Q1

Q1.1.1	Calculate the equivalent dose, *H* to the breast.
0.3 pts	0.2 mSv

Q1.1.2	Calculate the effective dose, *E*.
0.3 pts	0.024 mSv

Q.1.1.3	Calculate the CTDI_{vol}
0.3 pts	25 mGy

Q1.1.4	Calculate the dose length product, DLP.
0.3 pts	1500 mGy.cm

Q1.2.1	In a PET-CT scan, a positron emitted by a Fluorine-18 radionuclide collides with an electron, producing two gamma photons of 511 keV each. Calculate the wavelength of the 511 keV gamma photons.
0.3 pts	2.43 pm

Q1.2.2	If the distance between the source and the sensors differs by 40 cm, with one at x cm and the other at (x + 40) cm, what is the time difference between the two gamma photons reaching opposite detectors?
0.3 pts	1.33 ns

Q1.2.3	Calculate the required activity (MBq) for the patient based on her mass.
0.2 pts	Required Activity=337.5 MBq at 9.00 AM

Q1.2.4	Determine the volume (ml) of FDG solution needed to prepare based on the activity at Q1.2.3.
0.2 pts	Volume=0.5625 ml

Q1.2.5	Calculate the initial activity of the prepared dose at 9:00 AM, considering the decay from 7:00 AM to 9:00 AM.
0.5 pts	At 9:00 AM, A=158.52 MBq

Q1.2.6	Determine the remaining activity at 9:30 AM and check whether the dose is still sufficient for the scan.
0.5 pts	Activity at 9:30 AM, A=131.19 MBq Dose is insufficient at 9:30 AM

Q1.2.7	What is the minimum radioactive concentration of the stock that will be required to be prepared at 7:00 a.m if the injection is to be given at 9:30 a.m
0.5 pts	Required concentration = 643.2MBq / 0.56ml ≈1,148.6MBq/ml

Q1

Q1.3.1	A nuclear medicine pharmacist prepares Lutetium-177 (Lu-177) radiopharmaceutical in a hot lab. If the radiation dose rate at 1 meter from a 7.4 GBq source is 4.5 mSv/h, calculate the minimum thickness of lead shielding required to reduce the exposure level to ≤ 0.1 mSv/h. (Use lead HVL for Lu-177 gamma emissions: 0.06 cm).
1.0 pts	Thickness=0.33 cm of lead

Q1.3.2	Calculate the volume of Lu-177 solution needed to achieve the required activity of 50 MBq in the final preparation.
0.3 pts	Volume=0.25 ml

Q1.3.3	Calculate the volume of saline solution required to reach the total volume of 5 ml.
0.3 pts	V_{saline} = 4.75 ml

Q1.3.4	If the half-life of Lu-177 is 6.65 days, determine how much activity will remain in the prepared sample after 4 hours.
1.0 pts	After 4 hours, activity is 49.14 MBq

Q1.3.5	Lu-177 undergoes radioactive decay according to the equation: Lu-177 → Hf-177 + β^- + energy What types of radiation are emitted in this process?
0.2 pts	Beta and gamma radiation

Q1.3.6	What role does the specific molecule (e.g. peptide or antibody) attached to Lu-177 play in its theraupeutic action?
0.1 pts	It binds to receptors that are overexpressed on cancer cells, directing the radiation to the tumour.

Q1.3.7	Why is the use of receptor-targeting molecules important for minimizing damage to healthy tissues?
0.2 pts	It ensures that radiation is delivered directly to cancer cells, reducing the unintended exposure of healthy cells to radiation.

Q1.3.8	What is the main advantage of Lu-177 therapy compared to chemotherapy, a treatment that uses non-targeted chemicals to kill rapidly dividing cells?
0.1 pts	Lu-177 delivers targeted radiation to tumour cells.

Q1.3.9	How does Lu-177 therapy minimize damage to surrounding healthy tissue compared to external beam radiation?
0.1 pts	Lu-177-based therapy targets cancer cells specifically through receptor binding.

Q1.3.10	Calculate the Decay Factor during the 4-hour period before injection.
0.5 pts	Decay factor=0.9828

Q1.3.11	Determine the activity at the time of patient injection.
0.3 pts	Corrected activity=540.54 MBq

Q1.3.12	Calculate the actual activity received by the patient.
0.3 pts	Actual activity=502.54 MBq

Q1.3.13	Verify if the administered dose activity is within the standard therapeutic range of 4-7 GBq (4000-7000 MBq)
0.4 pts	The administered dose activity is not within the standard therapeutic range of 4-7 GBq

Q1.3.14	Patient is treated with Lu-177-DOTATATE for neuroendocrine tumors. Given a dose of 7.4 GBq, Estimate the absorbed dose to the lung if the residence time is 10 hours and the S-value (radionuclide dose factor) is 0.12 $mGyMBq^{-1}h^{-1}$.
0.5 pts	Absorbed Dose= 8880 mGy or 8.88 Gy

Q1.3.15	A patient receiving I-131 therapy for thyroid cancer is required to stay in isolation for a specific period. Assuming the initial administered activity is 5 GBq, calculate when can the patient be released based on the 5 μSv/h at 1 meter guideline? • For I-131, the exposure rate constant (Γ) = 0.0595 $\mu Sv \cdot m^2 MBq^{-1} h^{-1}$ • I-131 Half-life = 8.02 days
1.0 pts	t = 47.3 hours ≈ 2 days

Q2.1.1	What is the total solar surface area of a Starlink satellite facing the Sun? Assume the solar array can be maneuvered to face the Sun directly.	0.2 pts
Answer	104.960 m^2 (10% error acceptable)	0.2

Q2.1.2	The irradiance intensity is inverse proportional to the square of distance, $I \propto 1/r^2$. If the irradiance at the surface of the Sun is equal to the blackbody radiation at 5772K, how much irradiance does a satellite like Starlink receive? You may assume the radius of the Sun, $r_s = 6.9598 \times 10^5 km$ and 1 Astronomical Unit [A.U.] is $149,597,890\ km$. You may also assume that the distance from the Sun to the Satellite is equivalent to the distance between the Sun to the center of the Earth.	0.5 pts
Answer	1362.169 Wm^{-2} (10% error acceptable)	0.5
	Total	

Q2.1.3	How many photons are received by the Satellite Solar cells per m^2 area? Ignore geometrical factors of the solar cells. You may answer in terms of k_B, T, c, h and π.	1.0 pts
Answer	$\frac{2\pi k_B^3 T^3}{h^3 c^2}\zeta(3)\Gamma(3)$ or $\frac{2\pi k_B^3 T^3}{h^3 c^2} 2.404113806$ or $2.923844100 \times 10^{26}$ photons/m^2 (10% error acceptable)	1.0

Q2.1.4	What is the solar spectrum wavelength corresponding to the Silicon p-n junction band gap energy?	0.3 pts
Answer	1100.126 nm (10% error acceptable)	0.3

Q2.1.5	The ultimate efficiency of the solar cell is defined as the percentage of the electric power produced by the solar cell over the amount of power received from the Sun. What is the ultimate efficiency of the Solar Cells?	1.5 pts
Answer	Ultimate efficiency is 43.90% (Answer between 39.51% to 48.29% is acceptable)	1.5

Q2.1.6	What is the Q-value of the following decay process in units of MeV?	0.5 pts
Answer		
a)	Pu-238 decay	
	Q=5.593 MeV	0.1
b)	Am-241 decay	
	Q=5.638 MeV	0.1
c)	Np-237 decay	
	Q=4.958 MeV	0.1
d)	Np-238 decay	
	Q=1.291 MeV	0.1
e)	Np-238 decay	
	Q=0.722 MeV	0.1

Answer within 10% is acceptable

Q2.1.7	The Cassini mission carried with it, three GPHS-RTG modules on board the space probe. How many kilograms of PuO2 does the space probe carry? [Given: $N_A = 6.022141 \times 10^{23}$]	1.0 pts
Answer	Total PuO_2 mass is 30.183551 kg (Answer within 10% error is acceptable)	1.0

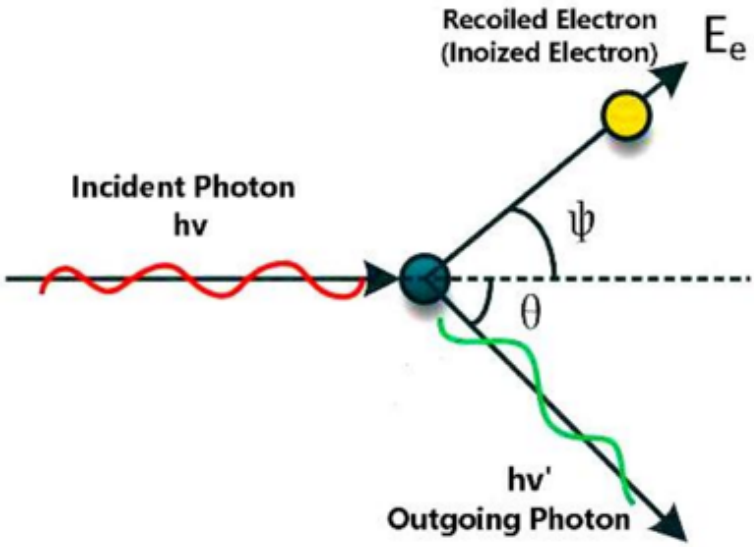
Q2

Q2.2.1	Write the relativistic and nonrelativistic expressions for the kinetic energy of a particle of mass m_0. Show that when $v \ll c$, the relativistic expression reduces to the nonrelativistic expression of the kinetic energy of the particle.	0.5 pts
Answer	The nonrelativistic expression for the kinetic energy of a body of mass m_0 moving with velocity v is given by $KE = \frac{1}{2} m_0 v^2$ The relativistic expression for the kinetic energy of a of a body of rest mass m_0 moving with velocity v is given by $KE = mc^2 - m_0c^2 = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2$ For $v \ll c$, $KE = m_0c^2\left(1 + \frac{v^2}{c^2} + \ldots\right) - m_0c^2$ For $v \ll c$ higher order terms can be neglected $KE = m_0c^2 + \frac{m_0v^2}{2c^2}c^2 - m_0c^2 = \frac{1}{2}m_0v^2$	0.5

Q2.2.2	Use the above expressions to show that the relationship between the relativistic momentum, kinetic energy, and the rest mass energy, is given by $p = (1/c)\sqrt{(KE)^2 + 2(KE)E_0}$	0.75 pts
Answer	Start from the relativistic expression of mass $$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$ Squaring both sides and multiplying by c^4 gives $$m^2\left(1 - \frac{v^2}{c^2}\right)c^4 = m_0^2c^4$$ $$m^2c^4 - m^2v^2c^2 = m_0^2c^4 \quad (1)$$ As $E_{tot} = mc^2$, $E_{rest} = m_0c^2$, $p = mv$, and $E_{total} = KE + E_{rest} = KE + m_0c^2$ Equation (1) can be written as $$E_{tot}^2 - p^2c^2 = E_{rest}^2$$ or $$p^2c^2 = E_{tot}^2 - E_{rest}^2 = E_{tot}^2 - m_0^2c^4 \quad (2)$$ Now Equation (2) can be written as $$p^2c^2 = (KE + m_0c^2)^2 - m_0^2c^4$$ $$p^2c^2 = (KE)^2 + 2(KE)m_0c^2$$ $$p = (1/c)\sqrt{(KE)^2 + 2E_0(KE)}$$	0.25 0.25 0.25

Q2.2.3	Find the energy of the gamma-ray emitted in this reaction. Also, calculate the kinetic energy and recoil velocity of the deuteron.	1.25 points
Answer	The Q-value of the neutron capture reaction is given by $$Q = [m({}^1_1H) + m_n - m({}^1_1H)]\text{amu} \times 931.5\left(\frac{\text{MeV}}{\text{amu}}\right)$$ So, $$Q = [1.007825 + 1.008665 - 2.014102]\text{amu} \times 931.5\left(\frac{MeV}{amu}\right)$$ $$Q = 2.224422\ \text{MeV}$$ Let m_D, P_D, and v_D be the mass momentum and velocity of the Deuterium respectively. Let p_γ be the momentum of the gamma ray emitted from the excited Deuterium nucleus. According to the law of conservation of momentum $$0 = P_D - P_\gamma \quad \text{or} \quad P_D = P_\gamma \quad \text{or} \quad m_D v_D = \frac{E_\gamma}{c}$$ According to the law of conservation of energy $$Q_{value} = \frac{1}{2} m_D v_D^2 + E_\gamma$$ $$Q_{value} = \frac{m_D^2 v_D^2}{2m_D} + E_\gamma$$ $$2.224422 = \frac{E_\gamma^2}{2m_D c^2} + E_\gamma$$ Now $2m_D c^2 = 4.028204 \times 931.5 = 3752.2720$ so we get the quadratic equation $$(0.0002665052)E_\gamma^2 + E_\gamma - 2.224422 = 0$$ Only the positive root of this equation gives the energy of gamma emitted (the other root gives negative energy) $$E_\gamma = 2.223104879\ \text{MeV}$$	0.1 0.15 0.5

	The recoil energy of the nucleus is given by $(KE)_D = 2.224422\ \text{MeV} - 2.223104879\ \text{MeV} = 0.00131712\ \text{MeV} = 2.110027475 \times 10^{-16}\text{J}$ $m_D = 2.014102 \times 1.66054 \times 10^{-27}\text{kg} = 3.34449694 \times 10^{-27}\text{kg}$ $v_D = \sqrt{\frac{2(KE)_D}{m_D}} = \sqrt{\frac{2(2.110027475 \times 10^{-16})}{3.34449694 \times 10^{-27}}} = 355{,}216.9231\ m/s$	0.5
	Total	

Q2.2.4	Using laws of conservation of energy and momentum show that the relationship between the energy of the incident gamma ray (E_γ) and energy of the scattered gamma ray E'_γ is given by $E_\gamma - E'_\gamma = \frac{E_\gamma E'_\gamma}{m_e c^2}(1-\cos\theta)$	1.75 points
Answer	From the solution of part (ii) we can find the relationship between the momentum and energy of massless particles like photons and is given by $p = \frac{E_\gamma}{c}$ The momentum of the incoming photon $= E_\gamma/c$ The momentum of the scattered photon $= E'_\gamma/c$ Let us assume that the photon is scattered through an angle θ, and the electron recoils through an angle ψ as shown in the figure below  	0.25 0.5 0.5

	The law of conservation of momentum in the horizontal and the vertical directions gives $$\frac{E_\gamma}{c} + 0 = \frac{E_\gamma}{c}\cos\theta + P\cos\psi$$ $$0 = -E'_\gamma \sin\theta + P\sin\psi$$ where P is the momentum of the electron The last two equations can be written as $$Pc\cos\psi = E_\gamma - E'_\gamma\cos\theta$$ $$Pc\sin\psi = E'_\gamma\sin\theta$$ By squaring and adding these equations one gets $$(Pc)^2 = (E_\gamma)^2 - 2E_\gamma E'_\gamma\cos\theta + (E'_\gamma)^2 \qquad (i)$$ The law of conservation of energy gives $$E_\gamma = E'_\gamma + (KE)_e$$ From the solution of Q2.2.2 $$(Pc)^2 = (E_\gamma - E'_\gamma)^2 + 2(E_\gamma - E'_\gamma)m_e c^2 \qquad (ii)$$ By comparing equations (*i*) and (*ii*), we get $$(E_\gamma)^2 - 2E_\gamma E'_\gamma + (E'_\gamma)^2 + 2(E_\gamma - E'_\gamma)m_e c^2 = (E_\nu)^2 - 2E_\nu E'_\nu\cos\theta + (E'_\nu)^2$$ $$(E_\gamma - E'_\gamma)m_e c^2 = E_\gamma E'_\gamma(1-\cos\theta)$$ $$(E_\gamma - E'_\gamma) = \frac{E_\gamma E'_\gamma}{m_e c^2}(1-\cos\theta)$$	0.5

Q2.2.5	Using the result of Q2.2.4, derive Equation (2.4)	0.75 pts
Answer	From the solution of Q2.2.4, one gets $$m_e c^2\left(\frac{1}{E'_\gamma} - \frac{1}{E_\gamma}\right) = 1 - \cos\theta$$ $$\cos\theta = 1 - m_e c^2\left(\frac{1}{E'_\gamma} - \frac{1}{E_\gamma}\right)$$ From the figure one can see that $E_\gamma = E_1 + E_2$ and $E'_\gamma = E_2$ So, $$\cos\theta = 1 - m_e c^2\left(\frac{1}{E_2} - \frac{1}{E_1 + E_2}\right)$$	0.25 0.25 0.25

Q3.1.1	The construction of a Rn-Be neutron source requires the extraction of Rn-222 from Radium. By bubbling air through a Radium solution, we are able to extract radon gasses from it. If we add 10 grams of RaBr2 to 100mL of water, what is the molarity (moles per Litre) of the solution?	0.25 pts
Answer	0.259 M (Answers between 0.255 to 0.265 M is acceptable)	0.25

Q3.1.2	What is the decay constant in [s^{-1}] for Ra-226?	0.25 points
Answer	$1.372 \times 10^{-11} s^{-1}$ (Answers between $1.2 \times 10^{-11} s^{-1}$ and $1.5 \times 10^{-11} s^{-1}$ is acceptable)	0.25
	Total	

Q3.1.3	What is the decay constant in [s^{-1}] for Rn-222?	0.25 points
Answer	$2.097 \times 10^{-6} s^{-1}$ (Answers between $1.90 \times 10^{-6} s^{-1}$ to $2.1 \times 10^{-6} s^{-1}$ is acceptable)	0.25
	Total	

Q3.1.4	If we accumulate the Radon gas for an hour by bubbling air into this solution, how much atoms of Rn-222 do we expect to get at most? Assume we are able to collect every single Radon atom produced.	0.75 points
Answer	7.675×10^{15} atoms (Answers between 7.0×10^{15} atoms and 8.0×10^{15} atoms are acceptable)	0.75
	Total	

Q3.1.5	We need around 100 mCi for our Radon-Beryllium source, how many millilitres of Radon Bromide solutions do we need to extract Rn-222 from?	0.25 points
Answer	229.837 mL (Answers between 225 and 235 mL are acceptable)	0.25
	Total	

Q3.1.6	U-238 is said to have been synthesized early in the formation of the Universe. What is the ratio of U-238 to Pb-206 you would expect to see in a primordial rock formed around 4.5 billion years ago?	0.25 points
Answer	The ratio of U-238 to Pb-206 is 1:1 (The ratio of 0.9:1.1 or 1.1:0.9 is acceptable as well)	0.25
	Total	

Q3.1.7	What is the Q-value in [MeV] of the Rn-222 decay?	0.1 points
Answer	5.591 MeV (Only exact answers acceptable)	0.1
	Total	

Q3.1.8	How much kinetic energy does the alpha particle from Rn-222 possess?	0.6 points
Answer	5.490 MeV (Answers between 5.4 MeV to 5.6 MeV can be accepted)	0.6
	Total	

Q3.1.9	What is the maximum kinetic energy of the neutron produced by this Radium-Beryllium source?	0.3 points
Answer	11.192 MeV (Answers between 11.1 MeV to 11.3 MeV is acceptable)	0.3
	Total	

Q3.1.10	Natural Uranium consisted predominantly of the isotope U-238. What is the excitation energy of the Uranium after it received a **thermal neutron**?	0.2 points
Answer	4.806 MeV (Only exact answers are accepted)	0.2
	Total	

Q3.1.11	Will the U-238 fission at this excitation energy (after excitation by thermal neutron)? Answer either YES or NO only.	0.1 points
Answer	NO	0.1
	Total	

Q3.1.12	If Fermi were to use his Radon-Beryllium neutron source to excite U-238 and a neutron did hit U-238, will it fission? Answer either YES or NO only.	0.1 points
Answer	YES	0.1
	Total	

Q3.1.13	How close can the alpha particle born from Rn-222 decay approach a U-238 atom assuming we are using point charge approximation of the nucleus. Write down the distance in units of femto-meter.	0.3 points
Answer	48.262 fm (Answers between 48 fm and 49 fm are acceptable)	0.3
	Total	

Q3.1.14	Even though this alpha particle are not able to reach U-238, it is able to reach the Be-9 nucleus. What is the Coulomb repulsion energy between Be-9 and the alpha particle at touching distance? Assume none of the alpha particle experience quantum tunnelling.	0.3 points
Answer	2.617598 MeV (Answers between 2.610 MeV to 2.620 MeV is acceptable)	0.3
	Total	

Q3.2.1	The multiplication factor (k) is the ratio of the number of neutrons produced in the current generation of fission, N_i over the number of neutrons produced in the previous generation, N_i, $k = \frac{N_i}{N_{i-1}}$ It is often used in the reactor to measure its criticality. Multiplication factor, $k = 1$ is the critical point, when the reactor is able to sustain the production of neutron. If it drops to below 1, the reactor is said to be subcritical, each generation will produce less neutrons than the previous generation. If the multiplication factor of the reactor is 0.95, what is the total number of neutrons released when 100 initial neutrons are introduced into the reactor?	0.3 points
Answer	2000 (Only exact answers acceptable)	0.3
	Total	

Q3.2.2	Calculate the sum of neutrons generated after 10 generations (round to the nearest integer)	0.2 points
Answer	1258 neutron (Answers between 1257 neutron and 1258 neutrons are acceptable)	0.2
	Total	

Q3.2.3	Calculate the sum of neutrons generated after 20 generations (round to the nearest integer)	0.2 points
Answer	3307 neutrons (Answers between 3306 to 3307 neutron is acceptable)	0.3
	Total	

Q3.2.4	Prompt neutron generation time is the average time between the emission of prompt neutrons from one fission and the initiation of the next fission by those neutrons. If the prompt generation time is $40\mu s$, what is the total number of neutrons generated after 0.1 seconds? Assume only 10 neutrons are introduced initially into the reactor, with this being the first generation in a reactor with $k = 1.05$.	0.3 points
Answer	1.880519×10^{55} (Answers between 1.8×10^{55} to 1.9×10^{55} neutrons are acceptable.	0.3
	Total	

Q3.2.5	Assuming a reactor is operating at a steady state. Suddenly, the reactor is scrammed (shut down). The delayed neutron precursor concentration for a particular isotope is given by, $$C(t) = C_0 e^{-\lambda t}$$ where C_0 is the initial concentration and λ is the decay constant. If the mean lifetime $(T = 1/\lambda)$ of the precursors is 20 seconds, and the initial precursor concentration is $C_0 = 10^{12}$ nuclei/cm^3, calculate the number of delayed neutrons released per second at $t = 40$ seconds. Assume that each decaying precursor releases one neutron.	0.2 points
Answer	6.766764×10^9 neutrons/s (Answers between 6.5×10^9 to 7.0×10^9 neutrons/s are acceptable)	0.2
	Total	

Q3.2.6	Delayed neutrons play a crucial role in reactor control. Their behaviour is best modelled using six groups of precursor isotopes, each with a different decay constant (λ_i), mean lifetimes (T_i) and yields (β_i). The parameters for each group are given in the table below. Fill up the missing decay constant data.	0.6 points

Answer

Group	$\lambda_i\left[s^{-1}\right]$	$T_i[s]$	β_i
1	0.01244	80.4	0.000215
2	0.03040	32.9	0.001425
3	0.07576	13.2	0.00127
4	0.00831	4.38	0.002565
5	0.96154	1.04	0.000845
6	3.61011	0.277	0.00027

Each correct answer is 0.1 points

(Answers within 1% error is acceptable)

Q3.2.7	Calculate the initial reactivity, if the reactor period is 15s and the prompt neutron generation time is negligibly small. Give your answer in units of pcm.	0.6 points
Answer	2.393810 pcm (Answers between 2.1 pcm to 2.5 pcm are acceptable)	0.6
	Total	

Q3.3.1	Given a 30 MW HTGR that operates with a core power density of 2.5 W/cm^3, estimate the total number of TRISO particles required to fuel the reactor. Note that 99.2% of the core consists of graphite moderators, coolant channels, and voids. Assume that each TRISO particle contributes equally to the power density.	0.4 points
Answer	2.28×10^8 (5% error is acceptable)	0.4
	Total	

Q3.3.2	Calculate the total mass of 8% enriched uranium in a single TRISO particle if the kernel consists of UO_2 with a density of 10.5 g/cm^3 and a diameter of 500 µm. Note: $M_{U235} = 235.044\text{g/mol}$, $M_{U238} = 238.051\text{g/mol}$, and $M_0 = 16.000\text{g/mol}$.	0.8 points
Answer	0.6057 mg (Answer within 5% error is acceptable)	0.8
	Total	

Q3.3.3	Calculate the total initial uranium inventory in a 12 m^3 HTGR core.	0.4 points
Answer	$1.381 \times 10^5 g$ (Answers within 5% error is acceptable)	0.4
	Total	

Q3.3.4	Determine the burnup in %FIMA (Fissions per Initial Metal Atom) for a 30 MW HTGR assuming constant power operation for 2 years. Note: Avogrado's number, $N_A = 6.022 \times 10^{23}$ atoms/mol, Energy released per fission $\approx 3.204 \times 10^{-11}$ J.	0.8 points
Answer	16.89 % (Answers within 5% error is acceptable)	0.8
	Total	

Q3.3.5	In an experimental HTGR setup, TRISO fuel particles that have been irradiated to a burnup of ~10% FIMA were heated at $1600\ ^{\circ}C$ for 300 hours to simulate an accident scenario. The fuel initially generated $1.200 \times 10^{-6}\,\mathrm{mol}$ of Cs-137 per gram of uranium. At the end of the experiment, only $2.400 \times 10^{-9}\,\mathrm{mol}$ of Cs-137 was measured to have escaped through the SiC layer from a sample containing 5.000 g of uranium. Determine the percentage of Cs-137 retention and explain its implication. Express your answer with 4 significant figures.	0.4 points
Answer		
	Total	

Q3.3.6	Derive the expression for time t from the above equation. Assume that the argument for the error function $\ll 1$.	0.8 points
Answer	$t = \frac{x^2}{\pi D}\left(\frac{n_0}{n_t}\right)^2$	0.8
	Total	

Q3.3.7	Given that the diffusion coefficient of Krypton in SiC is $6.5 \times 10^{-20}\mathrm{m}^2\mathrm{s}^{-1}$, estimate the time required for 1% of Krypton to escape through a $35\mu\mathrm{m}$ thick SiC layer.	0.4 points
Answer	$6 \times 10^{13} s = 1.96 \times 10^6 y$ (Answers within 5% error is acceptable)	0.4
	Total	

Q4

Q4.1.1	Calculate the average energy released per second by the generator after 72.4 years.	1.0 pts
Answer	$T_{1/2} = 4$ half-life	0.3
	Cu-244 mass after 72.4 years is **468.75 g OR 31.25 g**	0.35
	Generator release **1312.5 J/s OR 87.5 J/s** after 72.4 years	0.35
	Total	1.0

Q4.2.1	The total mass of Uranium-238 present in the deposit.	0.5 pts
Answer	10,000 kg	0.5 pts

Q4.2.2	The number of moles of Uranium-238 in the deposit. (Given atomic mass of U-238 = 238 g/mol)	1.0 pts
Answer	4.202×10^4 moles	1.0 pts

Q4.2.3	The total activity (in becquerels) of Uranium-238 in the deposit. (Half-life of Uranium-238 = 4.468 × 10^9 years, Avogadro's number = 6.022 × 10^{23} atoms/mol)	1.0 pts
Answer	1.24×10^{11} Bq	1.0 pts

Q4.3.1	Calculate the total mass of Thorium-232 present in the deposit.	0.5 pts
Answer	120,000 kg	0.5 pts

Q4.3.2	Determine the number of moles of Thorium-232 in the deposit.	0.5 pts
Answer	5.172×10^5 moles of Th-232	0.5 pts

Q4.3.3	Calculate the total number of Thorium-232 atoms in the deposit.	0.5 pts
Answer	3.115×10^{29} atoms	0.5 pts

Q4.3.4	Using the decay law, calculate the total activity (in becquerels) of Thorium-232 in the deposit.	0.5 pts
Answer	4.87×10^{11} Bq	0.5 pts

Q4.4.1	How many half-lives have passed?	0.5 pts
Answer	0.080 half-lives	0.5 pts

Q4.4.2	Estimate the age of the uranium ore sample.	0.5 pts
Answer	357.6 million years or 3.58 x 10^8 years	0.5 pts

Q4.4.3	If the sample originally had 100 g of Pb-206 (U-238 decay product), how much Pb-206 is present today?	0.75 pts
Answer	123.36 g of Pb-206	0.75 pts

Q4.4.4	What is the ratio of Pb-206 to U-238 in the sample now?	0.75 pts
Answer	Pb-206/U-238 ratio in mol = 0.226 in mass = 0.26	0.75 pts

Q4.5.1	Calculate the number of atoms of each isotope in the sample?	1.0 pts
Answer	1.77 $\times 10^{22}$ of U-238 atoms	0.3 pts
	7.21 $\times 10^{21}$ of Th-232 atoms	0.3 pts
	5.12 $\times 10^{20}$ of U-235 atoms	0.3 pts

Q4.5.2	Determine the total activity (Bq) of the sample?	1.0 pts
Answer	103540 Bq or 103.54 kBq or 1.04 × 10^5 Bq	1.0 pts

B2.1.1	Write down the mathematical relation between derivative of gamma ray intensity with respect to the attnuator thickness, dI/dx and the gamma beam intensity.	0.1 points
Answer	$\frac{dI}{dx} = -\mu I$	0.1
	Total	

B2.1.2	Derive a linear equation (in the form of $y = mx + c$, in which m is the slope of the line and c is the y intercept of the line) that relates gamma ray intensity, I with the initial intensity, I_0 and the attenuator thickness, x.	0.2 points
Answer	$\frac{dI}{dx} = -\mu I$ $\int \frac{dI}{I} = -\mu \int dx$ $\ln I = -\mu x + \ln I_0$	0.2
	Total	

B2.1.3	Derive the exponential relation between the gamma ray intensity, I and the attenuator thickness.	0.2 points
Answer	$\ln I - \ln I_0 = -\mu x$ $\frac{I}{I_0} = e^{-\mu x}$ $I = I_0 e^{-mux}$	0.2
	Total	

B2.1.4	A half-value layer (HVL) is the thickness that attenuates a beam to 50% of its original intensity. How many HVLs would you expect is needed to ensure that 99% of the gamma beam is absorbed by the attenuator material?	0.2 points
Answer	6.643856 HVL (10% error acceptable)	0.2
	Total	

B2.1.5	A mean free path is the average distance travelled by gamma ray before its first interaction. What is the mean free path in cm based on the plot?	0.1 points
Answer	1.517541 cm (10% error acceptabble)	0.1
	Total	

B2.1.6	What is the decay constant of Cs-137? Phrase your answers in units of s^{-1}.	0.2 points
Answer	$7.321 \times 10^{-10} s^{-1}$ (Answers between $7.0 \times 10^{-10} s^{-1}$ and $8.0 \times 10^{-10} s^{-1}$ can be accepted)	0.2
	Total	

B2.1.7	Given what we know about the efficiency of this detector, what is the actual number of photons emitted by the source?	0.5 points
Answer	1957.842043 cps (10% error acceptable)	0.5
	Total	

B2.1.8	Estimate the activity in μ Ci printed on the sticker of the collimated Cs-137 check source.	0.5 points
Answer	$0.076678 \mu Ci$ (Answers between 0.07 and 0.08 $$\mu$Ci are acceptable)	0.5
	Total	

B2.1.9	The XRS detector of GOES measures x-ray intensities with wavelengths between 1 to 8 Å. What is the maximum photon energy this instrument can measure?	0.1 points
Answer	12.398 keV (1% error acceptable)	0.1
	Total	

B2.1.10	What is the minimum photon energy the XRS detector can measure?	0.1 points
Answer	1.5498 keV (1% error acceptable)	0.1
	Total	

B2.1.11	What is the mass attenuation coefficient of the polyurethane for photons with energy of 5 keV?	0.2 points
Answer	$28.308\,\mathrm{cm}^2\,\mathrm{g}^{-1}$ (1% error acceptable)	0.1
	Total	

B2.1.12	What is the mass attenuation coefficient of aluminum for photons with energy of 5 keV?	0.1 points
Answer	$193.4\,\mathrm{cm}^2\,\mathrm{g}^{-1}$	0.1
	Total	

B2.1.13	If an X45 solar flare were to happen, what is the intensity of a 5 keV photon penetrating past the wall of the Destiny module of the ISS? You may assume that the increase of x-ray intensities between 1 to 8 Å is the same.	0.2 points
Answer	1.819×10^{-167} (Alternative answer of 0.00 acceptable along with 10% error)	0.2
	Total	

B2.1.14	If we replace the polyurethane shielding with more aluminum instead, how much more aluminum do we need to add to the total aluminum thickness? Current total thickness of aluminum wall is 0.6 cm.	0.1 points
Answer	0.032402 cm (10% error acceptable)	0.1
	Total	

B2.1.15	How much will the cost increase if we replace the current design with a simple single aluminum wall that has the same shielding capabilities against 5 keV x-ray? Let us assume that the cost of a Falcon 9 flight is USD 2,720 per kg and the cost of the polyurethane foam is negligible. To reduce the burden of calculation, also assume only the cylindrical walls are changed but the plate at both ends are kept the same.	0.2 points
Answer	USD 4313.11 (10% error acceptable)	0.2
	Total	

B2.2.1	What is the kinetic energy of this $0.85c$ proton? Express it in units of either MeV or Joule.	0.5 points
Answer	842.568 MeV or 0.135 nJ (10% error acceptable)	0.5
	Total	

B2.2.2	A critical electronic chip in the spacecraft fails if it absorbs 1 mJ of energy. If 10,000 protons with the speed of $0.85c$ impact the chip per second. Estimate how many seconds it takes before failure occurs if every single proton is absorbed.	0.3 points
Answer	740.741 seconds (Answer between 740 to 741 seconds are acceptable)	0.3
	Total	

B2.2.3	How far would be the following proton energies penetrate into the human tissue in units of cm?	0.2 points
a)	10 MeV proton	
	0.127935 cm (0.05 points, 10% error acceptable)	
b)	20 MeV proton	
	0.445543 cm (0.05 points, 10% error acceptable)	
c)	50 MeV proton	
	2.341304 cm(0.05 points, 10% error acceptable)	
d)	100 MeV proton	
	8.134783 cm (0.05 points, 10% error acceptable)	

B2.2.4	Estimate the distance the proton can travel upon entering an astronaut's body after it passed through the spacecraft shielding. Assume the proton starts with the maximum energy (600 MeV) and loses 250 MeV in the shielding of the spacecraft. Also assume no energy loss when it travels through air inside the spacecraft.	0.5 points
Answer	65.340645 cm (10% error acceptable)	0.5
	Total	

B2.2.5	The stopping power of proton, $S(E)$ is related to the continuous slowing down approximation (CSDA) range via the relationship $R_{CSDA}(E) = \int_0^E \frac{dE}{S(E)}$ At high proton energy, the continuous slowing down range is approximately equal to the projected range. Estimate the stopping power in human tissue for this 350 MeV proton? The units of stopping power are $[\mathrm{MeV.cm^2/g}]$.	0.5 points
Answer	3.344176 MeV cm^2/g	0.5
	Total	

B2.2.6	The sharp peak you see in the energy deposition of proton is called the Bragg peak. If a 100 MeV neutron was shot from the same source, will it have a Bragg peak as well?	0.3 points
Answer	No Bragg peak	0.3
	Total	

B2.2.7	What is the possible merit of using protons in destroying cancer cells instead of x-rays and neutrons? Choose the correct answer. A. Protons are significantly cheaper to produce and deliver than X-rays and neutrons B. Protons can penetrate deeper into the body to reach the cancer cells C. We can deposit most of the proton energy exactly at the estimated location of the cancer cells D. Protons do not cause any damage to healthy tissues	0.2 points
Answer	C	0.2
	Total	

B2.2.8	Where do you expect to find the Bragg peak, if the tissue is shot with 100 MeV proton?	0.2 points
Answer	8.135 cm inside tissue or 13.135 cm from origin	0.2
	Total	

B2.2.9	Where do you expect to find the Bragg peak, if the tissue is shot with 800 MeV proton?	0.3 points
Answer	No Bragg peak	0.2
	Total	

B3.1.1	Find the amount of $^{235}_{92}U$ fissioned daily to operate this reactor.	0.5 points
Answer	$1.0525 \times P\left(\frac{\backslash \mathrm{mathr} grams}{\mathrm{day}}\right)$	0.5
	Total	

B3.1.2	Find the amount of $^{235}_{92}U$ consumed daily to operate this reactor.	0.5 points
Answer	$1.0525 \times P\left(\frac{\backslash \mathrm{mathr} grams}{\mathrm{day}}\right)$	0.5
	Total	

B3.1.3	How many kilowatt hours are there in one-megawatt day?	0.25 points
Answer	2.4×10^4 kWh	0.25
	Total	

B3.1.4	What is the specific burnup of the fuel at shutdown?	0.75 points
Answer	30586.47734 MWd/ton	0.75
	Total	

B3.1.5	Calculate the specific burnup of a CANDU reactor.	0.75 points
Answer	7.6464409 MWd/kg Equivalent to 7646.4409 MWd/ton	0.75
	Total	

B3.1.6	Find an expression for the amount of feed material in terms of the mass of product material.	0.5 points
Answer	$$M_F = \left(\frac{X_P-X_T}{X_F - X_T} M_P \right)	0.5
	Total	

B3.1.7	Using the burnup calculated in (Question B3.1.5), find the resource utilization for a CANDU reactor.	0.75 points
Answer	0.00804	0.75
	Total	

B3.1.8	Using the burnup calculated in (B3.1.4), find the resource utillization of a light water reactor (LWR). Which reactor has better fuel utillization?	1.0 points
Answer	LWR fuel utillization 0.005875096	0.5
	CANDU has better fuel utillization	0.5
	Total	

B3.2.1	Why do we choose Fluorine instead of Chlorine for the molten salt reactor? Choose your answer from one of the following options: i. Chlorine has a large absorption cross section at thermal energies; thus, it is less economical. ii. Fluorine forms a stronger chemical bond with both Beryllium and Lithium than Chlorine; thus, it is more stable at higher temperatures.	0.25 points
Answer	(i)	0.25
	Total	

B3.2.2	Estimate how many atoms of Li-6 are there in the IMSR given that the natural abundance of Li-6 is 7.5% and Li-7 is 92.5%	0.75 points
Answer	2.524×10^{26} nuclei (Answers between 2.45×10^{26} to 2.55×10^{26} nuclei are acceptable)	0.75
	Total	

B3.2.3	The height of the IMSR core is 4.0 m and the radius is 1.95 m. If the entire 278.5 kg of molten salt is in the reactor core, how much is the volume of the core are taken up by the reactor core components in percentage to the whole volume.	0.75 points
Answer	99.716% (Answers between 99.7% to 99.8% are acceptable)	0.75
	Total	

B3.2.4	The average thermal total cross section of Li-6 is 944.8 barns, its thermal elastic cross section is 798.0 mb, thermal neutron capture cross section is 38.9 mb and tritium production is 940.3 barns. How long would it take to burn out all the Li-6 in the molten salt? You may round it the closest number of days.	1.0 points
Answer	22 days (Answers between 21 to 22 days are acceptable)	1.0
	Total	

B3.2.5	Choose the wrong statement regarding the principle of how molten salt reactors differ from conventional nuclear reactors A. Molten salt reactors can use liquid salt as both coolant and fuel. B. Molten salt reactors with fluorine salts can use fast neutrons to transmute fission products. C. Molten salt reactors can recycle nuclear fuel, significantly reducing the volume of waste. D. Molten salt reactors with chlorine salts cannot use thermal neutrons to transmute fission products.	0.25 points
Answer	B	0.25
	Total	

B3.2.6	If 232 g of Th-232 are added into the fluoride salt, how much Th-233 atoms do we have after an hour of irradiation. Assume that the loss of Th-233 by fission, capture and decay in this period are negligible.	0.5 points
Answer	8.904×10^{18} atom (Answers between 8.5×10^{18} atom to 9.0×10^{18} atom are acceptable)	0.5
	Total	

B3.2.7	Th-233 can then be separated into isotope X. What is isotope X?	0.25 points
Answer	Pa-233	0.25
	Total	

B3.2.8	How much U-233 atoms do we expect to get, 30 days after separation?	1.25 points
Answer	4.783×10^{18} (Answer between 4.0×10^{18} to 5.0×10^{18} are acceptable)	1.25
	Total	

Q6

Q6.1.1	Calculate the Q-value in [MeV] of the Rn-222 alpha decay
0.2 pts	$Q = 16.374 - 8.358 - 2.425 = 5.591\ MeV$ **ANSWER: 5.591 MeV** **(Only exact answers are accepted)**

Q6.1.2	Calculate the kinetic energy of the alpha particle emitted from the decay of Rn-222.
1.0 pts	In this alpha decay, the total kinetic energy is equal to its Q-value, thus: $Q = \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_1v_1^2 \quad (1)$ From conservation of momentum $m_1v_1 = -m_2v_2$ $v_1 = -v_2\frac{m_2}{m_1} \quad (2)$ Squaring Eq. (2) $v_1^2 = v_2^2\left(\frac{m_2}{m_1}\right)^2$ Since $m_1 = \sim 4u$ and $m_2 = \sim 218u$, then $v_1^2 = v_2^2\left(\frac{m_2}{m_1}\right)^2 = v_2^2\left(\frac{218u}{4u}\right)^2 = v_2^2(2970.25) \quad (3)$ Inserting Eq. (3) into Eq. (1), $5.591 = \frac{1}{2}(218u)v_2^2 + \frac{1}{2}(4u)(2970.25\ v_2^2)$ Because $1u = 931.494\ MeV/c^2$, then $5.591 = 1.0153 \times 10^5\ \text{MeV}\frac{v_2^2}{c^2} + 5.5335 \times 10^6\ \text{MeV}\frac{v_2^2}{c^2}$ $\frac{v_2^2}{c^2} = 9.9218 \times 10^{-7}$ $v_2 = 0.00099608173c$ Thus, using Eq. (3), we can calculate the speed of the alpha particle $\frac{v_1^2}{c^2} = \frac{v_2^2}{c^2}\ 2970.25$ $v_1^2 = (9.9218 \times 10^{-7})(2970.25) = 0.00294702264\ c^2$ $v_1 = 0.05428648676c$ Since $v_1 \ll c$, relativistic effects can be ignored, and the kinetic energy of alpha particle is $K_1 = 0.5(4)(931.494\ MeV/c^2)(0.00294702264\ c^2)$ $K_1 = 5.490\ \text{MeV}$ **ANSWER: 5.490 MeV** **(Answers between 5.4 MeV to 5.6 MeV can be accepted)**

Q6.1.3	What is the maximum possible kinetic energy of the neutrons produced from an reaction?
0.2 pts	We first calculate the Q-value of the $\alpha + {}^{9}_{4}Be \rightarrow n + {}^{12}_{6}C$ reaction, $Q = 2.425 + 11.348 - 8.071 - 0.00 = 5.702\ \text{MeV}$ Given that the alpha particle that induced the reaction has $K_1 = 5.490\ \text{MeV}$, the total available energy is: $E_{av} = Q + K_1 = 5.702 + 5.490 = 11.192\ \text{MeV}$ Because this process involved at least three end states, it is very difficult to employ momentum conservation to calculate the kinetic energy of the neutron. However, assuming maximum energy transfer to the neutron (i.e., carbon recoils minimally), the maximum possible neutron energy is ~11.192 MeV. **ANSWER: 11.192 MeV** **(Answers between 11.1 MeV to 11.3 MeV is acceptable)**

Q6.1.4	Natural uranium consists predominantly of the isotope U-238 with a natural abundance of 99.27%. What is the excitation energy of the compound nucleus that absorbed a thermal neutron?
0.2 pts	The Q-value of U-238 receiving a thermal neutron, $(E = \sim 0)$ $Q = \Delta_{U238} + \Delta_n - \Delta_{U239}$ $Q = 47.309 + 8.071 - 50.574 = 4.806\ \text{MeV}$ Assuming U-238 is at ground state, the excitation energy of the compound nucleus after receiving a neutron $E_x = 4.806\ \text{MeV}$ **ANSWER: 4.806 MeV** **(Only exact answers are accepted)**

Q6.1.5	Based on the empirical data provided in Table 2, which uranium nuclide can be fissioned with thermal neutrons?
0.2 pts	**U-235** can undergo thermal fission because its neutron binding energy (6.8 MeV) exceeds its threshold energy (6.5 MeV) ($\boldsymbol{B_n} > \boldsymbol{E_{th}}$). U-238 cannot, as its binding energy (5.5 MeV) is below its threshold (7.0 MeV).

Q6.1.6	If the Rn-Be neutron source is used to bombard a natural uranium sample, which uranium nuclide will fission?
0.2 pts	**Both U-235 and U-238 can fission.** U-235 fissions with thermal and fast neutrons. U-238 requires neutrons above 7.0 MeV, which are present in the Rn-Be spectrum (up to ~11 MeV).

Q6.2.1	Measurements on an experimental thermal nuclear reactor show that for every 100 fast neutrons produced from fission, as the neutrons slow down, 8 neutrons are lost due to leakage, but no neutrons are absorbed. After becoming thermalized, another 22 neutrons leak out. Among the thermal neutrons that are absorbed within the reactor, 62% are absorbed in the fuel, while the rest are absorbed in non-fuel materials. The reproduction factor (η) or neutrons produced per absorption in fuel is 2.07. Express your answer with 5 significant figures for the following. (a) Calculate the multiplication factor (κ) and reactivity in pcm (ρ) of the reactor based on these observations. What is the reactor's criticality state? (b) A neutron reflector is added in the reactor system to reduce the thermal neutron leakage by half; what is the new value of κ and ρ ? Did the criticality state change?
0.6 pts	$N_{i-1} = 100$ neutrons 8 leak during slowing down → 92 remain 25 thermal neutrons leak → 67 remain Fuel absorption → $0.61 \times 67 = 40.9$ Neutron reproduction → $40.9 \times 2.07 = 84.601$ $k = \frac{N_i}{N_{i-1}} = \frac{84.601}{100} = \mathbf{0.84601}$ **(0.84551 to 0.84651 are accepted)** $\rho = \frac{0.84601-1}{0.84601} = -0.18202 \frac{\Delta k}{k} = \mathbf{-1820.2\ pcm}$ **(-1827.2 to -1813.2 are accepted)** The reactor is **subcritical**.

Q6.2.2	A neutron reflector is added in the reactor system in Q6.2.1 to reduce the thermal neutron leakage by half; what is the new value of k and ρ? Did the criticality state change?
0.6 pts	$N_{i-1} =$ 100 neutrons 8 leak during slowing down → 92 remain 12.5 thermal neutrons leak → 79.5 remain Fuel absorption → $0.61 \times 79.5 = 48.5$ Neutron reproduction → $48.5 \times 2.07 = 100.38$ $k = \frac{N_i}{N_{i-1}} = \frac{100.38}{100} = \mathbf{1.0038}$ **(1.0035 to 1.0041 are accepted)** $\rho = \frac{1.0038-1}{1.0038} = 0.0037856 \frac{\Delta k}{k} = \mathbf{37.856\ pcm}$ **(34.878 to 40.833 are accepted)** Yes, the reactor is now **supercritical**.

Q6.2.3	Given $l_p = 1 \times 10^{-4}$ seconds, calculate the reactor period for the two reactor systems described in Q6.2.1 and Q6.2.2.
0.6 pts	For 3.2.1: $T = \frac{l_p}{\rho} = \frac{10^{-4}}{-0.18202} = -5.494 \times 10^{-4}\text{s}$ For 3.2.2: $T = \frac{l_p}{\rho} = \frac{10^{-4}}{0.0037856} = 2.642 \times 10^{-2}\text{s}$

Q6.2.4	For the reactor system in Q6.2.2, by what factor will the neutron population increase if the time elapsed is 0.3 second?
0.3 pts	$\frac{N(t)}{N_0} = e^{t/T} = \exp\left(\frac{0.3\text{ s}}{2.642 \times 10^{-2}\text{s}}\right) = e^{11.36} = 3.491 \times 10^{16}$

Q6.3.1	Given a 30 MW HTGR that operates with a core power density of 2.5 W/cm3, estimate the total number of TRISO particles required to fuel the reactor. Note that 99.2% of the core consists of graphite moderators, coolant channels, and voids. Assume that each TRISO particle contributes equally to the power density.
0.4 pts	Total radius of TRISO particle: $r = 0.25 + 0.1 + 0.04 + 0.035 + 0.04 = 0.465\ \text{mm} = 0.0465\ \text{cm}$ Volume of TRISO particle: $V_{TRISO} = \frac{4}{3}\pi(0.0465)^3 = 4.21 \times 10^{-4}\text{cm}^3$ Thermal power output: $P_{th} = 30\ \text{MW} = 3.0 \times 10^7\text{W}$ Core power density: $\rho_P = 2.5\text{W/cm}^3$ The volume of the core is obtained by: $V_{core} = \frac{P_{th}}{\rho_P} = \frac{3.0 \times 10^7\text{W}}{2.5\ \text{W/cm}^3} = 1.2 \times 10^7\text{cm}^3$ The core volume occupied by TRISO fuel is: $V_{TRISO,tot} = (0.8\%)(1.2 \times 10^7\text{cm}^3) = 9.60 \times 10^4\text{cm}^3$ The number of TRISO particles that will fill the core volume is $N = \frac{V_{TRISO,tot}}{V_{TRISO}} = \frac{9.60 \times 10^4\text{cm}^3}{4.21 \times 10^{-4}\text{cm}^3} = \mathbf{2.28 \times 10^8}$

Q6.3.2	Calculate the total mass of 8% enriched uranium in a single TRISO particle if the kernel consists of UO_2 with a density of 10.5 g/cm³ and a diameter of 500 µm. Note: $M_{U235} = 235.044$ g/mol, $M_{U238} = 238.051$ g/mol, and $M_O = 16.000$ g/mol.
0.8 pts	Density of UO2: $$\rho_{UO2} = 10.5\ \mathrm{g/cm^3}$$ Kernel radius: $$r = \frac{500\ \mu\mathrm{m}}{2} = 0.025\ cm$$ Molar masses: $M_{U235} = 235.044$ g/mol, $M_{U238} = 238.051$ g/mol, $M_O = 16.000$ g/mol. The molar mass of 19.7% enriched UO2: $M_U = 0.08M_{U235} + 0.92M_{U238} = 237.810$ g/mol $M_{UO_2} = M_U + 2 \cdot M_O = 237.459 + 2(16.000) = 269.810$ g/mol The mass fraction of enriched UO2: $$f_U = \frac{M_U}{M_{UO2}} = \frac{237.459}{269.810} = 0.8814$$ The total mass of UO2 in the kernel: $$m_{UO_2} = \rho V = \rho\left(\frac{4}{3}\pi r^3\right) = (10.5)\left(\frac{4}{3}\pi 0.025\ \mathrm{cm}^3\right) = 6.872 \times 10^{-4}\mathrm{g}$$ The mass of uranium in the TRISO kernel is then: $$m_U = f_U \cdot m_{UO2} = (0.8814)(6.872 \times 10^{-4}\mathrm{g})$$ $$m_U = \mathbf{6.057 \times 10^{-4}g = 0.6057\ mg}$$

Q6.3.3	Calculate the total initial uranium inventory in a 12m3 HTGR core.
0.4 pts	Number of TRISO particles in the core: $N = 2.28 \times 10^8$ particles Mass of uranium in a single TRISO kernel: $m_U = 6.057 \times 10^{-4}$g Total initial uranium mass: $m_{U,tot} = N \cdot m_U = (2.28 \times 10^8)(6.057 \times 10^{-4}) = \mathbf{1.381 \times 10^5\ g}$

Q6.3.4	Determine the burnup in %FIMA (Fissions per Initial Metal Atom) for a 30 MW HTGR assuming constant power operation for 2 years. Note: Avogadro's Number, $N_A = 6.022 \times 10^{23}$ atoms/mol, Energy released per fission ~3.204×10^{-11} J
0.8 pts	The initial number of metal atoms: $N_{atoms} = \frac{m_{U,tot}}{M_U} N_A = \left(\frac{1.381 \times 10^5 \text{ g}}{237.810 \text{ g/mol}}\right)\left(6.022 \times 10^{23} \frac{\text{atoms}}{\text{mol}}\right)$ $= 3.496 \times 10^{26}$ atoms The total energy produced for 2 year continued operation is $E_{tot} = P \cdot t = \left(2.0 \times 10^7 \frac{\text{J}}{\text{s}}\right)\left(3 \text{ y} \cdot \frac{365 \text{ d}}{\text{y}} \cdot \frac{24 \text{ h}}{\text{d}} \cdot \frac{3{,}600 \text{ s}}{\text{h}}\right)$ $= 1.892 \times 10^{15}$ J Given the energy released per fission $E_f = 3.204 \times 10^{-11}$J/fission, the total number of fissions is obtained as: $N_f = \frac{E_{tot}}{E_f} = \frac{1.892 \times 10^{15}\text{J}}{3.204 \times 10^{-11}\text{J/fission}} = 5.906 \times 10^{25}$ fissions The %FIMA is therefore: $\%FIMA = \frac{N_f}{N_U} = \frac{8.858 \times 10^{25}}{3.496 \times 10^{26}} = \mathbf{16.89\%}$

Q6.3.5	In an experimental HTGR setup, TRISO fuel particles that have been irradiated to a burnup of ~10% FIMA were heated at $1600°C$ for 300 hrs to simulate an accident scenario. The fuel initially generated 1.200×10^{-6} mol of Cs-137 per gram of uranium. At the end of the experiment, only 2.400×10^{-9} mol of Cs-137 was measured to have escaped through the SiC layer from a sample containing 5.000 g of uranium. Determine the percentage of Cs-137 retention and explain its implication. Express your answer with 4 significant figures.
0.4 pts	Cs-137 generated per g U: 1.2×10^{-6} mol/g U mass: $m_U = 5$ g Cs-137 escaped: 2.4×10^{-9} mol $\%Retention = 1 - \%Escaped$ $= 1 - \frac{2.400 \times 10^{-9}\ \text{mol}}{(5.000\ \text{g})(1.200 \times 10^{-6}\ \text{mol/g})} \times 100$ $= \mathbf{99.96\%}$ Implication: A 99.96% retention of Cs-137 at 1600 °C for 300 hours shows that the SiC layer in TRISO fuel is **highly effective at containing fission products**. Even under accident conditions, **radiological release remains minimal**.

Q6.3.6	Derive the expression for time from the above equation. Assume that the argument for the error function is << 1.
0.8 pts	Simplified Fick's law: $$n_t = n_0 \mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$ Maclaurin series: $$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\left(z - \frac{1}{3}z^3 + \frac{1}{10}z^5 + \cdots\right)$$ Assuming: $z \ll 1$, the Maclaurin expansion of the error function can be truncated: $$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}z$$ Hence, Fick's second law can be approximated as: $$n_t = n_0\left(\frac{2}{\sqrt{\pi}}\right)\frac{x}{2\sqrt{Dt}}$$ $$\frac{n_t}{n_0} = \frac{x}{\sqrt{\pi D t}}$$ Solving for t: $$\boldsymbol{t = \frac{x^2}{\pi D}\left(\frac{n_0}{n_t}\right)^2}$$

Q6.3.7	Given that the diffusion coefficient of Krypton in SiC is $6.5 \times 10^{-20}\ \mathrm{m^2 s^{-1}}$, estimate the time required for 1% of Krypton to escape through a 35 μm thick SiC layer.
0.4 pts	$\frac{n_0}{n_t} = \frac{1}{1\%} = 100$ $x = 35\mu\mathrm{m} = 3.5 \times 10^{-5}\mathrm{m}$ $D = 6.5 \times 10^{-20}\ \mathrm{m^2/s}$ $t = \frac{(3.5 \times 10^{-5}\mathrm{m})^2}{\pi(6.5 \times 10^{-20}\ \mathrm{m^2/s})}(100)^2 = \mathbf{6 \times 10^{13}\ s = 1.96 \times 10^6\ y}$

Q 6.2.5	Since the amount of neutrons in a reactor is proportional to its power output, briefly explain the implication of the result from Q6.2.3.
0.3 pts	