACT

The level set method is commonly used in conjunction with X-FEM for the simulation of crack propagation [1,2] since, by its nature, the X-FEM doesn't embed the crack geometry description in the numerical model. The method allows to easily track the crack surface and the crack front shape and position by means of two orthogonal level sets. Consequently to the propagation, the new crack surface and position are determined by an update of these two level sets. The update is governed by Hamilton-Jacobi type differential equations [2]: first the level sets are updated in accordance with the crack propagation speed field given by a propagation law and then they are reinitialised to keep their signed distance property and reorthogonalised to remain ortoghonal.

In case of a 3D crack propagating in mixed mode at high kink angles, the integration of the differential equations governing the level sets update reveals to be problematic in terms of robustness and performance. In order to overcome these issues, the whole update process is first reviewed and explicit polynomial equations are proposed for the update of both level sets as an alternative to the usual differential equations. Then, for the integration of the differential equations governing the reinitialisation and reorthogonalisation, a robust 3D upwind finite difference scheme applied to an auxiliary regular grid is formulated and analysed in details. Particular attention is given to the enforcement of boundary conditions. Finally a fast localisation of the level set update domain, based exclusively on geometrical considerations, is formulated. This allows to increase the overall performance and robustness of the update process. Some concluding examples show that the resulting modified approach is able to easily and fast simulate 3D mixed mode propagation with kink angles up to 90 degrees.

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Modeling damage brittle ductile fracture transition with combined X-FEM cohesive zone .

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Key Words: *extended finite element method, damage, cohesive zones, ductile fracture.*

ABSTRACT

This presentation is devoted to show new results of X-FEM methods on two important points for practical applications.
The first point is a simple and efficient way to model damage cracking transition. The case of a micro structurally long crack will be presented. A strategy to switch from a localizing damage model to an energy equivalent cohesive zone model shall be presented. This cohesive law is then inserted in an X-FEM framework. Examples of application to crack propagation in ductile regime shall be presented and compared to experimental results.
The second one is a new propagation law for ductile fracture. An analytical model for fragile ductile transition during propagation shall be developed and its application to 2D experiment interpretation shall be presented.

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HOW BAD XFEM BEHAVES AT ESTIMATING FLUXES CLOSE TO THE INTERFACE AND HOW TO IMPROVE THAT BEHAVIOR

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Key Words: *extended finite element method, error estimation, erosion.*

ABSTRACT

The eXtended Finite Element (XFE) method [1] has been developed to account implicitly for physical interfaces in Finite Element (FE) problems. In this method, the interface is represented by a level-set, the mesh does not necessarily match the interface, and enriched functional bases are used on the elements that are cut by the interface. For a small additional computational cost, and limited complexity, impressive improvements in the accuracy (in terms of global energy norm) can be observed with respect to FE results. We note here that the improvements are not so clear in terms of fluxes close to the interface [2,4], while this quantity of interest is oftentimes the main issue, in particular because it generally drives the evolution of the interface. In erosion problems [3,4] for example, the interface marks the limit between the solid phase and the eroding fluid, and the shear force exerted by the fluid on the solid (related to the fluxes) is what controls the erosion process. An improvement to the classical XFE formulation is also proposed [2,4], that ensures that fluxes are equally well evaluated. The results discussed here are particularly relevant when the material parameters of the two phases are very different.

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Simulation of Strong Tangential Discontinuity for XFEM Shear Band Evolution

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Key Words: *extended finite element method, shear bands, strain localization.*

ABSTRACT

The shear band is a thin region of high sheared materials, which is developed during severe plastic deformation of solids. Once a shear band takes place, the region within the band undergoes considerable large plastic shear straining which leads to fracture. The outside region, however, behaves in an almost rigid manner. As a result, a clear slipping of two sides of this region in the opposite directions can be observed.

Numerical simulations of shear bands with conventional finite element formulation are fraught with serious difficulties. Shear bands cannot be accurately captured because of the directional dependency of finite element approximation, so the band is forced to follow predefined directions such as the element edges or diagonals. Furthermore, the width of the shear band is far smaller than the characteristic dimensions of the structure, so it decreases with the mesh refinement. Therefore, the classical finite element method suffers from severe mesh dependency in the analysis of shear bands.

This paper presents a strong tangential discontinuity formulation for shear band formation and propagation in the formwork of the extended finite element method (XFEM). Shear banding is associated with material instability which can occur in a plane with nearly zero plastic moduli. As a result, the loss of stability of the boundary value problem is used as a criterion for shear band initiation and its direction. This paper introduces a new enrichment function for the shear band by adding the tangential extra degree of freedom to the classical finite element nodal displacements. This enables the elements to deform freely along the band direction. A proper traction-separation law is used to capture the real phenomena, This helps the model to simulate the dissipated energy within the band in the plastic deformation procedure and reproduces the correct fields in its vicinity. Finally, examples of shear band development are presented to demonstrate the capabilities of the proposed method.

Reproducing Polynomial Boundary Particle Methods for Numerical Solutions of Boundary Integral Equations

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Key Words: *Meshless methods, Boundary Integral Equations (BIE), The Closed form reproducing polynomial particle(RPP) shape functions; Boundary Element Method (BEM); Moving Least Squares Method; Reproducing Singularity particle shape functions; Boundary Node Method (BNM)*

ABSTRACT

Meshless methods have been developed to alleviate the difficulties arising in the conventional Finite Element Method (FEM)[1]. Many people have applied meshless methods to the Boundary Element Method (BEM) due to the added benefit of a decrease in dimensionality of the problem. However, most of these methods involve using approximation functions, such as Moving Least Squares (MLS)[2], which lack the Kronecker delta property causing much difficulty in prescribing essential boundary conditions. Recently, in order to strengthen the effectiveness of meshless methods, Oh et al. Developed meshfree Reproducing Polynomial Particle (RPP) shape functions and patchwise RPP and Reproducing Singularity Particle (RSP) shape functions with use of a flat-top partition of unity[3]. All of these approximation functions satisfy the Kronecker delta property.

In this talk, I will show that meshfree RPP, patchwise RPP, and patchwise RSP shape functions effectively handle Boundary Integral Equations (BIE) with (or without) domain singularities when used in the context of BEM. There are many differences between two dimensional and three dimensional problems in practice, such as singular integration and mapping techniques to approximate the singularity. Theories and numerical examples will be presented for problems without singularities and problems with singularities such as the Motz problem and the Laplace problem on an L shaped domain in two dimensions and the edge singularity problem for the three dimensional L shaped domain.

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BRIDGING SCALES WITH A GENERALIZED FEM

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Key Words: *extended finite element method, generalized finite element method, multiscale, fracture.*

ABSTRACT

A major challenge in computational mechanics is the modeling of events in which sharply varying scales are present in a single system or phenomena. Predictive capability requires modeling multiscale phenomena simultaneously. The mathematical homogenization theory has been used extensively since the 1970s as a tool for analyzing multiscale response of materials. Several assumptions of this theory are not valid in critical regions of high gradients, where the macroscale fields can vary considerably and, in particular, in regions of evolving microscale damage.

The generalized FEM (GFEM) is an instance of the partition of unity method which has its origins in the works of Babuška et al. [1] and Duarte and Oden [2]. The GFEM has been successfully applied to the simulation of dynamic propagating fractures, polycrystalline and fiber-reinforced microstructures, porous materials, etc. All these applications have relied on closed-form enrichment functions that are known to approximate well the physics of the problem. However, for many classes of problems–like those with material non-linearities or involving multiscale phenomena–enrichment functions with good approximation properties are not amenable to analytical derivation.

In this talk, we present a GFEM based on the solution of interdependent global (structural) and fine-scale or local problems. The local problems focus on the resolution of fine-scale features of the solution in the vicinity of, e.g., evolving fracture process zones while the global problem addresses the macro-scale structural behavior. Fine-scale solutions are accurately solved using an *hp*-adaptive GFEM and thus the proposed method does not rely on analytical solutions. These solutions are embedded into the global solution space using the partition of unity method. The boundary conditions for the fine-scale problems are provided by the available solution at a simulation step. The solutions of these problems are used, in turn, to build the GFEM solution space for the next simulation step. The proposed methodology enables accurate modeling of problems involving multiscale phenomena on meshes with elements that are orders of magnitude larger than those required by the FEM. This leads to considerable computational savings when compared with available methods for this class of problem. Numerical examples demonstrating the approximation properties of the proposed GFEM and its computational performance are presented.

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INDUSTRIAL APPLICATIONS OF XFEM FOR 3D CRACK PROPAGATION WITH MORFEO/CRACK AND ABAQUS

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Key Words: *extended finite element method, fracture mechanics, industrial applications.*

ABSTRACT

This contribution presents a commercially available software product called *Morfeo/Crack* which implements the extended finite element method for the simulation of 3D crack propagation under high-cycle fatigue in linear elastic fracture mechanics. The emphasis is put on the connection of *Morfeo/Crack* with *Abaqus* for industrial applications.

Morfeo/Crack for Abaqus relies on the implementation of the XFEM method natively available in Abaqus since version 6.10. The functionality of Abaqus is however limited to the calculation of stationary cracks. *Morfeo/Crack for Abaqus* enhances *Abaqus* and is capable of performing crack propagations in complex geometries. The method is based on calling *Abaqus/Standard* at each propagation step. Between each step, it reads the *Abaqus* solution, recovers an richer, improved XFEM solution in a small area surrounding the crack using a tailored integration rule, accurately computes the stress intensity factors which determine the crack advance and updates the *Abaqus* input file with the new crack position. Moreover, *Morfeo/Crack for Abaqus* profits from the nice and intuitive user interface *Abaqus/CAE* since it is integrated in the latter as a plug-in for the definition of the initial crack position and the specific data for fatigue crack propagation.

Morfeo/Crack capabilities are illustrated with several industrial applications.

An explicit dynamics extended finite element method with standard critical time step

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Key Words: *explicit dynamics, mass lumping, dynamics fracture.*

ABSTRACT

We present a framework based on the eXtended Finite Element Method and explicit dynamics for the simulation of dynamic crack growth. First, a new mass lumping technique is proposed for arbitrary enrichment functions. We focus on the case of singular enrichment functions for cracks on piece-wise linear elements and on the case of free boundary and hole enrichment functions for quadrilateral and hexahedral elements. Based on a systematic study for these cases, we propose reasonable critical time step estimates. For singular enrichment, respectively hole enrichment, 50%, respectively 66%, of the standard finite element critical time step is sufficient for accurate explicit dynamics simulations.

Second, we propose a partitioned element-by-element Stable-Explicit / Explicit time integrator. The element partitioning is based on the implicit-explicit element-by-element time integrator proposed by Hughes and Liu [2]. For the non-enriched elements, we use the central difference explicit time integrator. For the enriched elements we use the explicit unconditionally stable time integrator proposed by Chang [1]. The stability of the method is demonstrated by the energy method. It shows that the overall stability is conditional and governed by the explicit group. Combining the general mass lumping formula with this time integrator, and defining features such as cracks, holes and free boundaries using enrichment allows us to use a regular mesh with standard finite element critical time step. Moreover, the use of a regular mesh, without any constraint from the enrichment on the critical time step, allows us to minimize the computational time and maximize accuracy. Comparisons with previously published numerical simulations and experiments illustrate the performance of the method.

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PUFEM combined to wavelet based ILU preconditioners for solving short wave problems in the frequency domain

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Key Words: *Finite elements, plane waves, PUFEM, scattering problem, GMRES, ILU factorization, DWT.*

ABSTRACT

This work deals with efficient preconditioned Krylov subspace iterative methods combined to the Discrete Wavelet Transform (DWT) of Daubechies family, for the numerical solution of large linear systems arising from the approximation by PUFEM of the time harmonic elastic and acoustic wave problems. At high frequency, the PUFEM performs well on coarse mesh grids and allows to drastically reduce the total number of degrees of freedom while maintaining good accuracy even for a discretization level of around 2 degrees of freedom per wavelength [1,2]. However, it leads to large systems highly indefinite and extremely ill-conditioned, with a large bandwidth growing linearly with the total number of approximating plane waves. These likely imply the failure of many standard preconditioned iterative methods to get satisfactory accuracy and convergence. Due to the block sparse form of PUFEM final matrix, the DWT is performed blockwise and the resulting linear system is solved by the restarted Generalized Minimum RESidual method (GMRES) with ILU preconditioners allowing fill-in elements. Numerical results dealing with wave scattering for Helmholtz and elastic problems show the good performance of the proposed wavelet based ILU preconditioners in improving the rate of convergence of GMRES, for high numbers of approximating plane waves and multi-wavelength sized elements.

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LOCKING-FREE ANALYSIS OF SHEAR DEFORMABLE BEAMS BY COUPLING FINITE ELEMENT AND MESHFREE METHODS

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Key Words: *shear-locking, finite element and meshfree coupling, matching field approach.*

ABSTRACT

It is well-known in numerical analysis that in the thin-beam limit of shear-deformable beams, stress oscillations and reduction in the optimal convergence rate, a phenomenon called shear-locking, may occur due to inconsistencies in the approximations of the transverse displacement and rotation fields. In order to eliminate shear-locking, so-called matching field strategy [1,2] can be adopted by using mesh-free methods in which the approximation functions of the rotation field are constraint to be identical to the derivatives of the approximation functions of the transverse displacement field. On the other hand, using finite element modelling for some portions of the structure, where finite element analysis is accurate, is computationally less expensive and thus coupling of mesh-free and finite element methods may be advantageous in some cases [3].

Recently [4,5] a meshfree and finite element coupling technique that satisfies the constraint conditions of the matching field approach was developed by complementing the incomplete bases of primitive functions that are associated with the finite element nodes, hence mixed interpolation. For both transverse displacement and rotation fields, the proposed technique produces approximation functions that satisfy the Kronecker delta property at the required nodes of the meshfree region, so that finite elements can be connected or the boundary conditions can be imposed to these nodes directly when the matching field approach is adopted to be able to eliminate the shear-locking. Examples show that the developed technique produces accurate results while the locking behaviour is completely alleviated and CPU time can be saved during the numerical analysis.

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Debonding Propagation Analysis of FRP Reinforced Beams by the Extended Finite Element Method

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Key Words: *extended finite element method, debonding, FRP, strengthened beams*.

ABSTRACT

Composite materials have been extensively employed in many kinds of engineering applications such as structural strengthening of existing structures [1]. FRP-plates are externally bonded to the tension face of a concrete or steel beam as a flexural reinforcement. Debonding failures are usually caused by high concentrations of normal and shear stresses at the end of the bonded plate.

Fracture mechanics approach based on linear elastic fracture mechanics (LEFM) is generally used for the simulation of debonding failures in FRP-strengthened structures [2]. In this category the energy release rate G or the J integral are evaluated numerically and compared with the relevant critical fracture energy.

One of the numerical methods used for fracture analysis of such problems, is the finite element method. This method requires remeshing techniques as the crack propagates. The extended finite element method enhances the conventional FEM capabilities by avoiding the requirement of mesh to conform to weak or strong discontinuities and the necessity of adaptive remeshing techniques in crack growth problems.

In this study, propagation analysis of debonding problems in FRP strengthened beams by means of recently proposed bimaterial orthotropic enrichment functions (Esna Ashari and Mohammadi [3]) is investigated within the extended finite element method. The domain interaction integral method is adopted to numerically evaluate the stress intensity factors while mixed mode loading conditions are assumed. The efficiency of this method is examined through some numerical examples.

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ACCURATE EVALUATION OF K IN XFEM USING ERROR ESTIMATION IN QUANTITIES OF INTEREST BASED ON EQUILIBRATED RECOVERY

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Key Words: *extended finite element method, error estimation, upper bounds, recovery.*

ABSTRACT

In this work we propose an *a posteriori* recovery-based error estimation procedure which considers the stress intensity factor K as the quantity of interest for extended finite element (FE) approximations. In general, error estimators in quantities of interest have been based on residual techniques, and so far, there is no available procedure which considers an equilibrated recovery technique that can be used in standard or enriched FE frameworks. The technique proposed herein is based on the enhanced superconvergent patch recovery technique presented in [1] to evaluate the recovered stress fields $\boldsymbol{\sigma}^*$ of the primal and dual problems, which are used to obtain the error estimate. To improve the quality of the recovered fields we decompose the raw stress field obtained from the finite element approximations into singular and smooth parts, and enforce the fulfilment of boundary and internal equilibrium equations. The results indicate an accurate estimation of the error in K for benchmark problems with exact solution.

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Cohesive Zone Models And Path-following Methods In The Extended Finite Element Formulation (XFEM)

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Key Words: *cohesive zone models, snap-back, path-following, instability*

ABSTRACT

Cohesive zone models account for interaction forces between two sides of a crack. The XFEM formulation proposed by [1] is a meshless method for crack modelling. The multiaxial cohesive law presented in [2] was implemented in the XFEM formulation of *Code_Aster*, a software for thermo-mechanics openly developed at EDF R&D. Cohesive-related terms have been added to the XFEM-adapted continuous formulation for frictional contact proposed by [3]. The fixed-point searching loop over contact status was disabled, so that contact could be addressed with a penalisation term in the cohesive law itself.

When subjected to cohesive forces, cracked structures exhibit strain-softening while the crack propagates. Thus, several path-following methods were adapted to XFEM to compute full response of the structure, including snap-backs and failure. [4] offers an overview of the existing methods. Three of them were re-formulated for XFEM: the imposed degree of freedom, the arc-length and the elastic prediction controls.

The results of elementary uniaxial traction and shear tests were successfully compared with the analytical solution in order to validate the implementation. A more elaborate test of a square plate under traction with a circular hole not located at the center of the plate was performed to compare FEM and XFEM methods, stress out some issues that XFEM raises and explain the way they can be addressed. The XFEM test gave the same results than its FEM equivalent, thus validating the process.

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X-FEM INVESTIGATION OF A CRACK PROPAGATION INFLUENCED BY QUENCHING RESIDUAL STRESSES

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Key Words: *quenching, residual stresses, crack trajectory.*

ABSTRACT

An application of the eXtended Finite Element Method is proposed for the simulation of a crack propagation on a quenched thick high yield stress steel plate (0.4x3x5 m). From the industrial point of view, the quenching process induce effects the role of which can be very important on the in service behavior of mechanical components. In this application, the thick plate must be sawed after quenching but a crack propagation is observed just after the beginning of the saw step due to residual stresses. The aim of this work is the numerical study of the crack trajectory after the simulation of the quenching process which provides the evolution of residual stresses at any point in the structure. For symmetry reasons, only a quarter of a cross-section has been analyzed. Considering the dimensions of the plate, the mechanical calculations have been performed under a generalized plane strain assumption. The results are compared with a remeshing technique based on the classical Finite Element Method.

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Three-dimensional h-adaptive XFEM for two-phase incompressible flow

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Key Words: *Two-phase flow, incompressible, adaptive, XFEM.*

ABSTRACT

This study develops the h-version of the Extended Finite Element Method (XFEM) [1,2] applied to the simulation of two-fluid incompressible flow in both two and three dimensions. The h-XFEM employs a multilevel adaptive mesh refinement realized via hanging nodes on 1-irregular meshes [3]. The mesh is refined in the vicinity of the interface to ensure a high resolution of the interface position and also the ability to capture steep gradients. The sign-enrichment is employed for the XFEM approximation which accurately accounts for the jump in the pressure field within the elements without mesh manipulation. The level-set method is used for the implicit representation of the interface which can readily handle topological changes. The reinitialization of the level- set is realized by solving a differential equation to steady state in order to recover the signed- distance property. For the modelling of the surface tension, the Laplace-Beltrami technique is employed which avoids the explicit computation of the curvature. The accuracy of the flow solver is demonstrated by computing the error norms in both pressure and velocity for several two- and three-dimensional test cases.

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ANALYSIS OF CRACKED ORTHOTROPIC MEDIA USING THE EXTENDED ISOGEOMETRIC ANALYSIS (XIGA)

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Key Words: orthotropic materials, crack, extended isogeometric analysis (XIGA), crack tip enrichment functions, stress intensity factor.

ABSTRACT

Isogeometric analysis (IGA), introduced by Hughes et al. [1], has attracted growing interests of many researchers in various engineering areas, but there are rare efforts at fracture problems [2]. Recently, Ghorashi et al. [3] extended the IGA to analyze stationary and propagating mixed mode cracks in isotropic materials using some concepts of the extended finite element method (XFEM). This method, dubbed as the “extended isogeometric analysis (XIGA)”, has been found to be an efficient approach in analysis of crack problems and benefits from advantages of both IGA and XFEM. The main advantage of this method is the ability of taking into consideration the existence and propagation of a crack without any explicit meshing of the crack surfaces. Moreover, superior accuracy and convergence rates are obtained.

Orthotropic materials, such as composites, have many practical applications in various engineering problems. In this paper, the orthotropic crack tip enrichment functions, proposed by Asadpoure et al. [4], are adopted in XIGA. To assess the validity and accuracy of the proposed approach, several 2D problems are analyzed and the stress intensity factors, computed by the interaction integral, are compared with those of other available numerical and (semi-) analytical methods.

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A Numerical Estimation of a Fully Plastic *J*-integral Using X-FEM Based MORFEO Software

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Key Words: *Plastic J-Integral, Extended Finite Element Method, MORFEO.*

ABSTRACT

This paper aims at developing a numerical approach to estimating a fully plastic *J*-integral by using the X-FEM based MORFEO software and Ainsworth analytical forms [1]. The latter are expressed by means of J_e, the elastic part of the elastic plastic *J*-parameter, and $A(\sigma_{ref}, \epsilon_{ref})$, a function of the reference stress, the yield stress and the reference strain.

Thus, for a given crack size, the elastic part J_e is evaluated from a straightward stress factor analyses which were performed with the linear solver of the (X-FEM) MORFEO software. Then, a reference stress function $A(\sigma_{ref}, \epsilon_{ref})$ is applied to this elastic part in order to derive J_{ep}, the plastic component of the *J*-parameter. The reference stress is computed, for the same crack size and structure, by using the nonlinear solver of MORFEO. The reference strain is obtained from the corresponding material curve.

The proposed numerical approach is illustrated by considering two analytic test cases. First, a centre-cracked (2D) plate under tensile stress field is considered. The numerical values of the *J*-parameter computed by MORFEO are in very good agreement with the analytical ones. Moreover, a comparison to the He-Hutchinson analytic expression [2] is also provided for this test case. Then, a (3D) crack problem with circumferential crack in a hollow cylinder is considered. The computed J_{ep} parameters are shown here to be more sensitive to the numerical estimation of the reference stress. Therefore, a small discrepancy between these parameters and the analytical values is noticed.

Furthermore, the J-Δa diagram is carried out to evaluate a critical crack size for the centre-cracked plate. This evaluation is made by comparing the computed J_{ep} parameters to the material J-R curve (*J*-resistance curve). The perspective of this work is to deal with large scale plasticity fracture problems of rocket engine components.

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XFEM APPLIED TO INVERSE WAVE PROBLEMS

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Key Words: *extended finite element method, inverse problems, waves, identification, crack, obstacle, flaw, NDT, time arrival, time reversal.*

ABSTRACT

One class of applications in which the eXtended Finite Element Method (XFEM) exhibits its significant power is the solution of a certain class of inverse wave problems that involve the identification of flaws or obstacles from information measured remotely. Solving the inverse problem involves the repetitive solution of a large number of forward problems, with different obstacle geometries. XFEM is particularly powerful in this context in that it allows the use of a single regular mesh for all these forward problems.

The need to identify obstacles arises in the practice of Non-Destructive Testing (NDT). Ultrasonic NDT is one such method: the tested specimen is subjected to an acoustic wave field and the reflected wave is measured and provides information on flaws contained in the specimen. The technique proposed here, which is made efficient by the use of XFEM, assists in the process of flaw identification by using *a computational model* side by side with the physical NDT system.

In the proposed method, a computational model of the specimen containing various candidate flaws is constructed, and an optimization scheme finds the "best flaw" among them. The mathematical problem to be solved is an *inverse problem*, which is known to be ill-posed. The optimization is performed using a genetic algorithm.

We apply the method to three different problems: time-harmonic identification of an inclusion, time-harmonic identification of a crack [1] and time-dependent identification of a crack [2]. We discuss the performance of the method when applied to these three problems. In addition, we present the application of XFEM in the framework of *time reversal*, a different technique that can be used for obstacle identification via the process of refocusing.

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Efficiency of different domain energy integrals with XFEM and level sets

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Key Words: *extended finite element method, domain integrals, convergence, SIF, curved cracks.*

ABSTRACT

The aim of this study is the analysis of the convergence rates achieved with different domain energy integrals and the corresponding stress intensity factors (SIF) when solving crack problems with the extended finite element method (XFEM).

Domain integrals, based on the J-integral and the interaction integral [1] are widely used for SIF extraction and provide high accurate estimations with finite element methods. There are other approaches which also use a domain integral formulation as the newly introduced neutral action integrals [2] and the application of the field decomposition method to separate the different SIF contributions [3].

The use of the XFEM and level set functions [1] simplifies the analysis of curved cracks in two dimensions and non-planar cracks in three dimensions [4]. However, it is necessary to understand the effect of the level set function definition and the auxiliary fields used in domain integrals for problems with curved cracks. Moreover, not all the domain integrals are suitable to the study of curved or non planar cracks [2, 3].

As some former works show, despite the good accuracy of domain integrals, convergence rates are not optimal and convergence to the exact solution is not always assured at least in curved cracks [5, 6]. The comparison between the different domain integrals is realized on a convergence rate basis. Our study shows the suitable application field and limitations for each domain integral considered.

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Rationalised computational time in fracture simulation: adaptive model reduction and domain decomposition

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Key Words: *Model order reduction, domain decomposition method, fracture mechanics.*

ABSTRACT

In early stage design in engineering, it is common to have to solve large mechanical problems many times, varying the input parameters. This is done to check the effect each parameter can have on the system. This can be a very time-consuming analysis. Model order reduction(MOR) is one way to reduce significantly the computational time by reducing the number of unknowns. This is done by looking for the solution in a reduced space spanned by only a few well-chosen basis vectors. It was applied for characterization of human faces back in 1987 by Sirovitch *et al.*[1]. In the context of solid mechanics involving material nonlinearities, this method performs poorly because any small perturbations of the problems parameters has the effect of changing significantly the evolution of the fracture, hence requiring a large number of global basis vectors. Ryckelynck *et al.* [2] and Kerfriden *et al.* [3] tackled this issue by performing global corrections of the basis vectors during the computation. These schemes can however be prohibitively expensive as they are based on global evaluation of the error over the domain. In this paper, we will by-pass the difficulty by coupling MOR together with domain decomposition methods (DDM). DDM are described in for example [4]. The essential idea of DDM is that the domain studied is divided into several subdomains leading to independent subdomains that can be solved independently. Using this framework, the fracture will be isolated on some subdomains, allowing us to apply MOR on all the other subdomains, which do not undergo high non-linearities. It is also a way to speed-up the computations since parallel computing can be used.

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Adaptivity in Multiscale Simulations using XFEM

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Key Words: *extended finite element method, adaptivity, dislocations, fracture.*

ABSTRACT

The Extended Finite Element Method (XFEM) has become an important tool in many areas of engineering analysis [1] and as a result will likely play a significant role in the evolving field of multiscale analysis. Two topics will be discussed in this presentation: model adaptivity and fracture.

Adaptivity within multiscale simulations is important to ensure that simulations are both accurate and computationally possible. As a case in point, we will discuss the application of adaptive algorithms in the context of XFEM-BDM simulations of dislocations, where dislocations cores are resolved by Molecular Dynamics and the slip discontinuities aways from the cores are resolved by an XFEM-based continuum model. Due to applied loads, dislocations propagate [2,3]. This necessitates an adaptive strategy where subregions of the continuum model must be refined into an atomistic model to accommodate the propagation dislocations. Furthermore, in order for the simulations to remain tractable, the subdomains vacated by the dislocations must be coarsen into continuum models.

Application of the XFEM to adaptive multiscale simulations of cracks will necessitate the answering of fundamental questions in mechanics with regards to the relationship between fracture energy, micro-cracks, and macro-cracks, such as: is fracture energy a scale dependent quantity and if so how is the fracture energy related a different scales?

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GLOBAL-LOCAL X-FEM FOR 3D NON-PLANAR FRICTIONAL CRACK

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Key Words: *global-local X-FEM, 3D frictional contact algorithm, tribological fatigue.*

ABSTRACT

Three-dimensional crack growth simulations require both an accurate geometrical modeling of the crack and front shapes and a precise quantification of interface displacement and traction fields. Tribological fatigue like rolling fatigue, fretting fatigue involve three-dimensional crack problems in which the interfacial crack behavior is mainly governed by complex sequences of contact/friction states. In this context, enriched finite element methods (coupled for instance with a level set modeling of the possible non-planar crack shape) are very well suited to model discontinuous physical behaviors independently of a given initial mesh. These enrichments avoid the mesh compatibility of the crack with the bulk, the re-meshing and the field interpolation when dealing with crack propagation modeling. However, many cases require to impose constraints on the enriched interfaces: Dirichlet boundary conditions, contact or frictional interfaces, etc... Unfortunately, imposing these constraints involves two drawbacks: On the one hand, it imposes to discretize the crack interface to address displacement and traction fields using interface elements based on bulk finite elements cut by the crack. Hence it involves a mesh dependency between the interface and the bulk. This work presents the key procedures to undertake the crack face contact problem when using X-FEM under a global-local approach. The use of the locally two-scale approach in a three field weak formulation ensures that sufficiently refined crack faces can be incorporated into the numerical models, avoiding an unaffordable refinement of the bulk mesh at the component level and thus keeping the spirit of the X-FEM. The need of the stabilization for the solution in the contact tractions is evidenced, especially for contact problems where sliding is important. For that purpose, a dedicated non-linear solver is introduced. A thorough numerical verification of the proposed methodology is presented. The combination of the three-scale X-FEM model and the non-linear solver enables the accurate resolution of the crack face frictional contact with a low computational cost and good stability properties. The application of the procedure to a 3D fretting fatigue test is then presented. The correlation with experimental testing is performed, taking into consideration the actual crack resulting from the tests by means of automated 3-D crack geometry reconstruction. The contact state evolution is presented and gives an idea of the potential of the methodology developed, which is capable of analyzing several cracks simultaneously with high accuracy while keeping a reasonable computational cost thanks to the multi-scale approach. Such an approach can also be applied to a wide range of engineering applications implying complex frictional effects on 3D crack propagation.

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An extended velocity-pressure finite element pair for 3D incompressible liquid-liquid / liquid-gas flows

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Key Words: *incompressible two-phase flow, 3D, level set, adaptivity, error analysis*

ABSTRACT

In this talk 3D incompressible flow simulations of two-phase systems (liquid/liquid or liquid/gas) with finite elements on adaptive multilevel tetrahedral grids are considered, where a level set technique is applied for interface capturing. Compared to one-phase flows, the velocity and pressure field in two-phase flow problems are smooth in the interior of each phase Ω_1, Ω_2, but undergo certain singularities at the interface Γ between the phases. E. g., the pressure has a jump across Γ due to surface tension. The velocity is continuous across Γ, but different viscosities of the fluid phases induce a kink of the velocity field at the interface. The latter is especially the case for liquid-gas systems where the viscosity ratio is rather large.

The approximation of such functions with classical finite elements (conforming or non-conforming) leads to poor $\mathcal{O}(\sqrt{h})$ convergence, if the finite element grid is not aligned to the interface (which is typically the case for interface capturing methods like level set or VOF). The application of suitable extended finite element methods (XFEM) can provide optimal approximation properties. We present a Heaviside enrichment of the pressure space [2] yielding second order convergence of the $L_2(\Omega)$ pressure error [4] and a ridge enrichment [3] of the velocity space leading to first order convergence of the $H^1(\Omega_1 \cup \Omega_2)$ velocity error. A theoretical analysis of the methods leads to Strang-type lemmata bounding the L_2 pressure and H^1 velocity errors. The convergence rates obtained for a static bubble and a flat film test case applying our 3D two-phase flow solver DROPS [1, 5] confirm the theoretical results.

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Simulation of Shear Deformable Plates using Meshless Maximum Entropy Basis Functions

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Key Words: *maximum entropy, MaxEnt, meshless methods, FSDT, Mindlin-Reissner plates, mixed-variational forms, plates, shells*

ABSTRACT

First-order Shear Deformable Plate Theory (FSDT) is widely used throughout engineering practice to simulate structures with planar dimensions much larger than their thickness. Meshless [1] methods have seen use in the literature as a method for discretising the FSDT equations and hold numerous advantages over traditional mesh based techniques [2,3]. A recent advance in the area of meshless methods are Maximum Entropy approximants (MaxEnt) [4]. MaxEnt combines many properties of various prior meshless approximants such as a *weak* Kronecker-delta property, seamless blending with Delaunay triangulations, high continuity, and convexity.

In this work MaxEnt along with other meshless approximants have been implemented in a hybrid object-oriented Python/C++/Fortran computer simulation for the simulation of static deflection, free vibration and linear buckling of FSDT plates. The relative performance and ease of implementation of each of the methods will be discussed. The causes of shear locking along with the merits of various alleviation techniques will be covered, including matching fields method [2], mixed-variational formulations and construction of higher order polynomial basis via both intrinsic and extrinsic (partition of unity) methods. Convergence results show that MaxEnt provides in most cases similar and in some cases superior behaviour to MLS and RPIM approximants when used to discretise the FSDT equations.

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Computational Fracture Mechanics using XFEM in SAMCEF: validation on an industrial test case

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Keywords: *SAMCEF, Fracture Mechanics, XFEM, Aeronautics, Slat.*

ABSTRACT

The goal of this article is to study an industrial fracture mechanics problem simulating a crack growth evolution in an aeronautical part. The computation was done using SAMCEF, a general-purpose analysis software that covers a wide range of applications, mainly in the aeronautical, automotive, and energical fields [1].

Even though SAMCEF has included XFEM capabilities since 2005, it was until recently limited to two-dimensional problems. However, since 2010, SAMCEF can be used in combination with the Morfeo/Crack XFEM plug-in developped by Cenaero that features three-dimensional crack propagation under fatigue loading [2,3].

The part studied in this article is a slat track structure submitted to an aircraft spectrum loading. This structure has been tested at the Belgian aeronautical company Sonaca. In particular, this article will focus on the study of the influence of several model parameters, e.g., the size of the initial crack, the mesh refinement, and the propagation step. Since experimental results are available for this part, detailed comparison will be performed between numerical and experimental results in order to provide the user with guidelines as to which model parameters lead to the most accurate results.

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Recovery based error estimation for the discretization error of twoscale crack calculations

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Key Words: *extended finite element method, recovery based error estimation, multiscale.*

ABSTRACT

Using the eXtended Finite Element Method, cracks can be modelled independent of the mesh. The resulting stresses especially at crack tips vary due to the mesh size which can be expressed by the computation of discretization errors. To end up in accurate results it is important to reduce the discretisation error in the domain of interest. The presented technique is based on a Zienkiewicz and Zhu [1] error estimator. To recover enhanced stresses a least sqaure fit method is used. For mesh refinement a relative error of the L_2 norm of stresses is applied to decide which elements have to be refined. Different relative error values are tested for the refinement and, regarding the error of the global energynorm, it is shown that the overall error convergences to zero. As a quantity for the quality of the error estimator the effectivity index θ is calculated. $\theta = 1$ means that the estimated error and the exact error are the same, thus values near one are desirable.

Mesh refinement is realised using hanging node strategies. Due to the enrichments of the XFEM elements here some special cases occur. Thus hanging nodes are not allowed to be enriched or to depend on enriched nodes.

Twoscale computations enable the possibility to achieve improved results in domains of interest. Therefore the domain around a crack tip is defined as a microdomain. The boundary conditions are given by a projection of the macroscale displacements onto the microscale boundary [2] and the stresses are given back from the microscale to the macroscale. The discretization error is considered in the microscale computation.

Diffenrent numerical examples for single- and multiscale computations are tested to verify the implemented error estimator.

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DEVELOPMENT OF A NEW HYBRID MODEL FOR THERMOMECHANICAL MODELING OF THIN LAYERS

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Key Words: *joints, thin layers, extended finite element method, matched asymptotic expansions.*

ABSTRACT

Mechanical structures often contain thin layers which are employed to insure sensitive functions like bonding or coating. Through the mechanical performances they provide, these techniques are strongly used in most advanced industrial fields (aircraft, aerospace, automotive, furniture, ...). Consequently there is an increasing need for both modeling approaches and computation methods in order to design and simulate the thermomechanical behavior of such heterogeneous structures.

Most of the theoretical approaches and numerical methods developed for this aim are limited due the difficulty to take into account thin layers. Indeed, the later must be modeled by using assumptions in theoretical approaches, or by meshing with a large number of finite elements to achieve accurate results by numerical methods. Enhancing a previous work [1], we describe here a new model in order to overcome these difficulties. A Matched Asymptotic Expansions based on analytical approach [2] is used to define the enrichment parameters which will be integrated in the formulation of the X-FEM numerical method [3]. This hybrid model is then used to describe the influence of joints on the thermo-elastic behavior of an axisymetric brazed assembly. Thermomechanical fields obtained are compared, on one hand, with the result given by the analytical model and on the other hand with that obtained by numerical solution under ABAQUS code. Using this new approach, the extended finite elements algorithm is simplified and highly accurate results are obtained without mesh refinement in the vicinity of the thin layer.

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XFEM modeling of interface damage in heterogeneous geomaterials

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Key Words: *extended finite element method, imperfect interface, interface damage.*

ABSTRACT

The macroscopic mechanical behavior of most geomaterials like concrete are strongly influenced by the imperfect interface properties between matrix and mineral inclusions. Experiments show that the macroscopic damage is directly related to evolution of such interfaces for instance sliding and debonding. Recently, the research on imperfect interface gains a lot of interest due to the rapid development of nanomaterials. There exist many models, which predict linear relations between the imperfect interface properties and the mechanical properties of composite. However, few models exist so far, which study the damage evolution of imperfect interfaces. Therefor there is need to develop new approaches that account for the nonlinear behavior of imperfect interface and its influence on the macroscopic mechanical behavior of heterogeneous materials.

In our paper, we focus a class of cement-based materials largely used in various constructions. We first propose a damage model for the description of progressive degradation of imperfect cohesive interfaces between the matrix and inclusions. The model is then implemented in a 3D computer code based on the extended finite element method in order to investigate the macroscopic responses of materials due to the interface damage. The concept of average strain and average stress is used and extended to geomaterials with multi inclusions. A series of numerical simulations are performed. The numerical results show that there exists a critical threshold of macroscopic stress behind which there is a significant decrease of mechanical strength in the material. Such a decrease of macroscopic strength is inherently due to the progressive damage of cohesive interfaces. Further, the numerical results coincident with the theoretical ones obtained for some specific cases. The macroscopic behaviors of materials with different size and different volume fraction of inclusion are also compared and analysed.It is showed that the the interface damage evolution is highly affected by the inclusion size and volume fraction.

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XFEM-ANALYSIS OF STATIONARY MAGNETIC FIELDS AND MAGNETO-STRUCTURAL COUPLING

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Key Words: *XFEM, weak discontinuities, magnetic fields, coupled problems.*

ABSTRACT

Over the past decade XFEM has been applied to a variety of mechanical and non-mechanical problems involving weak and strong discontinuities [1]. The authors have previously implemented XFEM to model the local heterogeneous material structure in a representative volume element [2] required in the multi-scale analysis of the mechanical material behaviour of composite materials. For an efficient modelling, an automated model generation procedure that determines the required element types, locates the material interfaces within an element domain and performs the subdivision into integration subdomains has been developed.

Motivated by the need to simulate novel composites with magnetorheologically controllable stiffness properties, the XFEM approach is now extended to the analysis of stationary magnetic fields. Analogous to the mechanical field problem a material interface represents a weak discontinuity in the field of the magnetic potential which is taken into account by the modified abs-enrichment [3].

By combining the XFEM modelling of material interfaces for the mechanical and magnetic field problem, the effect of magnetic forces on the mechanical structure can be analysed. As both the structure and its surrounding have to be meshed in order to obtain a proper solution for the magnetic field quantities, large displacements of the mechanical structure can lead to large deformations and serious distortions of the surrounding mesh. In this context XFEM can remove the limitations of standard finite element methods if the moving interface is tracked on the fixed mesh. The same applies for electro-structural coupling as outlined in [4].

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Numerical calculation of physical fields in homogeneous media with isolated heterogeneous inclusions

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Key Words: *heterogeneous media, integral equations, Gaussian approximating function, Teoplitz matrix, fast fourier transform*

ABSTRACT

The problem of calculation of physical fields in homogeneous media with heterogeneous inclusions, cracks, etc. has many important engineering applications. This problem may be reduced to the solution of volume integral equations in the regions occupied by the inclusions. But the method of volume integral equations has well known drawbacks: the matrices of the corresponding discretized problems are usually non-sparse, and calculation of the elements of these matrices involves numerical integration. In result, the numerical solution has rather high computational cost. The method proposed in this work allows reducing the computational cost of the numerical solution of many important volume integral equations of mathematical physics.

The method is based on the use of radial Gaussian approximation functions. The theory of approximation by Gaussian and other similar functions was developed in works of V. Maz'ya and G. Schmidt [1]. The actions of integral operators of many problems of mathematical physics on such functions are presented in simple analytical forms. As a result, the time of calculation of the elements of the matrix of the discretized problem is essentially reduced in comparison with the methods incorporated conventional approximating functions (e.g., polynomial splines). The method is mesh free, and only coordinates of the nodes – centres of the Gaussian functions - are initial data for carrying out the method.

Essential saving in the computational time is achieved for regular grids of approximating nodes. In this case, the matrix of the discretized system proves to have the Teoplitz structure, and the Fast Fourier Transform algorithms may be used for calculation of matrix-vector products. This essentially reduces the time of the iterative solution of the discretized problem.

In this work, the solutions of 3D-problems of elasticity for an infinite homogeneous isotropic medium containing arbitrary heterogeneous inclusions are presented. The numerical results are compared with exact solutions. The 3D-problem of scattering of electromagnetic waves on inclusions of arbitrary shapes is also considered [2]. Some specific features of the method and details of its application are indicated and discussed. It is shown that the method allows to obtain numerical solutions in a reasonable time for the grids with several millions nodes using modest modern workstations.

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Fully Coupled Modeling of Interaction Between Hydraulic and Natural fractures in Naturally Fractured Reservoirs by XFEM: The Effect of Pore Pressure Change

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Key Words: *Hydraulic Fracturing,Extended Finite Element Method,Naturally fractured reservoirs.*

ABSTRACT

Hydraulic fracturing is a common technique used to stimulate the production of oil and gas wells. Typically, fluids are injected underground at high pressures, the formations fracture, and the oil or gas flows more freely out of the formation. Hydraulic fracturing in naturally fractured reservoirs can greatly differ from hydraulic fracturing in conventional reservoirs due to interaction of induced hydraulic fracture with existing natural fractures. Natural fracture plays an important role in hydraulic fracture propagation when it interacts with natural fracture. In this study, the interaction of natural fracture and induced hydraulic fracture has been studied according to the pore pressure change. The growing hydraulic fracture may open or cross the natural fracture. Hydraulic fracture propagation involves coupling of mechanical deformation of the formation caused by the pressure inside the fracture, fluid flow within the fractures and fracture propagation. While the rock deformation is governed by linear elasticity, the fracture propagation process is modeled using the linear elastic fracture mechanics (LEFM). Coupling of fluid flow and geomechanical deformations are based on the poroelastic theory developed by Biot. In this paper, a 2D fully coupled model based on the Extended Finite Element Method (XFEM) has been developed to simulate hydraulic fracture interaction with pre-existing natural fracture in a poroelastic formation. The motivation behind applying XFEM are the desire to avoid remeshing in each step of the fracture propagation, being able to consider arbitrary varying geometry of natural fractures and the insensitivity of fracture propagation to mesh geometry. The problem is to determine the effect of pore pressure on hydraulic fracture interaction with natural fracture. This model is of great use to improve hydraulic fracturing operation in naturally fractured reservoirs.

Modeling of crack propagation and fluid flow in multi-phase porous media using a modified X-FEM technique

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ABSTRACT

Hydraulic fracturing in the partially saturated soil is of great interest in widely different fields of geotechnical engineering [1, 2]. The hydro-mechanical modeling is consisted of a multi-phase porous medium composed of a deformable solid skeleton and fluid phases filling the pore spaces of the soil. The mathematical model of multi-phase porous medium consists of the balance equations of mass and linear momentum and of the appropriate constitutive, and is based on the generalization of the Biot theory, which was extended to deal with non-linear material behavior and partially saturated conditions [3]. In this paper, modeling of crack propagation and fluid flow is presented based on the extended finite element analysis of the deforming porous medium interacting with the flow of two immiscible compressible wetting and non-wetting pore fluids. The governing equations involving coupled fluid flow and deformation processes in unsaturated soils are derived within the framework of the generalized Biot theory. The displacements of the solid phase, the pressure of the wetting phase and the capillary pressure are taken as the primary unknowns of the formulation.

In order to model the crack propagation in multi-phase porous medium, a modified X-FEM technique is used in the analysis of fluid flow in fractured porous media. The fluid exchange between the fractured media and porous medium is taken into account, and the flow equation in the cavity is assumed as the average value over the cross-section. The governing equations of the multi-phase problem are solved using the extended finite element method together with the Newmark method is used for the time integration. In order to model the crack opening and fluid flows in the X-FEM, the displacement field of solid skeleton and the normal derivative of pressure in fluid phases must be discontinuous from one face to other face of the cavity. Hence, the displacement field is enriched by the Heaviside function and the pressure fields of fluid phases are enriched by distance functions. In order to perform an appropriate local partition of unity enrichment, an improved construction of the elements is proposed in the blending area. Finally, numerical examples are presented to demonstrate the efficiency of proposed model for crack propagation in fractured porous media.

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SIMULATION OF CRACK PROPAGATION IN HIGH STRENGTH AUTOMOTIVE STEEL SHEETS USING HYBRID TREFFTZ METHOD

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Key Words: *hybrid trefftz method, crack propagation, car crash, press-hardened steel.*

ABSTRACT

Extensive crash simulations play an important role in modern vehicle development. Due to inertial effects at high strain rates and computational efficiency these simulations are done with explicit finite element solvers. To achieve stable solution and acceptable computational times the element sizes need to be kept of specific minimum element sizes. Local stress concentrations in the vicinity of geometrical inhomogeneities, like crack tips, cannot be predicted sufficiently with standard finite elements of that size.

This paper presents the Hybrid Trefftz Method as a promising approximation technique to overcome this problem. Introducing Trefftz-Elements based on trial functions, which satisfy the governing differential equations and partial boundary conditions exactly [1], a high prediction quality for the strains and stresses in the crack tip region is achieved. Using Conformal Mapping it is also possible to derive trial functions for the simulation of kinking cracks. The Trefftz-Elements can be coupled with standard finite elements via an extended variational approach [2].

In order to simulate the crack propagation in press-hardened steels, a model describing the materials fracture toughness based on the crack tip opening displacement (CTOD) is implemented. The associated fracture mechanical experiments to parameterize this model are presented. Besides a comparison of experimental and simulation results future enhancements and applications are discussed.

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PRESSURE AND SHEAR PLANE WAVE DECOMPOSITION IN FINITE ELEMENTS FOR ELASTIC WAVE PROBLEMS IN INHOMOGENEOUS MEDIA

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Key Words: *finite elements, plane wave basis, layered media, Lagrange multipliers.*

ABSTRACT

The objective of this work is to extend the ability of the multi-wavelength sized finite elements, developed earlier [1] for homogeneous elastic media, to solve elastic wave problems in layered media. This is achieved by applying the plane wave basis decomposition to the elastic wave equation in each layer, assumed to be homogenous and isotropic. The displacement field is expressed as a combination of propagating planar pressure and shear waves. At the interface between two layers, necessary continuity conditions are enforced through the use of Lagrange multipliers which are also approximated using the plane wave decomposition approach [2].

The proposed model will be validated by solving typical problems of practical interest such as those encountered in soil wave propagation and scattering. The first problem deals with elastic waves propagating in a homogeneous medium and hitting a plane free surface. The second test problem considers reflection and transmission of elastic waves at a plane interface between two elastic media. Last, wave scattering by an elastic inclusion in an infinite elastic medium will be considered.

The developed finite elements are capable of containing many wavelengths per nodal spacing and hence allow us to relax the traditional requirement consisting of around ten nodal points per wavelength. This leads to huge savings in computing effort with the mesh of the computational domain being kept unchanged for a wide range of frequencies.

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An eXtended Finite Element Method for Maxwell's equations

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Key Words: *extended finite element method, edge finite elements, Maxwell's equations, error estimates.*

ABSTRACT

Electromagnetic testing is a commonly used analysis tool in science and industry to detect cracks and other defects in conducting structures, as for example the wings of aircraft. The precise simulation of the electromagnetic field near cracks is necessary in order to develop numerical methods and algorithms for the tests.

In this talk, a new eXtended Finite Element Method is presented based on *edge finite elements* which are commonly used in the discretization of Maxwell's equations. Contrarily to classical eXtended Finite Element Methods which are based on nodal Lagrange Finite Elements, the degrees of freedom for edge finite elements are related to the *edges* (and not to the nodes) of the mesh [3].

The model problem that we are interested in, is derived from the time-harmonic Maxwell's equations and simulates the behaviour of the electric field near an emerging crack. The simulation of the electromagnetic field near geometric singularities is always challenging since the singularity is much stronger as, for example, in fracture mechanics: if r denotes the distance to the crack tip, the electromagnetic field behaves as $\mathcal{O}(r^{-1/2})$ and is thus unbounded [1]. Classical edge finite elements on a geometry fitting mesh are able to handle this situation provided the mesh is sufficiently refined near the crack tip which results in heavy computations. Here, according to the XFEM strategy, we take into account the singularity and the discontinuity of the field in the discretization space defined on a structured mesh independent of the crack geometry.

We prove error estimates and show some numerical experiments illustrating the theory and the performance of the method [2]. Finally, we will discuss the generalization of the method to other models in electromagnetic theory like the eddy current model.

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MODAL REDUCTION OF A VIBROACOUSTIC PROBLEM FOR A PARAMETRIC STUDY USING XFEM

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Key Words: *extended finite element method, vibroacoustic, modal reduction.*

ABSTRACT

Noise reduction for interior domain in transport industry has become more and more important for the last decades. During the design process, several numerical investigations are performed in order to optimize geometries and material properties leading to large time computation. The aim of this work is to be able to analyze efficiently different configurations of structures immersed in the acoustic domain and there influence on the noise level in the cavity. An important assumption in this work is that embeded structures such as seats in a plane cabin for instance, have no thickness in the acoustic domain.

The structures are immersed arbitrarily within the acoustic mesh allowing to use the same acoustic mesh during the whole parametric study. The xfem is used to take into account the structure influences in the acoustic domain by enriching the pressure by a Heaviside function [1]. Since the standard part of the approximation does not change when structures are moved, it can be reduced into its modal basis leading to a much smaller diagonal system for the standard part:

$$p(M) = \sum_{j=1}^{n} \Phi_j(M)\alpha_j + \sum_{i\in\mathcal{E}} N_i(M)H(\phi(M))A_i$$

where $\Phi_j(M)$ is the j^{th} acoustic mode of the empty cavity, n is the number of kept modes, α_j is the unknown associated to the j^{th} mode, $\mathcal{E}$ is the set of enriched nodes, $N_i(M)$ is the shape function associated to node i, $\phi(M)$ is the signed distance to the structures, H is the Heaviside function and A_i is the additional nodal unknown coming from the enrichment.

Finally, the system in the frequency domain to solve is reduced to the number of:
(i) kept modes for the acoustic domain: small diagonal system, independant of the structure positions;
(ii) enriched d.o.f.: small since only a few nodes are enriched;
(iii) structures d.o.f.: small since the structures are 2D, moreover a modal reduction can easily be applied to the structures.

The main aspect of the proposed reduction method is mainly to be able to use the same projection basis for the acoustic domain whatever the structure positions are: only the coupling terms between fluid and structures have to be computed for each configuration.

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High order eXtended Finite Element Method and Level Sets: Uncoupling geometry and approximation

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ABSTRACT

Recently, the use of high order approximation has been promoted in the X-FEM context [1, 2, 3]. These contributions were dedicated to the construction of strategies that allow to recover optimal convergence rates for moderately high order approximations (linear, quadratic and cubic). The main issue comes from the necessity of controlling geometrical approximations accuracy to maintain the optimal convergence of the approach. The objective of the present contribution is to study the application of the X-FEM to the context of p finite elements where p reach high values (typically $p = 10$). Thanks to this approach, complex geometries can be treated by means of a high order approximation defined on a coarse mesh. A strategy is proposed for uncoupling the geometrical description of the domain of interest and the approximation. This strategy, which is derived from [2, 4] is based on an uniform coarse mesh that defines the high order approximation of the mechanical fields and an adapted one that defines the geometrical features by means of level-sets. The geometric and the approximation meshes are connected through a shared quadtree/octree structure. Numerical examples involving level-set based parts, convergence studies, mechanical computations and numerical homogenization show good promise for this approach in both 2D and 3D.

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Isogeometric Analysis with Boundary Element Methods: comparison with quad-tree finite element method with implicit domain definition

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Key Words: *isogeometric analysis, boundary element method, NURBS quad-tree finite element method.*

ABSTRACT

The concept of isogeometric analysis (IGA) was introduced by Hughes *et al.,* [1] in 2005 to integrate the traditional finite element method (FEM) [2] and Non-Uniform Rational B-Splines (NURBS) [3] [4] which are standard in computer aided design (CAD) software. The key idea of IGA is to approximate the unknown fields with the same basis functions as that used to generate the CAD model, thereby suppressing the need for mesh generation and regeneration (as the design evolves) and, perhaps more importantly, permitting an exact representation of the geometry. Additionally, the accuracy of the solutions can be improved with hpk-refinement without changing the exact geometry. The Boundary Element Method (BEM) [5] comes from the discretization of the boundary and, for linear elastic problems, affords a cheap solution. Although most work on IGA has been performed within a finite element context, we find it is natural to combine it with the BEM since only a representation of the surface of the problem is required. Therefore, we propose the isogeometric boundary element method (isoBEM), which inherits the advantages of the IGA and the boundary element method. The proposed formulation is tested on two dimensional elastic problems and compared with a newly proposed quad-tree finite element method with implicit domain definition.

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Crack Propagation and Coalescence in an Adaptive Multiscale Framework

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Key Words: *multiscale, crack propagation, crack coalescence, adaptive*

ABSTRACT

The multiscale projection method [1] designed for localization phenomena and domains with stress concentrations and high stress gradients is extended to crack propagation and crack coalescence. Typical and important industrial applications are brittle ceramic materials where microcrack nucleation and propagation in the vicinity of propagating macrocracks can be observed.

Crack propagation as well as crack coalescence are calculated accurately and in detail on the microscale where all microstructural features are considered explicitly. Thus microcracks and macrocracks may coalesce with each other. Crack propagation is only considered for the crack with the most critical stress intensity factor. The possibly new macrocrack configuration including branches [2] due to crack coalescence needs to be projected from the microscale onto the macroscale. On the macroscale the macroscopic boundary value problem needs to be solved incorporating all the necessary information from the microstructural features.

Applying the corrected XFEM [3] it is necessary to modify the ramp function blending out the crack tip enrichment functions without violating the partition of unity concept. This is necessary on the macroscale to account for the accurate description of microcracks and macrocracks and their coalescence.

Within the presented multiscale framework, crack propagation automatically leads to a model adaption by an adjustment of the microscale domain according to the zone of interest where the macroscopic stresses as well as the stress gradients are high. In case of multiple macrocracks many fine scale domains need to be defined.

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Two X-FEM methods and two constitutive law for convergence analysis within a nonlinear incompressible elasticity framework

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Key Words: *extended finite element method, non-linear elasticity, incompressible, Cut-off.*

ABSTRACT

Although simulation tools have improved over the past years to overcome poblems dealing with linear fracture mechanics, very few tools exist to solve non-linear fracture mechanics problems. This work proposes a numerical analysis of stress and displacement fields in the vertex of a crack-tip in large strain fracture mechanics using e**X**tended **F**inite **E**lement **M**ethod [1].
We consider a plane strain problem in an incompressible isotropic hyperelastic media. Analysis of convergence are performed for two nonlinear constitutive law (**SaintVenant-Kirchhoff** and **Mooney-Rivlin**). The asymptotic functions obtained for a notch vertex in hyperelastic media by [2] can be restricted to the crack-tip case.

$$y_1(r,\theta) = rsin^2(\theta/2)$$
$$y_2(r,\theta) = \sqrt{r}sin(\theta/2)$$

Where y_1, y_2 are the components of the cartesian position in the euler configuration, r, θ are the radial coordinates of material point.
We perform a convergence analysis of two variant of eXtended finite element methods. The first one is a standard X-FEM with a fixed enriched zone [1], the second one is an X-FEM with a Cut-off function to ensure a smooth transition between enriched zone and non enriched zone [3]. Unless the X-FEM with a fixed enriched zone increase the computational cost, the X-FEM with cut-off function method gives optimal convergence results without increasing significantly the computational cost and without degrading the condition number of the tangent matrix.

- Discretized displacements for standard XFEM method

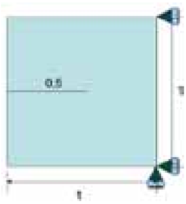
$$u_h = \sum_{i\in I} u_i\,\varphi_i + \sum_{i\in I_H} a_i\,H\,\varphi_i + \sum_{i\in I(R)}\sum_{\alpha=1}^{4} c_{\alpha,i}\,F_\alpha N_i$$

Where H Heaviside step function for discontinuties and F_α Functions taking into account singularities.

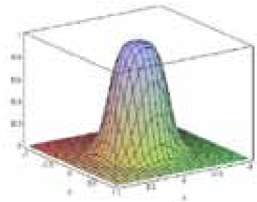
- Discretized displacements for XFEM with Cut-off functions

$$u_h = \sum_{i \in I} u_i \, \varphi_i + \sum_{i \in I_H} a_i \, \mathbf{H} \, \varphi_i \; + \sum_{i \in I(R)} \sum_{\alpha=1}^{4} c_\alpha \, F_\alpha \chi$$

Where H Heaviside step function for discontinuties, F_α Functions taking into account singularities and χ the Cut-off function ensuring smooth transition between enriched zone and non-enriched one.
The convergence results are obtained with tests in the following cracked specimen. We bloc only 3 dof



(a) Cracked specimen


(b) Example of Cut-off function

to avoid singularities belonging to Dirichlet Neumann boundary conditions crossing. To prevent the locking of the finite element approximation in the incompressible limit we use a mixed finite element method. However, such method introduce a critical condition for the stability of the formulation (inf-sup or LBB condition). Then we use a numerical inf-sup test for the stability of the formulation [4].
The first convergence result are in good agreement with previously published works. An improvement of the regularity of the enriched finite element approximation was observed.

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Recent advances in X-FEM based crack modeling in the context of poromechanics

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Key Words: *Extended finite element method, poromechanics, hydraulic fracturing, durability*

ABSTRACT

In durability oriented computational analysis of structures made of fully or partially saturated porous materials, discontinuities such as evolving cracks in concrete structures, joints in rocks or shear bands in soft soils, often serve as prominent channels of accelerated transport of pore water and hazardous or corrosive substances, respectively. On the other hand, in hydraulic fracturing processes of rocks, applied in the exploitation of natural oil and geothermal resources, water pressure is used to induce a controlled propagation of fracture zones. In computational analyses related to these areas, the highly accelerated moisture transport and the pressure acting within the opening discontinuities has to be taken into account. The paper is concerned with an Extended Finite Element model for the numerical representation of crack propagation in partially saturated porous materials. For the numerical analysis of the undamaged part of partially saturated porous materials, the theory of poromechanics [1] is used. The spatial discretization of the coupled hygro-mechanical problem is characterized by discontinuous approximations of the displacement field and of the moisture flux across the discontinuity. While for the representation of the displacement jump a classical, Heaviside-based enrichment strategy for cracks is employed [2], for the hygric sub-problem, a two-scale Extended Finite Element modeling strategy is used which is characterized by the superposition of the approximation for the moisture field at the macro scale with a local approximation along the crack, considering the local mass balance along the discontinuity [3]. For the approximation of the capillary pressure at the discontinuity an enhancement is proposed, which leads to a local increase of the capillary pressure at the discontinuity and represents an appropriate extension of the capillary pressure field for the simulation of transport processes across and along discontinuities. In the contribution, selected results of applications of the two-phase X-FEM formulation to crack propagation in concrete structures subjected to variable hygric conditions and comparisons with an alternative formulation, characterized by an embedded consideration of the increased moisture flux along cracks, are presented.

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Conform XFEM for automatic three dimensional crack propagation

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Key Words: *extended finite element method, 3D, conform, crack propagation.*

ABSTRACT

Stress intensity factors can be obtained as part of the solution by enriching finite elements with analytical solutions [1]. The eXtended Finite Element Method [2] uses more general asymptotic functions to enrich the approximation space. In the last decade, it has encountered a growing interest, in particular in the field of crack propagation. Indeed, the removal of the mesh conformation constraint is a great help for three dimensional crack modeling. In this framework, many tools have been developed to increase the performance: usage of level set method [2], integration technique [3], fast marching method [4].
In this contribution, the authors propose to take advantage of these developments in order to perform three dimensional automatic *conform* crack propagation using the XFEM with minimal mesh operations and starting from a non-conforming mesh. The authors will demonstrate the advantages of this method: removal of Heaviside enrichment, easy handling of contact, modal analysis, link with commercial tools, less problematic management of cracks coalescence. The performance of the method will be illustrated on simple and industrial-like cases with comparison to non-conforming XFEM.

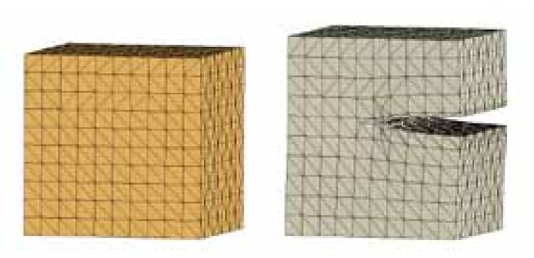

Figure 1: Initial mesh (left) and deformation on automatically generated conform mesh (right).

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THE THICK LEVEL METHOD (TLS) TO MODEL DAMAGE GROWTH AND TRANSITION TO FRACTURE

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Key Words: *damage, fracture, level set, localization, X-FEM*

ABSTRACT

The Thick Level Set method [1] is a new way to model the non-locality of damage growth. The iso-zero of a level set locates the separation front between the undamaged and the damaged zone. In the damaged zone, the damage variable is expressed as an explicit function of the distance to the front. Beyond a critical distance, l_c, the damage is set to 1. The TLS allows one to regularize purely local damage models.

From the theoretical point of view, the TLS model is self contained to initiate and grow cracks. The branching and merging of cracks is also handled. Numerical experiments will demonstrate these facts.

From a numerical point of view, it is interesting to introduce the X-FEM (with Heaviside enrichment) [2] in the TLS simulation to allow one to use large size elements when cracks are formed over long distances.

An efficient coupling between the TLS and the X-FEM will be discussed.

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ANISOTROPIC/ORTHOTROPIC XFEM FOR FRACTURE ANALYSIS OF STRUCTURES

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Key Words: orthotropic materials, crack, extended finite element method (XFEM)

ABSTRACT

Accurate study of crack stability and propagation in composites has retained its paramount importance for computational community because of vulnerability of these materials to cracking and delamination, which can cause severe performance and safety problems.

This paper discusses the recent developments of the extended finite element method (XFEM) for fracture analysis of orthotropic materials. The method is now being extended to other anisotropic applications such as modeling contact and interface, simulation of inclusions and holes, and nano/multiscale analyses.

This presentation covers all important computational topics in the field of fracture analysis of orthotropic/anisotropic materials, such as fracture analysis of stationary and propagating cracks in single and multilayer orthotropic composites, both in static and dynamic conditions, and for homogenous and functionally graded materials; all important aspects of accurate fracture analysis of composites.

The paper also addresses a number of recent developments in closely related topics. The results of anisotropic analysis of dislocations by new anisotropic enrichment functions are presented. This is expected to provide a concrete step forward in realistic simulation of dislocation dynamics.

Finally, a perspective for similar anisotropic enrichments in recently developed isogeometric analysis (IGA) is presented. Such anisotropic eXtended IsoGeometric Analysis (XIGA) has successfully been applied to analysis of various mixed mode fracture analysis of composite laminates.

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Development of Extended Finite Element Method to Solve Arbitrary Multiple Curved Cracks

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Key Words: *extended finite element method, multiple curved cracks, fracture mechanics, stress intensity factor, Abaqus.*

ABSTRACT

In this paper Extended Finite Element Method (XFEM) is employed to solve two dimensional solid structure problems including arbitrary multiple curved cracks. For this purpose, a level set framework is adopted to represent the multiple curved cracks locations. In order to distinguish of cracks tips, a procedure is also performed to locate the intersected points of the considered zero level set with another appropriate orthogonal zero level set. During the solution procedure, the cracks are discritized to a number of points and then the necessary values of the level set functions are calculated and stored for material nodes around the crack. Heaviside and a Branch function are also used to nodal enrichment in order to model discontinuities and singularities in the problem.

A simple XFEM code is developed based on proposed procedures and XFEM formulations. Numerical results of this code are verified with those presented in the literature. Case of two arbitrary curved cracks is solved numerically and SIF values at the crack tips are calculated based on the evaluation of the Crack Tip Opening Displacement (CTOD). Then J-integral methodology by ABAQUS is used to evaluate the SIF values of same problem. The results of XFEM solution have good agreement with those obtained in ABAQUS. This code especially when coupled with simplification of the practical structures will allow quick and reliable analysis whenever problems are found.

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ACCURATE AND OPTIMALLY CONVERGENT IMPLICIT GEOMETRY DEFINITION AND FINITE ELEMENT ANALYSIS PRESERVING SHARP EDGES AND CORNERS FROM PARAMETRIC PRIMITIVES

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Key Words: *Implicit objects, Level set, Curved boundaries and sharp edges, XFEM, Dirichlet.*

ABSTRACT

Recently, analysis techniques that do not require the generation of a conforming mesh have been developed to relax the complexity of mesh generation and to avoid completely the remeshing process. In the extended finite element method (X-FEM) [1], a background mesh or grid is used for representing the internal boundaries (cracks, material interfaces, voids) of the domain analysis and serves to construct the finite element interpolation space. The aim of this work is the implementation of a new approach in order to overcome some difficulties associated with mesh generation and avoids the need for boundary conforming meshes. The novel contributions are threefold: (1) present and validate a novel method to obtain the signed distance fields of parametric shapes on an unstructured mesh; (2) represent arbitrary solids implicitly, including sharp features using level sets and boolean operations; (3) impose arbitrary Dirichlet and Neumann boundary conditions on the resulting implicitly defined boundaries.

The methods proposed do not require local refinement of the finite element mesh in regions of high curvature, ensure the independence of the domains volume on the mesh, do not rely on boundary regularization, and are well suited to methods based on fixed grids. Numerical examples will be presented to demonstrate the robustness and effectiveness of the proposed approach and we will show that it is possible to achieve optimal convergence rates using a fully implicit representation of object boundaries. This approach is one step in the desired direction of tying numerical simulations to Computer Aided Design (CAD).

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HIGHER ORDER XFEM-MODELLING OF MATERIAL INTERFACES AND COHESIVE CRACKS

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Key Words: *higher order XFEM, material interface, cohesive zone.*

ABSTRACT

For a consistent lightweight design the consideration of the nonlinear macroscopic material behaviour of composites, which is amongst others driven by damage effects and the strain-rate dependent material behaviour of typical polymeric matrices, is required. Here, numerical homogenisation techniques are used to predict the effective material behaviour of the composite based on the simulation of a representative volume element (RVE). As a result of the complex geometry of textile reinforcing architectures, the application of the standard finite element method tends to result in an extensive modelling and meshing effort including problems related to distorted element shapes and a poor numerical condition of the system of equations to be solved. Therefore, the authors have implemented XFEM [1] to model the local heterogeneous material structure in an RVE [2]. For an efficient modelling procedure, an automated model generation procedure that determines the required element types, locates the material interfaces within an element domain and performs the subdivision into integration subdomains has been developed. It is based on linear shape functions which results in a piecewise linear approximation of the discontinuity. Describing curved material interfaces, this can lead to a discontinuous displacement field in adjacent elements with enriched displacement approximation and to a poor convergence rate. An improved approximation requires higher order shape functions [3]. For the description of curved material interfaces a higher order XFEM formulation including quadratic basis functions and integration subdomains which are consistent to the curved path of the discontinuity has been developed. In order to incorporate microscopic damage effects, the modelling procedure is currently enhanced to model the failure of the fibre-matrix interface. To this, a second enrichment term [4] accounting for the strong discontinuity is added to the displacement approximation. The mechanical behaviour in the fracture zone is represented by an exponential cohesive zone model.

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STRESS ANALYSIS OF A CFRP STIFFENED PANEL WITH A DELAMINATION BY XFEM

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Key Words: *composite laminate, delamination, buckling, laminated solid element, shell element*.

ABSTRACT

The extended finite element method (XFEM) is applied to perform stress analyses of CFRP laminate with a delamination. In the proposed method, a thin-walled structures containing a bonding part is modeled by solid or shell elements. The utilized eight-node isoparametric solid element is added with incompatible modes and enriched with the Heaviside and the asymptotic basis functions through the framework of XFEM. In addition, the four-node degenerated shell element is utilized and the nodes on the bonding part are enriched with the Heaviside and/or the branch functions in order to model the delamination. Moreover, the eight-node laminated solid element, which can model multiple layers using one element, is also utilized to reduce computational cost. The XFEM code based on the proposed method was developed and applied to elastostatic and linear buckling eigenvalue analyses of a CFRP stiffened panel with a delamination. It was shown that the proposed XFEM provides the appropriate results, which agrees with those by the conventional FEM analyses.

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ENRICHED ELEMENT FREE GALERKIN METHOD FOR GRADIENT ELASTICITY WITH APPLICATION TO FRACTURE MECHANICS

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Key Words: *meshless methods; computational methods; gradient elasticity; enriched element free galerkin method.*

ABSTRACT

The classical theory of elasticity being the long wavelength limit of the atomic lattice theory does not account for the existence of long-range interaction forces. Recent studies have shown that the long range interaction forces are significant when size effects are prominent. In order to improve the theory within the domain of continuum mechanics, Eringen [1] proposed a nonlocal integral type theory and subsequently, Aifantis [2] proposed a gradient type elasticity theory. The key idea behind these theories is to use a continuum approach which has the information regarding the behavior of the material micro-structure and this is accomplished with the constitutive equations. This paper presents a study of the use of partition of unity enrichment to model strong discontinuities (cracks) in gradient type elastic materials. New enrichment functions are developed based on the analytical solution presented by Aravas and Giannakopoulos [3]. For the numerical analysis, the eXtended Element Free Galerkin (XEFG) method has been used, because higher order gradients as required by the gradient theory can easily be incorporated in XEFG. Numerical results for the displacement, stress and strain fields around the crack are compared with available solutions.

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COMPUTATIONAL HOMOGENIZATION FOR MULTISCALE CRACK MODELLING

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Key Words: *computational homogenization, cracks, extended finite element method (XFEM), representative volume element (RVE).*

ABSTRACT

The concept of the RVE for quasi-brittle softening materials has been revisited in [1] in which we developed a new averaging technique coined the failure zone averaging scheme. The basic idea of the scheme is to do the averaging over a propagating damaged zone, rather than over the entire fine scale domain. By doing so, we obtained homogenized initially rigid cohesive laws which are independent of fine scale model size which allows us to state that an RVE exists for softening materials when averaging towards a coarse scale cohesive law. Departing from this result, we have devised a computational homogenization based multiscale crack framework in [2]. The basic ingredients of the method are as follows. The coarse scale model is a homogeneous elastic solid (with effective properties) containing a cohesive crack (modelled by XFEM). The behavior of that cohesive crack is coming from FE computations performed on a fine scale model. In the fine scale model, all heterogeneities are explicitly meshed and failure of microstructural constituents is modelled by a non-local continuum damage model. The coarse-fine scale coupling is done in the spirit of the multilevel FE framework which is better known as FE^2 methods. The method is objective with respect to coarse/fine scale discretizations and to the size of the fine scale sample. Numerical examples including a comparison with a direct numerical simulation (DNS) are given to demonstrate the performance of the method.

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Meshfree Particle Methods for Thin Plates

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Key Words: *Generalized product partition of unity. Partition of unity with flat-top, Reproducing polynomial particle shape functions, Kirchhoff plate model. Error estimates. Meshfree methods.*

ABSTRACT

We are concerned with meshfree particle methods for the solutions of the classical plate model. The vertical displacement of a thin plate is governed by a fourth order elliptic equation and thus the approximation functions for numerical solutions are required to have continuous partial derivatives. Hence, the conventional finite element method has difficulties to solve the fourth order problems. Meshfree methods have the advantage of constructing smooth approximation functions, however, most of the earlier works on meshfree methods for plate problems used either moving least squares method with penalty method or coupling FEM with meshfree method to deal with essential boundary conditions. In this paper, by using generalized product partition of unity, introduced by Oh et al [3], we introduce meshfree particle methods in which approximation functions have high order polynomial reproducing property and the Kronecker delta property. We also prove error estimates for the proposed methods. Moreover, to demonstrate the effectiveness of our method, results of the proposed method are compared with existing results for various shapes of plates with variety of boundary conditions and loads.

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Two-dimensional Dynamic Extended Finite Element Method for Simulation of Seismic Fault Rupture

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Key Words: *crack propagation, elastodynamics, XFEM, seismic source model*

ABSTRACT

Dynamics of spontaneous crack propagation is a considerable problem in seismology and earthquake engineering, revealing some physical aspects of the faulting phenomenon and the resulting ground motion.

Seismic fault rupture is basically assumed as a shear dominant elastic propagation of in-plane and/or anti-plane healing crack, with a kind of slip- or velocity-dependant frictional constitutive law. Propagation occurs due to a local impulse of stress drop from a datum stress field on a specific segment of the fault.

In order to simulate such a process, a two-dimensional dynamic eXtended Finite Element Method (XFEM) is used to model the mixed mode in-plane as well as anti-plane crack propagation in a homogenous and isotropic elastic continuum, with a simple slip-dependant friction law (i.e. Coulomb's friction law) on the slipping segments of the fault.

Large spatial and temporal scales of the fault rupture process, (i.e. several kilometers and duration normally in the range of a few seconds) result in choosing coarser meshes and larger time steps in the way that is not common in regular crack analyses. Therefore necessary conditions for numerical accuracy and stability of the results are also investigated.

The results are assessed to be consistent with available solutions in the literature.

XFEM coupling techniques for landslide-fluid interaction

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Key Words: *XFEM, Fluid-structure-interaction, Sharp interface, Interface conditions.*

ABSTRACT

Selected topographies on earth are threatened by sudden landslides on natural or artificial hillsides. During landslides natural granular materials exhibit phase transitions from solid-like to fluid-like behavior [1]. In order to describe such materials interacting with surrounding fluids a material formulation allowing a switch from solid- to fluid-like state is developed. The solid-like state of the granular is modeled as compressible elastic material in an eulerian framework, while the surrounding fluid is described by an incompressible newtonian fluid. Discretization of the balance equations is carried out with a stabilized space-time finite element method [2]. The domain of elastic material utilizes mixed-hybrid space-time elements while for the surrounding fluid standard velocity-pressure elements [3] are used. Interaction of both continua is described with an interface-coupled formulation where the level-set technique [4] is employed for the interface motion. The challenge of the introduced model is the coupling of fields with discontinuous state variables as well as fieldwise different state variables. Available coupling techniques in the context of the extended finite element method and based on localized mixed hybrid formulations [5] will be discussed and evaluated numerically by means of selected 2D examples.

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Direct estimation of generalized stress intensity factors using a multigrid X-FEM

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Key Words: *generalized SIF, Williams, multigrid, integral matching.*

ABSTRACT

Over the 15 last years, several numerical methods have been devised to make the simulation of crack propagation be more and more convenient, flexible, and easy to implement. Among them, the eXtended Finite Element Method (X-FEM [1]) has allowed a significant step in this direction. With only a few additional degrees of freedom (DOF), it is possible to use meshes that do not conform to the crack shape, and so there is no need to remesh the structure at every propagation step. The introduction of singular enrichments in the discrete field interpolation is also a major asset, because a standard FE convergence can be reached even when using standard linear elements in the vicinity of the crack tip. In this context, an alternative enrichment technique was proposed recently [2] which has the advantage to estimate directly and accurately the generalized Stress Intensity Factors (SIF) *without any post processing*. The model starts form a standard X-FEM description, but the singular enrichment differ. Indeed, near the crack tip, the finite elements are replaced by analytical linear elastic crack tip asymptotic fields [3] for the interpolation of the unknown displacement field. The two overlapping descriptions are coupled thanks to the Arlequin Method.

In this work, a non-overlapping variant of this method is proposed. The coupling is performed thanks to a Mortar technique on the interface. We will show that this variant, although easier to implement and to use (since there are less parameters), provides comparable efficiency and accuracy.

However, the methods discussed herein, as well as X-FEM, remain FE-based method, and thus require a sufficiently fine mesh in the zones of interests. In particular, if the mesh is such that its elements are larger than the scale of the crack, then it may not be adapted to take into account properly the effect of the crack in the simulation, even with discontinuous and singular enrichments. Another example concerns the simulation of curvilinear crack propagation. Sometimes, the radius of curvature of the crack may not be large enough so that we can no longer assume that the crack is right at least locally around the tip. In this situation, the standard singular enrichments functions, which are build for a straight crack, may not be appropriate. We choose not to resort to more sophisticated enrichment functions taking into account the curvature of the crack in order to have to handle a unique set of predefined modes. In these two previous situations, the problem can be solved by refining the mesh, to make it more suited to the crack geometry, despite the non-compatibility condition of X-FEM. Besides

the computational cost of remeshing, the refinement lead to a larger algebraic system which can yield a significant additional cost, which may, in some tridimensional complex cases, become prohibitive.

In a second part of the talk, we will present a multiscale method that attempts to circumvent the above limitations. A multigrid solver inspired from the multigrid-X-FEM method of [4] is developed and coupled to the non-overlapping method presented in the first section. The proposed method [5] bridges three characteristic length scales that can be present in fracture mechanics: the structure, the crack and the singularity at the crack tip. For each of them, a relevant model is proposed. First, a truncated analytical reduced-order model based on Williams expansion is used to describe the singularity at the tip. Then, it is coupled with a standard extended finite element (FE) method model which is known to be appropriate to the scale of the crack. A multigrid solver finally bridges the scale of the crack to that of the structure for which a standard FE model is often accurate enough. Dedicated coupling algorithms are presented and the effects of their parameters are discussed. The efficiency and accuracy of this new approach will be exemplified using several reference benchmarks.

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NUMERICAL COMPARISON OF XFEM AND EFGM SOLUTIONS FOR FRACTURE MECHANICS PROBLEMS

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Key Words: *XFEM, meshfree, EFGM, cracks*

ABSTRACT

Over the years, a variety of approaches have been developed in FEM for the analysis of the stress field at the crack tip. However, in FEM, a crack must coincide with the edge of the finite elements i.e. a conformal mesh is required besides special elements to handle crack tip asymptotic stress field. To overcome these difficulties, novel approaches such as (XFEM) and meshfree methods have been developed. XFEM allows the modeling of arbitrary geometric features independently of the finite element mesh, whereas meshfree methods do not require elements for discretization of problem domain.

In the present work, XFEM and meshfree element free Galerkin method (EFGM) are used to simulate few fracture mechanics problems, and are compared with each other. In EFGM, the domain of interest is discretized by nodes without physically having any crack in the domain, and the presence of a crack in the domain is ensured by intrinsic enrichment only [1-2]. In intrinsic enrichment, a regular basis used in standard EFGM has been enriched by four additional functions i.e. crack tip functions obtained from theoretical background of the problem [1-2], whereas in XFEM [3-4], local partition of unity (PUM) enrichment has been used for modeling a discontinuity i.e. regions near the discontinuities (cracks and material interfaces) are enriched. The numerical comparisons have been made on the basis of various parameters such as accuracy, rate of convergence, order of Gauss quadrature, and computational time. The model problems including plate having a center crack, an edge crack, a bi-material interface crack and multiple cracks are selected, and solved by both XFEM and EFGM.

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3D CRACK PROPAGATION IN INELASTIC MATERIAL

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Key Words: *extended finite element method, 3D, plasticity, cohesive zone model.*

ABSTRACT

This paper presents numerical methods developped in the Finite Element code Cast3m [1] to model crack propagation in inelastic materials. Compared to traditional methods, XFEM is a promising way to model crack propagation in FE code but some difficulties exist. Element integration is complex due to non-polynomial function; transition zone reduces the convergence rate; crack geometry update may be hard when sudden topological changes; fracture criteria should be adapted.

Crack geometry may be describe in an implicit (with a couple of level set functions) or explicit way (by meshing the crack independently of the structure mesh). Implicit way becomes uneasy to use when crack orientation change suddenly or when bifurcation occurs, and some tweaks are often needed [2]. At the opposite explicit description encouters difficulties to remain consistent with the structure edges, but update is not iterative and results from a translation $\underline{\Delta a}$ deduced from the fracture criteria. Geometry relations enables to calculate the local base (ψ, ϕ) if wanted. Moreover the crack mesh is a good support when local material law (cohesive, friction, ...) have to be used. With this methodology large ductile fracture is studied by Simatos [2]. Transition from damage to macro crack is obtained with the introduction of a cohesive zone thermodynamically equivalent.

Characteristic parameters (J, K_I, K_{II}, K_{III}) of linear fracture mecanics can be extracted by integral method $G\theta$. The virtual advance fields $\underline{\theta}$ is usually defined on a parallelepiped box into which mechanical fields are projected. As gradient of these field may evolve quickly, it is suggested here to build a field $\underline{\theta}$ conforming the structure discretization. Application to 3D mixed mode penny shape crack propagation is proposed and difficulties encountered discussed.

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Electrode Shape Change Simulations Based on the XFEM

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Key Words: *extended finite element method, level set method, electrode shape change.*

ABSTRACT

If the XFEM is recognized today in the scientific world as an important computational tool for complex applied mechanical problems, not the same may be said about the use of this method in the study of computational electrochemistry problems more particularly in the studies of the electrode shape change.

This paper aims to extend the combination of XFEM and LSM methods from structural mechanics to computational electrochemistry and to show the particular mathematical aspects that can arise on such problems for the two dimensional electrode shape changes during electroforming processes.

The computational model consists of two coupled problems: electrode shape change rate computation based on the potential model [1] and [2] which is solved with X-FEM and a moving boundary problem handled by LSM [2]. Validation of the electrode growth algorithm is made by comparison with experimental results and simulations in [1] and [2] obtained for the electrode growth in the vicinity of a singularity (incident angle between the electrode and insulator = 180^o)).

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B-spline XFEM for problems with discontinuities

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Key Words: *extended finite element method, B-spline, integrated design and analysis.*

ABSTRACT

We present a new XFEM based on B-splines. The B-splines, WEB-splines and subdivision basis functions are very attractive alternative to standard polynomial shape functions used in FEM [1-3], as they integrate design and analysis of structures. Identical basis functions are used for representation of complicated shapes and for further analysis in a Galerkin framework. We use the B-spline to approximate the standard (non-enriched) part of solution, while the enriched part obtained from XFEM is superimposed on this solution to get a discontinuous solution. We compare the numerical performance by solving some standard benchmark problems.

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S. Bordas, B. Karihaloo and P. Kerfriden

Partition of unity enriched methods for multiscale fracture and fluid structure interaction driven brittle and ductile failure

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Key Words: *Partition of Unity, Material Failure, Fracture, Cohesive Model, Localization.*

ABSTRACT

We will present partition-of-unity methods for modeling brittle and ductile material failure. Methods for continuum and structural (thin-shell) formulations will be presented. The structural PU-enriched formulation is based on higher order continuity of the underlying shape functions such that Kirchhoff-Love theory can be exploited in pristine form. Moreover, an efficient method to treat fluid-structure interaction for fracturing thin structures will be presented. Another focus is on partition-of-unity enriched multi-scale methods that treat fracture by first order principles, i.e. on the atomistic level and employ an enrichment to capture the discontinuity on the macroscopic scale. The methods will be tested at benchmark problems and validated by comparison numerical results to experimental data.

A partition of unity finite element method for simulating non-linear debonding and matrix failure in thin fibre composites and reinforced concrete

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Key Words: *partition of unity finite element method, thin fibre composites, reinforced concrete, damage*

ABSTRACT

The numerical analysis of large numbers of arbitrarily distributed discrete thin fibres embedded in a continuum is a computationally demanding process. In this contribution we review an approach [1,2] based on the partition of unity property of finite element shape functions that can handle discrete thin fibres in a continuum matrix without meshing them. This is made possible by a special enrichment function that represents the action of each individual fibre on the matrix. Our approach allows to model fibre reinforced materials considering matrix, fibres, and interfaces between matrix and fibres as individual phases, each with its own constitutive law. A series of examples will show the versatility of the approach in modelling thin fibre composites and reinforced concrete.

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IDENTIFICATION OF COHESIVE ZONE MODELS USING X-FEM AND DIGITAL IMAGES

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Key Words: *cohesive zone, identification, digital image correlation*

ABSTRACT

Since the pioneering work by Dugdale [1], cohesive zone models have been widely used. One of the reasons for this success is their ability to model non-linear fracture processes that is suitable for numerical simulations: mesh dependency is avoided as long as appropriate the size of cohesive elements is considered, computational cost is preserved as the non-linear processes are concentrated along lines in 2D, planes in 3D.... However, the length scale involved in cohesive fracture is usually small compared to the length of a crack or a structure. Therefore, the observation and/or identification of non-linear processes inside the cohesive zone is a challenging topic.

We propose a methodology based on the use of digital images and X-FEM. Digital Image Correlation (DIC) is a powerful method that allows to compute (in its 2D version) the displacement field on the surface of solid between two states. Its is based on the resolution of the passive advection of the grey level functions of a reference image f and a deformed one g:

$$f(z) = g(z + \boldsymbol{u}(z)). \tag{1}$$

In a first analysis, the displacement is searched for as a decomposition over Williams' series [2]. As shown in [3], it allows for extracting not only stress intensity factors but also the position of the crack tip. In a second step, X-FEM simulations with a cohesive model are performed using the displacement obtained by DIC as boundary conditions. The parameters of the cohesive law are then adjusted so that the gap between the crack tip position obtained within the first analysis and the one computed by the projection of the X-FEM displacement field onto the Williams' series is minimized. A test on a PMMA sample is performed and analyzed with the proposed methodology. The set identified parameters agrees well with values found in the literature. Further, the proposed methodology allows to access to the mechanical state inside the cohesive law whereas its length is far smaller than the usual resolution of DIC.

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A FAST AND ACCURATE SOLVER BASED ON CARTESIAN GRIDS FOR SHAPE OPTIMIZATION PROBLEMS

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Key Words: *cartesian grids, shape optimization, recovery technique*

ABSTRACT

The software for structural shape optimization is composed by a higher level controlled by the optimization algorithm and a lower level controlled by the numerical method used for the analysis of each different design, normally the Finite Element Method (FEM). In this work we present a technique for the improvement of the behavior of the optimization process based on the use of Cartesian Grids (CG) in the lower level. As the mesh does not conform the geometry, the stress field obtained directly from FEM has a poor accuracy along the boundary. So, another key ingredient of our method is the use of a recovery technique called SPR-C [1] which considerably improves the stress field, especially along the boundary of the domain. The SPR-C stress field has also been used to guide the *h*-adaptive process in each geometry, which is required to achieve the right convergence in the shape optimization process [2].

This work shows that the use of the proposed technique considerably improves the computational behavior of the optimization process: *a*) as all internal elements have the same shape, stiffness matrices and patches of the recovery technique are calculated only for boundary elements, *i.e.*, the complexity of some parts of the FEM analysis is (N-1)-D; *b*) as mesh and geometry are independent, information between individuals in the optimization process can be easily shared, thus avoiding mesh-boundary intersections along fixed geometries; *c*) CG have allowed for the development of a powerful data structure and very efficient algorithms.

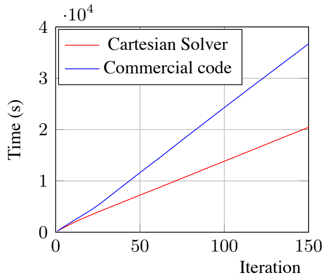

Figure 1: Comparison of Computational cost of optimization process

Figure 1 compares the evolution of the computational cost along the optimization process obtained with our Cartesian Solver, fully developed in MatLab®, with that obtained with Ansys® 11. The graph clearly shows the improvement provided by the proposed technique.

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ON THE USE OF RECOVERY TECHNIQUES FOR ACCURATE ERROR ESTIMATION AND ERROR BOUNDING IN XFEM

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Key Words: *XFEM, error estimation, error bounding, recovery techniques*

ABSTRACT

A posteriori implicit residual-type estimators have traditionally been the most commonly used techniques to provide bounds of the error of the finite element method, FEM. Recovery-based error estimators based on the ideas of Zienkiewicz and Zhu have been often preferred by practitioners, due to their simple implementation and robustness, but they were unable to provide guaranteed bounds, which is especially desirable in the context of goal-oriented adaptivity. In 2007, Diez *et al.*[1] circumvented this problem by means of the use of a nearly statically admissible stress field and correction terms, proposing the first recovery-based technique used to evaluate upper bounds of the error in energy norm for FEM. Then, in 2010, Ródenas *et al.* [2] enhanced this technique and adapted it to the extended finite element method, XFEM. In this paper we will show: *a)* how these techniques can also be applied to obtain sharp upper bounds in *goal-oriented* error estimation; and *b)* new enhancements of the recovery technique that provide sharp upper bounds of the error in energy norm avoiding the use of correction terms. The numerical results show that these kind of techniques are a clear alternative to the use of residual-type estimators in error bounding.

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Ductile fracture approach using the XFEM

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Key Words: *extended finite element method, ductile fracture, cohesive laws.*

ABSTRACT

The XFEM [1] has proven to be an optimal method to model brittle fracture: cracks can be modeled independently from the mesh, with a reduced computational cost when compared to other methods such as remeshing [2].

For ductile fracture, the XFEM can be combined with continuous models establishing the bridge between damage, softening and the final stages of material failure [3, 4, 5], however this subject is not so widely covered by the existing literature.

In this work, a continuous-discontinuous approach to ductile fracture is proposed, combining Lemaitre's model for isotropic damage [6] with the XFEM, in order to model the behavior of metallic materials in a complete way, considering plasticity, hardening, damage and rupture. The transition between the critically damaged areas and the macro-cracks is performed using a cohesive law, which is directly built from the bulk properties, without the need of identifying extra material parameters. This methodology allows an accurate transition of history variables and it is energetically consistent.

Numerical strategies to deal with locking and numerical integration are also adressed. In addition, special attention is given to the extension of the proposed formulation to large strains.

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Self-affine scaling of simulated intergranular cracks in brittle polycrystals

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Key Words: *generalized finite element method, crack roughness, self-affinity, polycrystals, brittle fracture*

ABSTRACT

We study the scaling properties of intergranular cracks through 2D numerical simulations of quasi-static crack propagation in brittle polycrystals. Crack paths are computed by means of a Generalized Finite Element Method (GFEM) for polycrystals [1]. The GFEM, contrary to classical Finite Element Methods, does not need a mesh generator to mimic the polycrystalline topology. Being described by means of discontinuous enrichment functions, grain boundaries can cut elements and grain junctions can be arbitrarily located within elements. Grain boundaries follow the Xu-Needleman cohesive law while grain interior obeys linear isotropic elasticity. Material properties are representative of alumina.

We have recently demonstrated [2] that intergranular crack paths in brittle polycrystals are not influenced by key cohesive law parameters. Considering these results, we characterize the statistical properties of a series of crack profiles obtained from representative polycrystals. Within a certain range of length scales, they exhibit self-affine properties characterized by a Hurst exponent close to 0.5. This result is compared with recent experimental findings in other disordered brittle materials. Our simulations shed light on the roughening mechanism in 2D brittle solids.

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A robust discontinuous-Galerkin-based extended finite element method for fracture problems with nearly incompressible elasticity

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Key Words: *extended finite element method, discontinuous Galerkin, locking.*

ABSTRACT

For nearly incompressible elasticity, volumetric locking is a well-known phenomenon with low-order (cubic or lower) finite element method methods, of which continuous extended finite element methods (XFEMs) are no exception. Legrain *et al.* [1] numerically investigated the stability of two-dimensional mixed methods involving XFEM displacement and pressure spaces. But a *displacement*-formulation XFEM which is both optimally convergent and robust still does not exist.

We will present the specific challenges of such construction, and our work towards this goal. Based on our earlier work of an optimally convergent discontinuous-Galerkin-based XFEM [2], the method herein consists in enriching a region surrounding the crack tip that contains a fixed ball, i.e., the enrichment zone does *not* shrink with the mesh parameter. To maintain the order of the condition number, the enrichment space consists of modes I and II asymptotic solutions *without* the use of partition of unity. The discontinuous Galerkin (DG) method is used between all neighboring elements, to allow the existence of a divergence-free subspace with the optimal approximation order.

We adopt a bilinear form similar to a non-symmetric interior penalty approach, which renders a stable method with a problem-independent stabilization parameter. Numerical examples demonstrate the absence of locking and show an improved convergence rate relative to the case with an (unenriched) DG FEM, i.e., $h^{\frac{3}{4}}$ versus $h^{\frac{1}{2}}$ for the stress and $h^{\frac{3}{2}}$ versus h for the displacement, both in the L^2-norm, where h is a measure of the mesh size.

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Correction of Incompleteness of Blending Approximation in XFEM

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Key Words: *partition of unity, PUFEM, blending, a priori knowledge, weighted XFEM.*

ABSTRACT

A lack of numerical accuracy in the original XFEM is caused by 'blending approximation' in the elements whose nodes are partially enriched. Although the weighted XFEM shows effective improvements of the accuracy for this problem, it was found that the influence of the blending approximation still remains in the numerical results. In order to fundamentally solve the problem of the blending approximation, we have reformulated an approximation of the XFEM based on the concept of the PUFEM, which assures the approximation accuracy, as the 'PU-XFEM'.

The PU-XFEM approximation $\mathbf{u}_{\text{ap}}(\mathbf{x})$ has the form of

$$\mathbf{u}_{\text{ap}}(\mathbf{x}) = \varphi_0(\mathbf{x})\mathbf{v}_0(\mathbf{x}) + \varphi_1(\mathbf{x})\mathbf{v}_1(\mathbf{x})$$

where the sets of $\varphi_0(\mathbf{x})$ and $\mathbf{v}_0(\mathbf{x})$, and $\varphi_1(\mathbf{x})$ and $\mathbf{v}_1(\mathbf{x})$ are the PU and the approximate functions, relating to the classical finite element approximation and the enrichment approximation, respectively. The blending elements have been redefined as the elements in the common part of the supports of the PU $\varphi_0(\mathbf{x})$ and $\varphi_1(\mathbf{x})$. In the interpolation error analysis for a one dimensional problem, it is shown that the incompleteness in the existing XFEMs has been overcome in the PU-XFEM.

The PU-XFEM is applied to two dimensional linear fracture mechanics. The numerical results show that a priori knowledge of the solution included in the enrichment near the crack tip can be accurately reproduced in the PU-XFEM.

Therefore, it is concluded that the PU-XFEM is a correction of the original XFEM or the weighted XFEM to obtain its essential features, by solving the problem of the blending approximation.

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X-FEM with large sliding contact-friction along branching surfaces

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Key Words: *contact, junction, large sliding*.

ABSTRACT

This presentation gives an example of X-FEM ability to represent both branching surfaces and large sliding contact-friction conditions. Starting from the results of [2] on large sliding frictionless contact with X-FEM in 2D, and on its extension to 3D and friction in [4], we focus here on the junctions by using the X-FEM enrichment proposed by [1]. The originality of this work is to take into account large-sliding contact condition on junction surfaces.

The kinematic space is based on the classical X-FEM enrichment strategy plus the additional junction function [1]. When an element is cut at least twice, we sequentially cut this element to create sub-domains where the functions to integrate are continuous. The contact-friction condition is satisfied with augmented Lagrange multipliers in a hybrid formulation. We take care of the LBB condition with X-FEM by setting the Lagrange unknowns to the nodes [3] and by using a vital edge approach to reduce the Lagrange multiplier space [3][4]. Large sliding over the junction is managed by pairing master-slave X-FEM elements [2]. The difference with [2] is that slave and master elements can have different kind of X-FEM enrichment.

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Modeling of Matrix Damage Initiation and Propagation in Clay-Nanocomposites using Extended Finite Element Method (XFEM)

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Key Words: *Nanocomposite, Extended Finite Element Method, Damage Mechanics.*

ABSTRACT

Due to the increasing use of clay nanocomposites in various industries, investigation of mechanical properties of clay nanocomposites has become of great interest in recent years. The addition of nanoclay improves material strength. However, one of the challenges is to enhance strength without reducing ductility. Currently, experimental observations show that ductility reduces with increasing percentage weight of montmorillonite clays.

Modeling the mechanical stiffness of clay nanocomposites is well documented in the literature [1]. However, damage propagation and initiation in nanocomposite is not clearly known yet. Several key mechanisms for energy dissipation have been identified in the fracture of nanocomposites. Matrix cracking and debonding of clay are the most dominant mechanisms of failure in epoxy-silicate nanocomposites [2].

The main aim of this paper is to model the matrix cracking mechanism in epoxy-silicate nanocomposite materials. Toward this goal, the extended finite element method (XFEM) is used as a solution approach. The advantage of this method is that the finite element mesh is completely independent of the trajectory of the cracks. A two-dimensional continuum representative volume element (RVE) model of silicate/epoxy nanocomposite is used to investigate the effects of matrix cracks on the damage properties of the nanocomposite. Periodic boundary conditions were applied on the model and a simple tension test was simulated. Modeling of damage initiation and propagation was conducted using the XFEM capabilities of the ABAQUS finite element software. Various clay weight fractions were considered. The numerical results show that the proposed damage model can qualitatively reproduce the effect of failure on the mechanical properties of clay nanocomposite materials.

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SUPPRESSING MESH GENERATION FOR LINEAR ELASTIC FRACTURE SIMULATION: ISOGEOMETRIC ANALYSIS WITH BOUNDARY ELEMENT METHODS

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Key Words: *isogeometric analysis, boundary element method, linear-elastic fracture mechanics*

ABSTRACT

The seminal work of Hughes et al. [1] which proposed the isogeometric concept of using the same functions that describe CAD geometry to approximate the unknown fields has developed into a thriving field of research. Much work has been carried out on the development of the technique using the ubiquitous Finite Element Method (FEM), but little has been seen on the Boundary Element Method (BEM) [2]. In fact, it is found that the BEM is a natural fit with the isogeometric concept, since in both CAD surface modelling and BEM only a description of the surface (boundary) of the domain is required.

The present authors have developed an isogometric BEM which uses NURBS to approximate both the geometry and the unknown displacement and traction fields. Since the NURBS functions are already prescribed for a given CAD model, *no meshing is required* and consequently, gains in efficiency are obvious. As in any BEM implementation, integration of singular functions play a key role, and in this work the subtraction of singularity method of Guiggiani et al. [3] is used. In addition, the location of collocation points is not immediately obvious, but we choose the Greville abscissae [4] which provide accurate results for the problems tackled here.

The method is applied to linear-elastic fracture mechanics where it is found that use of NURBS to describe the geometry facilitates the creation of crack surfaces. Results compare favourably with conventional BEM fracture routines, but the major advance of the present work is the ability to perform BEM fracture analysis on CAD models with no need for meshing.

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NUMERICAL SIMULATION OF FATIGUE CRACK PROPAGATION IN FGM USING XFEM

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Key Words: Functionally graded materials, fatigue, crack propagation, XFEM

ABSTRACT

The present work deals with the fatigue life estimation of cracked functionally graded material (FGM) plate having multiple discontinuities by extended finite element method (XFEM). XFEM [1-2] has been adopted for the simulation as it rules out a need of conformal mesh, and thus facilitates the modeling of fatigue crack growth without remeshing.

Various crack problems such as a major crack in the presence of multiple discontinuities (minor cracks, holes and inclusions) are chosen for the numerical simulations. The effect of these discontinuities on the fatigue life of cracked FGM plate is studied and analyzed in detail.

A rectangular plate ($100\text{ mm} \times 200\text{ mm}$) of graded alumina nickel alloy with $E = 397$ GPa and $\nu = 0.23$ is taken for the present study. An exponential gradation in material (graded alumina nickel alloy) property along x-direction is considered. Uniform traction is applied at the boundary of the domain along y-direction. The material properties of the inclusions are taken as $E = 20$ GPa and $\nu = 0.2$. The applied stress (σ) is assumed to vary from 50 MPa to 100 MPa. Nine noded Lagrangian quadrilateral elements are used for domain discretization. A mesh size of 91 and 183 nodes is used for the purpose of analysis in each case. Stress intensity factors (SIFs) are computed at the tip of the major crack using domain based interaction integral approach. These simulations show that the presence of holes and inclusions or their combination(s) in the domain has more significant effect on the SIF as compared to the presence of minor cracks.

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Examples of groundwater problems with discrete features and singularities.

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Key Words: *groundwater, fracture flow, mixed-hybrid finite elements*

ABSTRACT

One of the important problems in groundwater modeling is representing the fractures as conductive discontinuities, which solution with combination of 3D, 2D and 1D elements is done e.g. in [1]. We present application background to the submission by J. Brezina [2] to this conference a set of problem from groundwater flow with fractures and other singular features in the problem geometry, as motivation for future use of some generalizations of finite element method (the problems are not yet solved in this way).

- regional models of groundwater flow, where equivalent continuum until some scale and discrete features (faults and other planar conduits) are combined
- coupled model of stress and water flow in discrete fracture network in small scale differences how to treat the fractures in the elasticity problem and in the flow problem are shown together with upscaling questions (replacement of complex fracture network with combination of simpler network and continuum)
- calculation of tunnel water inflow if the tunnel is needed to be represented with 1D line in 3D space: such problem leads to solution with singularity, but we can ask for method which use 1D line as geometry plus real radius (of tunnel as cylinder) as parameter, such solution can be the analytical element method employing analytical solution [3]

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Partition-of-unity finite elements for large, accurate quantum-mechanical materials calculations[1]

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Key Words: *extended finite element method, enrichment, Kohn-Sham DFT, ab initio, electronic-structure*

ABSTRACT

Over the past few decades, the planewave (PW) pseudopotential method has established itself as the dominant method for large, accurate, density-functional calculations in condensed matter. However, due to its global Fourier basis, the PW method suffers from substantial inefficiencies in parallelization and applications involving highly localized states, such as those involving 1st-row or transition-metal atoms, or other atoms at extreme conditions. Modern "real space" approaches, such as finite-difference (FD) and finite-element (FE) methods, can address these deficiencies without sacrificing rigorous, systematic improvability but have until now required much larger bases to attain the required accuracy. In this talk, I will present our work on a new real-space FE based method [1,2] which employs modern partition-of-unity FE (PUFE) techniques [3] to substantially increase the efficiency of the real-space representation, thus decreasing the number of basis functions required correspondingly, by building known atomic physics into the FE basis: without sacrificing locality or systematic improvability. I will discuss the weak formulation of the required Poisson and Schrödinger problems in the Kohn-Sham solution, and the imposition of Bloch-periodic boundary conditions. Both pseudopotential and all-electron applications will be considered, with attention in the latter case to the approximation of the singular solution outside Sobolev space H^1 using a basis in H^1. Finally, I will highlight recent progress and open questions relating to efficient and accurate quadrature, development of nonlinear generalized eigensolvers, and parallelization. Initial results show order-of-magnitude improvements relative to current state-of-the-art PW and adaptive-mesh FE methods for systems involving localized states such as d- and f-electron metals and/or other atoms at extreme conditions.

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A framework for fracture modeling using implicitly defined enrichments over C^k partitions of unity simultaneously based on finite elements and mesh-free nodes

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Key Words: *C^k continuous approximations, boolean R-functions, fracture modeling.*

ABSTRACT

This work is concerned with the development of a procedure for to build discontinuous enrichments for ansatz spaces using boolean R-functions [1] on arbitrary polygonal supports. A C^k-continuous Partition of Unity [2] simultaneously based on finite elements and mesh-free nodes [3] is built in such a way that the position of the mesh-free nodes and its supports are defined to receive the discontinuous and singular enrichments. Thus, the topology of the discontinuity can be effectively constructed since all the closed semi-analytic sets may be represented implicitly by real valued functions with guaranteed differential properties [4]. The Shepard approximants capability of to couple different partitions of unity is exploited and the width of the support of the discontinuous enrichments may to vary in order to improve the precision of the unknowns fields, aiming to circumvent the problems related to the transition layer and blending, as discussed in [5] and [6]. Moreover, the integrations are performed only in global coordinates, avoiding errors due distortion of meshes. The purpose is to model discontinuities for two-dimensional problems of Linear Elastic Fracture Mechanics in such a fashion that the error inherent to C^0-continuous approximations and due the lack of accuracy on the transition layer between enriched and non-enriched portions of the domain can be reduced.

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DEVELOPPEMENT AND APPLICATIONS OF X-FEM AXISYMMETRIC MODELS IN FRACTURE MECHANICS

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Key Words: *extended finite element method, axisymmetric, stress intensity factor.*

ABSTRACT

Extended Finite Element Method (X-FEM), a standard finite element approximation enriched by additional special solutions using the framework of partition of unity, is increasingly used in fracture mechanics to compute the stress intensity factors (SIFs) for cracked structures. The introduction of the enriched solutions inside the elements without a need to mesh explicitly the crack helps to avoid tedious remeshing requirements involving with the crack propagation or complicated crack geometries. The X-FEM method has been developed in fracture mechanics for many types of models, except for the axisymmetric model (to the authors knowledge). A recent study at Electricit de France (EDF) and Politechnio de Milan (Italy) shows that the closed-form asymptotic solutions under plane strain conditions can be approximately used to represent the singularity of the crack tip under axisymmetric conditions. These solutions are therefore used to enrich the crack tip elements in the X-FEM axisymmetric models.

In this study, the X-FEM axisymmetric model is first implemented in Code_Aster, an open-source software developed by the EDF and devoted to mechanical finite elements analysis. The results obtained from the X-FEM models are then compared with the existing analytical solutions. The SIFs for axisymmetric cracks in rectangular plates obtained from the X-FEM models are in good agreement with the corresponding analytical solutions for cylindrical bars with penny-shaped and circumferential cracks under different loading conditions (Sih [1]). The sensitivity of the mesh density, the number of enriched functions and the size of the enriched zone is also studied. Next, the validated X-FEM models are employed to obtain the SIFs for spot welds and associated kinked cracks in cup specimens (Wang et al. [2]), which are widely used in the automotive industry to investigate the durability of the welds under dominant mode I loading conditions. The computational results obtained from the X-FEM models agreed well with the analytical solutions proposed by Pook [3] and those obtained from the traditional finite element method (in which cracks are explicitly modelled) [2]. Finally, the use of the X-FEM axisymmetric models to assess the integrity of cracked components and to estimate the fatigue life of the structures in different industries is discussed.

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AN IMPROVED ISOGEOMETRIC ANALYSIS USING THE LAGRANGE MULTIPLIER METHOD

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Key Words: *isogeometric analysis, essential boundary conditions, Lagrange multiplier method, NURBS.*

ABSTRACT

The recently developed Isogeometric analysis is increasingly used in engineering problems. It is based on application of Non-Uniform Rational B-Splines (NURBS) basis function for both the solution field approximation and the geometry description. Simple and systematic refinement strategies, exact representation of common and complex engineering shapes, robustness and higher accuracy are among the main superiorities of the method in comparison with the conventional finite element method. Nonetheless, it still suffers from a number of drawbacks. One of the major concerns with this method is finding an efficient approach to impose essential boundary conditions, which is rooted in the non-interpolating nature of NURBS and its lack of Kronecker delta property. Furthermore, imposition of inhomogeneous essential boundary conditions is not a straightforward task, as it is the case in a homogeneous one.

This paper proposes the use of Lagrange multiplier method to impose essential boundary conditions for improving the accuracy of isogeometric solution field. In the proposed approach, the Lagrange multipliers are interpolated on the *Greville abscissas* [1] corresponding to essential boundaries. Several 2D numerical examples are performed to investigate the efficiency of the proposed approach. The results demonstrate significant improvement in accuracy and rate of convergence in comparison with the direct imposition of essential boundary conditions.

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Mesoscopic modelling of masonry using embedded weak discontinuities based on partitions of unity

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Key Words: *masonry, mesoscopic model, weak discontinuities, partition of unity.*

ABSTRACT

The modelling of masonry has been a popular topic within computational mechanics for some years now. Two major groups of modelling approaches exist: macroscopic and mesoscopic models [1]. The macroscopic approach homogenises the mortar joints and bricks creating one orthotropic material, whereas the mesoscopic approach models the joints and bricks as seperate entities. In this contribution a two dimensional mesoscopic model will be developed, where mortar joints are modelled by embedded discontinuities using the partition of unity property of the finite element shape functions.

Unlike classical mesoscopic models, where joints are modelled using strong discontinuities (i.e. jumps in the displacement field) [2], the model developed in this paper uses weak discontinuities. A weak discontinuity introduces a jump in the strain field, allowing for failure to localise in a zone with finite width [3]. The thickness of this failure is in this case linked to the joint thickness. An advantage of this weak discontinuity approach is that the constitutive modelling can be performed in the general stress and strain spaces.

In this work, the embedded weak discontinuity is implemented using the partition of unity concept. Within this method, nodes are locally enhanced to enrich the solution with discontinuous modes ([4], [5]). Both the governing equations and the algorithmic aspects will be discussed. Special attention is given to the modelling of triple junctions and the dealing with the enhanced degrees of freedom.

A Drucker-Prager local damage model is used to describe the non-linear behaviour of the discontinuities. The global equilibrium path is traced using an energy release constraint function [6]. The performance of the developed masonry model will be demonstrated by the simulation and validation of three-point-bending tests and shear wall analysis.

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AN EXTENDED STIFFNESS DERIVATIVE TECHNIQUE TO EXTRACT THE STRAIN ENERGY RELEASE RATES BASED ON THE EXTENDED FINITE ELEMENT METHOD

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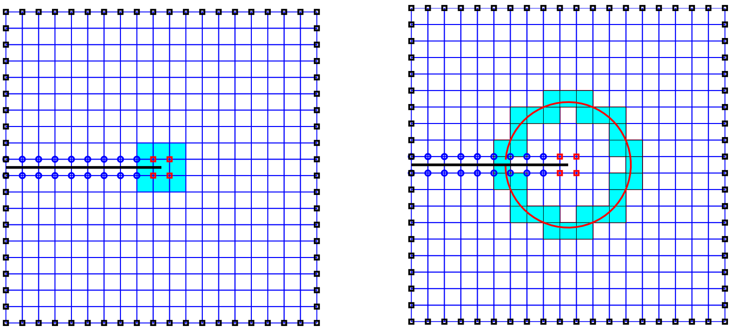
Key Words: *XFEM, stiffness derivative technique, J-integral, Strain Energy Release Rates, SIFs.*

ABSTRACT

An extended stiffness derivative technique to extract the Strain Energy Release Rates is proposed. The method is based on the extended finite element method (XFEM) and Parks classical stiffness derivative technique. The idea hinges on two XFEM properties: (i) the mesh is independent of the crack geometry, i.e. for a virtual crack extension Δa, the mesh is left unchanged and (ii) the asymptotic crack tip field is embedded in the mathematical formulation of the stiffness matrix. By using these properties, it is shown that the Strain Energy Release Rates may be computed analytically and on the fly, i.e. there is no need for virtual crack extensions and the finite difference approach in Parks method may be avoided.



(a) (b)

Figure 1: Illustration of two methods to extract the Stress Intensity Factors (or Strain Energy Release Rates). (a) proposed method, and (b) the J integral method

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QUASI-ALGEBRAIC MULTIGRID PRECONDITIONERS FOR FRACTURE PROBLEMS MODELED BY EXTENDED FINITE ELEMENT METHODS

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Key Words: *XFEM, multigrid, smoothed aggregation, coarsening, Schur complement*

ABSTRACT

An algebraic multigrid method is proposed that is suitable for the linear systems associated with modeling fracture via extended finite element method (XFEM). The new method follows naturally from an energy minimizing algebraic multigrid framework and is suitable to be used as a preconditioner to accelerate Krylov subspace based iterative methods. The key idea is the modification of the prolongator sparsity pattern to prevent interpolation across cracks. The proposed method is quasi-algebraic since the geometric levelset information relating to cracks are used to modify the prolongator sparsity pattern. It is shown that algebraic multigrid using these modified transfer operators is highly efficient when applied to the Schur complement of the XFEM stiffness matrix in which the enriched degrees of freedom are condensed out. In the proposed method, suitable modifications are made to avoid the explicit computation of the Schur complement. Numerical experiments illustrate that the resulting method is scalable since the convergence properties are relatively insensitive to the mesh density and to the number of cracks or their location.

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MECHANICALLY ENRICHED FRACTURE ELEMENTS FOR A HYDROMECHANICAL PROBLEM IN FRACTURED ROCKS

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Key Words: *extended finite element method, hydromechanical processes, fractured rock.*

ABSTRACT

For many applications in geotechnical engineering, such as oil exploration and deep geothermal reservoirs, analysis of mechanics related coupled processes, such as hydromechanical (HM), is important; particularly in fractures, where high hydraulic conductivity can significantly impact transport processes. For instance, changes in the in-situ stress field can affect opening or closing of fractures and hence alter preferential flow paths and hydraulic transmissivity in the system. Various numerical HM models, e.g. the interface elements (IE) approach, have been developed [1]. However a challenge still remains in treatment of complex geometries with pre-existing or newly generated fractures. Explicit representation of the fractures is required when an equivalent media or homogenization approach is not appropriate.

In this study, we present a numerical method partially adopting a theory of an extended finite element method (XFEM) [2] to include pre-existing fractures for modeling fully coupled HM processes. Our enrichment method is based on the work by [2] which has the capability of handling arbitrary discontinuities. Modeling a fully coupled HM problem becomes possible by (1) representing fractures as lower dimensional elements, (2) applying enrichment only at the fracture elements and (3) selecting fracture relative displacements as additional unknowns for enrichment. Although the XFEM does not require the inclusion of fracture geometries into a mesh, we have the geometries in it to use a single mesh for both flow and mechanical processes. This is feasible because we consider only pre-existing fractures and do not assume fracture growth.

For verification of the method, we have conducted simulations of a 2D fluid injection problem into a single fracture where fracture aperture varies depending on fluid pressure. The study shows that the proposed method can produce accurate results to a semi-analytical solution. Extension to three-dimensional spaces will be the focus of future work.

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S. Bordas, B. Karihaloo and P. Kerfriden

DOMAIN SPECIFIC LANGUAGES AND AUTOMATED CODE GENERATION: APPLICATION TO THE EXTENDED FINITE ELEMENT METHOD

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Key Words: *extended finite element method, abstract, conference.*

ABSTRACT

The development of a domain-specific language for solving partial differential equations is presented. A domain-specific language permits expressive computer input that is near-identical to formal mathematical notation, thereby allowing computer representations to be generated rapidly and with a minimum of errors. Moreover, domain-specific languages can implement algorithms for specialised manipulations. From the abstract representation provided by a domain specific language, computer code can be automatically generated by a compiler, and in the generation phase special optimisation strategies can be deployed.

The extension of a domain specific language and a form compiler to extended finite element applications will be elaborated, with an emphasis on the key abstractions used. The usefulness of such a framework will be demonstrated by considering a range of applications that involve different fields coming from different function spaces. This will include non-Lagrange finite element bases, such as H(div) and H(curl) conforming spaces. The applications are chosen to illustrate the ease with which computer code for complicated problems can be generated, and in particular for problems that are not straightforward to implement by hand.

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Study on the dynamic process of tsunami earthquake by FEM with liquid-solid coupling

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Key Words: *Tsunami modelling, Earthquake fault, Liquid-solid coupling, Interaction, Tsunami warning*

ABSTRACT

Tsunami induced by earthquake is an interaction problem between liquid and solid. Shallow-water wave equation is often used to model the tsunami , and the boundary or initial condition of the problem is determined by the displacement or velocity field from the earthquake under seabed, usually no interaction between them is considered in pure liquid model. In this study, potential flow theory and the finite element method with the interaction between liquid and solid are employed to model dynamic process of the earthquake and tsunami.

For modelling the earthquake, firstly initial stress filed to generate the earthquake is set up, and then the occurrence of the earthquake is simulated by suddenly reducing the elastic material parameters inside earthquake fault. It is different from seismic dislocation theory in which the relative slip on the fault is specified in advance.

The results of this study reveal that P and SP waves as well as surface wave can be found at the sea surface besides the tsunami wave. The surface wave with maximum amplitude of 0.55m arriving at the distance of 600km from the epicenter is earlier than the tsunami about 48 minutes. Therefore the surface wave may be taken as a kind of tsunami warning information, which is much earlier than that obtained from the seismograph stations on land. The tsunami speed on the open sea with a depth of 3km is 175.8m/s, which is a little greater than that predicted by long wave theory, and its wavelength and amplitude in average are 32km and 2m, respectively. After tsunami propagates to continental shelf, its speed and wavelength is reduced, but its amplitude becomes greater, especially, it can be up to 10m and run 55m forward in vertical and horizontal directions at sea shore, respectively.

The maximum accelerations at the epicenter on the sea surface and on the earthquake fault are 5.9m/s^2 and 16.5m/s^2, respectively, the later is 2.8 times the former. Therefore, sea water is a good shock absorber. The acceleration at the sea shore goes down 10 times as large as at the epicenter. However, the vertical velocity at the epicenter on the sea surface is only 1.4 times that on the fault. The maximum displacement at the fault is less than that at the epicenter on the sea surface. The difference between them is the amplitude of the tsunami at the epicenter.

Integrated layout design of multi-component system using XFEM

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Key Words: *extended finite element method, layout optimization, topology optimization, multi-component system.*

ABSTRACT

This work is an extension of research on integrated layout design of multi-component systems by Zhu et al (2009, 2010) using the XFEM proposed by Moës et al (1999). The idea of integrated layout design is to find a proper package design of components as well as the structural topology in a limited space simultaneously. However, there are some difficulties caused by remeshing in the integrated layout design within traditional finite element method framework. In the optimal design process, the movement of component leads to the variation of the finite element mesh so that a global or local remeshing approach is needed. In the previous work of Zhu et al, a local remeshing approach named embedded meshing method was applied. In addition, topology design variable could not be naturally defined on elements due to the mesh variation. This is why topology variables are defined on certain density point specified a priori over the design domain and the pseudo-density value of each element is then calculated using the value of the nearest density point to the centroid of the element.

In this paper, based on the combination of XFEM and the level set method proposed by Sukumar (2001), an efficient approach is developed to handle the material interface within a fixed mesh. In this way, the base structure is discretized with regular rectangular meshes and the topology design variable is defined on each element. The interface between component and base structure is described by a level set function and the structural response of the elastic system is calculated within the XFEM framework. Several design problems of maximizing structural stiffness are finally tested to validate the proposed method.

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GPU ACCELERATED XFEM AND ITS APPLICATIONS IN THE SIMULATION OF UNDERGROUND EXCAVATION

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Key Words: *extended finite element method, underground excavation, GPU, iterative solver.*

ABSTRACT

The simulation of the tunnel excavation is useful in the safety evaluation of tunnel engineering. Rock mass is an inhomogeneous material which includes fractures in the forms of joints, fissures, faults and microcracks. These discontinuities under complex conditions of stresses dominate the behavior of rock masses. The XFEM is utilized to model the fractured rock masses due to its advantage with minimal remeshing. However the condition number of the linear system of equations deteriorates in the new method and the total number of DOFs increases compared to the traditional finite element method. In order to reduce the computational time, GPU is used to accelerate the solution of the linear system of equations, which takes up a significant time in the overall computational time of XFEM. A GPU-based preconditioned conjugated gradient solver is integrated into XFEM, which is developed under the CUDA architecture proposed and developed by nVIDIA.

Stress variation under self-weight due to excavation is simulated. Influences of different patterns of pre-existing cracks to the rock mass failure modes are analyzed using the XFEM simulations. Then the creation and evolution of the excavation disturbed zone (EDZ) are studied in order to evaluate the safety of the tunnel excavation. Different tunnel excavation methods are compared with regard to the effectiveness in reducing disturbance. Numerical experiments with an NVIDIA TESLA C1060 are carried out to show the efficiency and speedup of different scale of numerical model, ranging from thousands of DOFs to millions of DOFs. Impacts of different scale of models to the accuracy of the simulation results are discussed as well.

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Modelling of imperfect interface effects via XFEM with developments of enrichment functions

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Key Words: *extended finite element method, enrichment functions, imperfect interfaces.*

ABSTRACT

The imperfect interface effects in composite materials, which may play an important and sometimes dominant role in mechanical and micromechanical behaviors, are of increasing interest to researchers in mechanics and materials science. From the analytical point of view, it is still extremely difficult or even impossible to take into account interfaces of complex shape. For engineering applications, numerical methods are then mandatory to predict the effective behavior and nonlinear processes of inhomogeneous materials such as composites containing imperfectly bounded inclusions.

The present work is devoted to the development of a simple and efficient numerical technique for modelling the effects of imperfect interfaces via XFEM [1]. Appropriate enrichment terms are proposed with clear physical meanings of both the basic and complementary nodal variables, which allows imposing Dirichlet boundary conditions correctly and efficiently. Numerical applications will be performed for problems of spring-layer imperfect interfaces where the displacement jumps are linked to the stress vector and taken into account by superposing to the standard finite element field a discontinuous enrichment term [2]. The numerical accuracy and computational convergence of the method are assessed via benchmarks with known analytical solutions.

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MODELING AND ENRICHED APPROXIMATIONS FOR MULTI-FIELD PROBLEMS IN ENGINEERING APPLICATIONS

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Key Words: *moving non-smooth solutions, multi-field problems, fluid-structure interaction.*

ABSTRACT

This contribution addresses the crucial step of selecting or creating mathematical models for the investigation of engineering processes with respect to implied solution characteristics and their proper approximation by computational methods such as the extended finite element method (XFEM).

Coupled multi-field problems in engineering can involve a number of very different and very localized physical and geometrical effects that need to be properly represented by the chosen mathematical model. Very often, the necessary depth of a production model related to time and length scales involved is not clear in advance and then requires iterative refinement. This situation is even more complex if model-dependent non-smooth solution characteristics are possible and tackled by advanced (extended) approximations.

The modeling and (enriched) approximation of stationary and transient (multi-scale) engineering problems involving nonlinear solid and fluid mechanics as well as boundary-coupled fluid-structure interaction are exemplarily discussed and evaluated with respect to model-extension sensitivity.

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