THEORETICAL

EXAMINATION

Answer All the Questions
(5 Hours)

1. Answer ALL questions preferably in **English**.
2. Write all your **answers including steps on the answer sheet**. You will not be allocated any time to write down your answers at the end of the exam.
3. Write your answers including steps using ink **pens**. Final answers must be written in ink to ensure clarity when the answer sheets are scanned.
4. You can answer the questions **in any order you like**, but make sure to manage your time carefully.
5. Your file contains 4 different signs (Toilet, Kit, Medic, Question). Raise the appropriate sign whenever you need assistance related to it. Toilet and Medic is self-explanatory. You can raise Kit for request extra stationary if you need; and Question can be raised when you received incomplete or incorrect question sheet.

Question 1.1 Imaging (X-Ray and CT Scan) (1.2 pts)

A 60-year-old woman visited a general practitioner and presented with complaints of increasing fatigue, a mild cough and difficulty in breathing. Over the past three months, she also observed a weight loss of approximately 10 kg, which she denied attributing to any intentional weight loss regimen. The attending doctor ordered an X-ray of the chest. Radiographically, there are few round opacities seen at the right hilar region as shown in Figure 1.

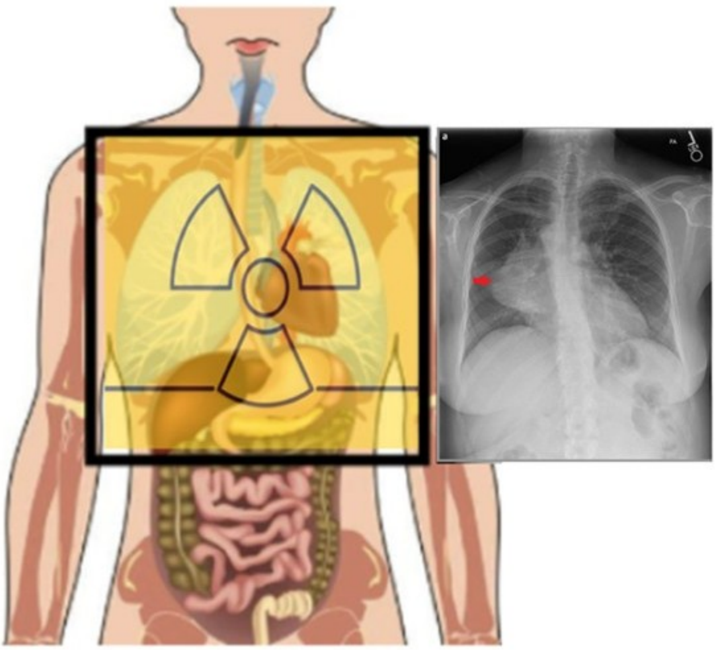


Fig. 1: A right middle lobe mass (arrow) chest X-ray of female patient.

According to the International Commission on Radiological Protection (ICRP) Publication 103, the tissue weighting factor for the breast is 0.12. The breast was exposed to an absorbed dose of 0.2 mGy from X-ray irradiation. Whereas the **radiation weighting factor for X-ray is 1**.

Q1.1.1	Calculate the equivalent dose, *H* to the breast.	0.3 pts

Q1.1.2	Calculate the effective dose, *E*.	0.3 pts

Unfortunately, the radiograph revealed only a large right perihilar mass. Therefore, a follow-up 256-slice computed tomography (CT) chest scan with contrast was suggested. The scan provided a detailed view of a mass with significant invasion of mediastinal structures.

In estimating radiation dose to the patient, CT Dose Index Weight ($CTDI_W$) and CT Dose Index Volume ($CTDI_{vol}$) can be used where $CTDI_{vol}$ represent the $CTDI_W$ divided by the pitch. Alternatively, another parameter to represent the total energy deposited in the patient is the Dose Length Product (DLP). It can be estimated by multiplying $CTDI_{vol}$ and scan length. For this patient, the chest CT Scan has the following parameters:

CTDI_w: 30 mGy

Pitch: 1.2

Scan Length: 60 cm

Q.1.1.3	Calculate the CTDI_{vol}	0.3 pts

Q1.1.4	Calculate the Dose Length Product (DLP).	0.3 pts

Question 1.2 Imaging (PET-CT) (2.0 pts)

Noticing the abnormality, the woman was referred to a respiratory clinic for further evaluation. She underwent a hybrid PET-CT scan, which provided detailed insights into cancer staging, complementing the earlier CT-guided biopsy. Positron Emission Tomography-Computed Tomography (PET-CT) combines functional (PET) and anatomical (CT) imaging to enhance diagnosis.

Fluorine-18 fluorodeoxyglucose (F-18 FDG) PET-CT confirmed a primary lung neuroendocrine tumor with metastases to the liver, bone, and thyroid, exhibiting high glucose metabolism as indicated by significant FDG uptake. Additionally, Ga-68 DOTATATE PET-CT demonstrated high somatostatin receptor expression, identifying the patient as a suitable candidate for Lutetium-177 (Lu-177) DOTATATE peptide receptor radionuclide therapy (PRRT) (Figure 2). Furthermore, increased iodine uptake in the thyroid was noted, and subsequent radioiodine imaging (I-131 scan) confirmed metastatic differentiated thyroid cancer (DTC) with iodine avidity.

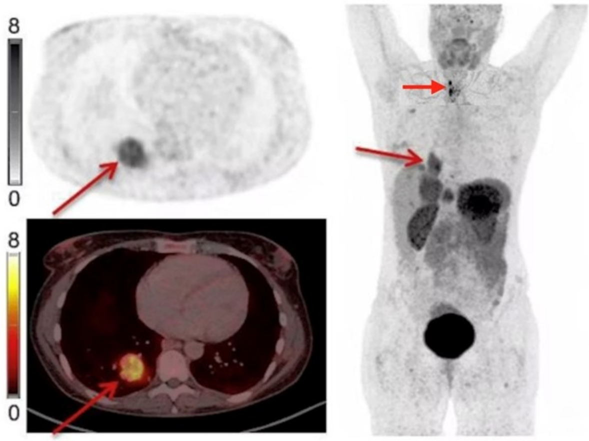


Fig. 2 Trans axial PET and fused PET/CT images of the patient. The arrow indicates the primary tumor in the lung and thyroid, along with a maximum intensity projection (MIP) image featuring color bars.

Q1.2.1	In a PET-CT scan, a positron emitted by a Fluorine-18 radionuclide collides with an electron, producing two gamma photons of 511 keV each. Calculate the wavelength of the 511 keV gamma photons.	0.3 pts

Q1.2.2	If the distance between the source and the sensors differs by 40 cm, with one at x cm and the other at (x + 40) cm, what is the time difference between the two gamma photons reaching opposite detectors?	0.3 pts

The patient is scheduled for an FDG PET-CT scan to evaluate a suspected lung tumor. The radiopharmacy prepares the Fluorine-18 FDG dose, and the following conditions are given:

Given data:

1. Patient’s mass: 75 kg
2. Required FDG dose for a patient: 4.5 MBq/kg
3. Stock FDG solution activity concentration at 7:00 a.m: 600 MBq/ml
4. The dose is prepared at 7:00 AM, but the injection will be scheduled to be performed at 9:00 AM.
5. Half-life of Fluorine-18: 110 minutes
6. Minimum required activity for an optimal scan at 9:30 AM: 250 MBq

Q1.2.3	Calculate the required activity (MBq) for the patient based on her mass.	0.2 pts

Q1.2.4	Determine the volume (ml) of FDG solution needed to prepare based on the activity at Q1.2.3.	0.2 pts

Q1.2.5	Calculate the initial activity of the prepared dose at 9:00 AM, considering the decay from 7:00 AM to 9:00 AM.	0.5 pts

Q1.2.6	Determine the remaining activity at 9:30 AM and check whether the dose is still sufficient for the scan.	0.5 pts

Q1.2.7	What is the minimum radioactive concentration of the stock that will be required to be prepared at 7:00 a.m if the injection is to be given at 9:30 a.m.	0.5 pts

Question 1.3 Personalized Nuclear Medicine Treatment Plan using Lutetium-177 Peptide Receptor Radionuclide Therapy (PRRT) and Iodine-131 Therapy. (6.3 pts)

Given the dual malignancy, the patient was scheduled for both Lutetium-177 (Lu-177) and Iodine-131 (I-131) therapy. This required careful coordination to maximize therapeutic benefits while minimizing radiation exposure risks.

The case was discussed in a multidisciplinary tumor board, where a **personalized nuclear medicine treatment plan** was developed. The treatment approach integrated Lu-177 therapy to target somatostatin receptor (SSTR) expressing tumors and metastases and I-131 therapy for the iodine-avid thyroid cancer to ensure effective tumor ablation. This strategy aimed to enhance overall treatment efficacy, control tumor progression, and ensure adherence to radiation safety protocols.

Recent advancements in nuclear medicine, particularly the use of radiopharmaceuticals for **targeted cancer therapy**, have opened new pathways for precision medicine. A new radiopharmaceutical developed in 2023, using Lu-177 as the radioactive isotope, has shown promising results in treating neuroendocrine tumors. The mechanism of action involves Lu-177 binding to specific receptors on neuroendocrine tumor cells, delivering a targeted dose of radiation while minimizing damage to surrounding healthy tissue.

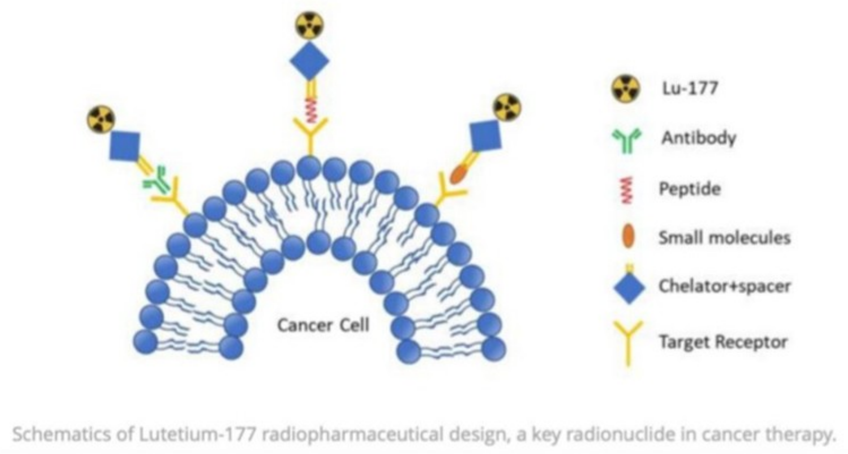


Figure 3: Schematics of Lutetium-177 radiopharmaceutical design, a key radionuclide in cancer therapy

Q1.3.1	A nuclear medicine pharmacist prepares Lutetium-177 (Lu-177) radiopharmaceutical in a hot lab. If the radiation dose rate at 1 meter from a 7.4 GBq source is 4.5 mSv/h, calculate the minimum thickness of lead shielding required to reduce the exposure level to ≤ 0.1 mSv/h. (Use lead HVL for Lu-177 gamma emissions: 0.06 cm).	1.0 pts

Before proceeding with the injection, the radiopharmacist is preparing a Lutetium-177 (Lu-177) radiopharmaceutical sample for a radiochemical purity test in the laboratory.

The following conditions are given at the moment of sample preparation:

1. The required activity for the test is 50 MBq
2. The initial activity concentration of the Lu-177 solution is 200 MBq/ml
3. The sample preparation requires a total volume of 5 ml, obtained by mixing the radiopharmaceutical with a saline solution.

Q1.3.2	Calculate the volume of Lu-177 solution needed to achieve the required activity of 50 MBq in the final preparation.	0.3 pts

Q1.3.3	Calculate the volume of saline solution required to reach the total volume of 5 ml.	0.3 pts

Q1.3.4	If the half-life of Lu-177 is 6.65 days, determine how much activity will remain in the prepared sample after 4 hours.	1.0 pts

Q1.3.5	Lu-177 undergoes radioactive decay according to the equation: Lu-177 → Hf-177 + β^- + energy What types of radiation are emitted in this process?	0.2 pts

Q1.3.6	What role does the specific molecule (e.g. peptide or antibody) attached to Lu-177 play in its theraupeutic action?	0.1 pts

Q1.3.7	Why is the use of receptor-targeting molecules important for minimizing damage to healthy tissues?	0.2 pts

Q1.3.8	What is the main advantage of Lu-177 therapy compared to chemotherapy, a treatment that uses non-targeted chemicals to kill rapidly dividing cells?	0.1 pts

Q1.3.9	How does Lu-177 therapy minimize damage to surrounding healthy tissue compared to external beam radiation?	0.1 pts

A nuclear medicine department prepares a patient-specific therapeutic dose of Lutetium-177 (^{177}Lu) for Peptide Receptor Radionuclide Therapy (PRRT) in a neuroendocrine tumor patient.

Given the following information:

- Initial syringe activity at calibration time: 550 MBq
- Calibration time: 10:00 AM
- Half-life of Lutetium-177: 6.65 days
- Patient injection time: 2:00 PM (4 hours after calibration)
- Remaining syringe activity after injection: 38 MBq

Q1.3.10	Calculate the Decay Factor during the 4-hour period before injection.	0.5 pts

Q1.3.11	Determine the activity at the time of patient injection.	0.3 pts

Q1 Cancer Detection

Q1.3.12	Calculate the actual activity received by the patient.	0.3 pts

Q1.3.13	Verify based on calculation if the administered activity is within the standard therapeutic range of 4-7 GBq (4000-7000 MBq)	0.4 pts

Q1.3.14	Patient is treated with Lu-177-DOTATATE for neuroendocrine tumours. Given a dose of 7.4 GBq, Estimate the absorbed activity to the lung if the residence time is 10 hours and the S-value (radionuclide activity factor) is 0.12 $mGyMBq^{-1}h^{-1}$.	0.5 pts

Q1.3.15	A patient receiving I-131 therapy for thyroid cancer is required to stay in isolation for a specific period. Assuming the initial administered activity is 5 GBq, calculate when can the patient be released based on the 5 μSv/h at 1 meter guideline? * For I-131, the exposure rate constant (Γ) = 0.0595 $\mu Sv \cdot m^2 MBq^{-1} h^{-1}$ * I-131 half-life = 8.02 days.	1.0 pts

Question 2.1 Satellite Power (4.0 pts)

Harvesting the solar energy from the Sun however becomes more challenging when we need to generate power for space probes going beyond the vicinity of earth. One such space probes are the Cassini space missions. This space probes were sent to Saturn and it manages to capture some of the best pictures of the planet and improved our understanding of the Solar System.

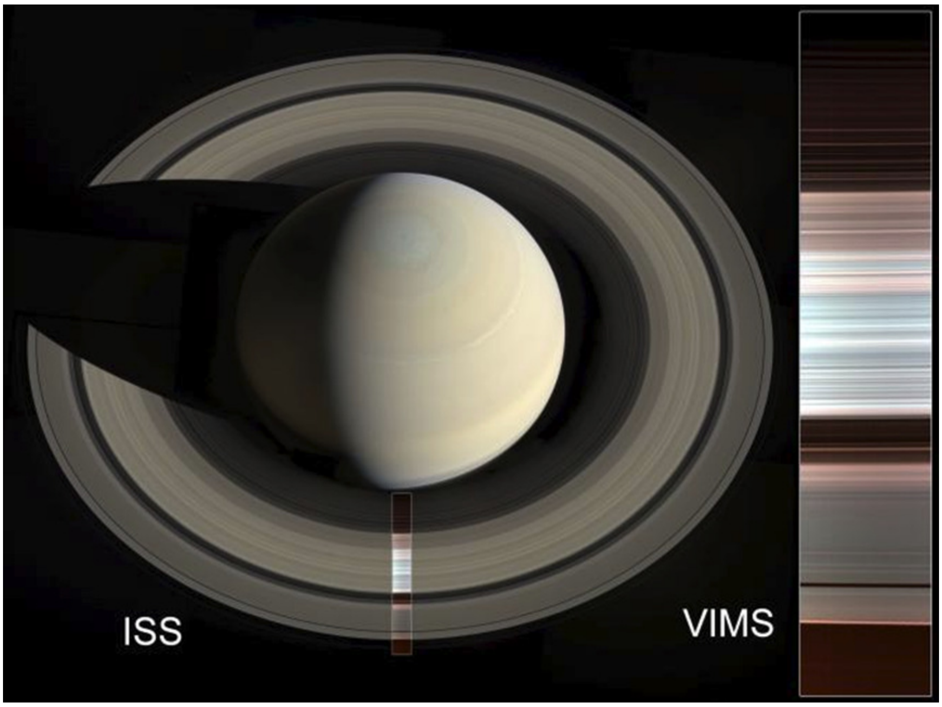


Fig. 2 Image of Saturn captured by ISS and VIMS instruments in Cassini. Source: NASA

Obviously, these missions need to be powered in order to operate the computers, cameras and other instruments of the probe. However, we can no longer rely on solar cells to power the equipment and charge the batteries. Thus, we had to rely on Radioisotope Thermoelectric Generator (RTG) to produce the energies required. RTG's harvest the heat generated by the alpha decay of $^{238}_{94}Pu$. In future missions, RTG's made of $^{241}_{95}Am$ are also studied by the European Space Agency (ESA).

Unfortunately, due to the low temperature generated by the decay, the efficiency of the RTG is quite low. The highest efficiency achieved by the RTG is around 7%. In nuclear reactions, the mass excess, defined as the difference between the actual mass of a nucleus and its mass number, is used to calculate reaction Q-values. The nuclear data related to this decay is given in the table below.

Isotope	Decay Type	Mass Excess [MeV]	Half Life [s]
$^{238}_{94}Pu$	Alpha	46.165	2.768×10^{9}
$^{239}_{94}Pu$	Alpha	48.590	7.608×10^{11}
$^{241}_{94}Pu$	Beta	52.957	4.522×10^{8}
$^{241}_{95}Am$	Alpha	52.936	1.365×10^{10}
$^{242}_{95}Am$	Beta	55.470	5.767×10^{4}
$^{234}_{92}U$	Alpha	38.147	7.747×10^{12}
$^{235}_{92}U$	Alpha	40.820	2.222×10^{16}
$^{238}_{92}U$	Alpha	47.309	1.408×10^{17}
$^{239}_{92}U$	Beta	50.574	1.407×10^{3}
$^{237}_{93}Np$	Alpha	44.873	6.766×10^{13}
$^{238}_{93}Np$	Beta	47.456	1.814×10^{5}
$^{239}_{93}Np$	Beta	49.312	2.036×10^{5}
$^{233}_{91}Pa$	Beta	37.490	2.331×10^{6}
$^{207}_{81}Tl$	Beta	-21.034	2.862×10^{2}
$^{34}_{14}Si$	Beta	-19.957	2.77
$^{4}_{2}He$	Stable	2.425	Stable
$^{2}_{1}H$	Stable	13.136	Stable
$^{1}_{1}H$	Stable	7.289	Stable
neutron	-	8.071	-

Q2.1.1	What is the Q-value of the following decay process in units of MeV?	2.0 pts
a)	Pu-238 decay	
b)	Am-241 decay	
c)	Np-237 decay	
d)	Np-238 decay	
e)	Np-239 decay	

The RTG's used in the Cassini mission is called General Purpose Heat Source RTG (GPHS-RTG) and each module produce around 300 W of electricity from the 4400 W of thermal energy released by the decay of ${}^{238}_{94}Pu$. The Plutonium in PuO_2 used in the Cassini RTG are enriched by Pu-238 to 82.2wt%. There are two ways to produce ${}^{238}_{94}Pu$, the first way is by bombarding ${}^{238}_{92}U$ with deuteron,

$${}^{238}_{92}U + {}^{2}_{1}H \rightarrow {}^{238}_{93}Np + 2n$$

With ${}^{238}_{93}Np$ eventually decaying to ${}^{238}_{94}Pu$ by beta decay

$${}^{238}_{93}Np \rightarrow {}^{238}_{94}Pu + e^{-} + \overline{\nu}_e$$

However, most ${}^{238}_{94}Pu$ are produced as part of the process in producing weapons grade plutonium. A portion of ${}^{237}_{93}Np$ can be bombarded with neutrons to produce ${}^{238}_{93}Np$ first before ${}^{238}_{94}Pu$ are produced by beta decay

$${}^{237}_{93}Np + n \rightarrow {}^{238}_{93}Np \rightarrow {}^{238}_{94}Pu + e^{-} + \overline{\nu}_e$$

In the reactor, ${}^{237}_{93}Np$ could be produced when ${}^{239}_{94}Pu$ are bombarded via the $(n, 2n)$ process. The production of ${}^{237}_{93}Np$ could also happen either from double neutron absorption by ${}^{235}_{92}U$ or by the reaction of ${}^{238}_{92}U$, in which ${}^{237}_{92}U$ are produced. From ${}^{237}_{92}U$, we can get ${}^{237}_{93}Np$ by beta decay. After separating Plutonium from other elements, the resulting mix can be enriched to get either a higher portion of ${}^{239}_{94}Pu$ for weapons or ${}^{238}_{94}Pu$ for RTG, pacemakers and other utilities.

Q2.1.2	The Cassini mission carried with it, three GPHS-RTG modules on board the space probe. How many kilograms of PuO2 does the space probe carry? [Given: $N_A = 6.022141 \times 10^{23}$]	2.0 pts

Question 2.2 Momentum of a Photon and Compton Camera (6.0 pts)

Conservation of energy is one of the most important laws in physics. We know that in classical physics, a system's total energy remains constant. In relativistic physics also, the energy remains constant, however, its definition is altered to include the possibility of mass changing to energy, as in the reactions that occur within a nuclear reactor. Consequently, we learn that several fundamental quantities are related in ways not known in classical physics. All of these relationships have been verified experimentally and have fundamental consequences. According to Einstein's famous relationship, the energy and mass are related by the equation

$$E = mc^2. \quad (2.1)$$

Where E is the total energy of the particle, c is the speed of light, and m is the mass of the particle moving with velocity v. The mass m of the moving body is related to mass m_0 of the body when it is at rest through the following expression

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}. \quad (2.2)$$

This equation indicates that as the velocity of a body increases towards the speed of light, its mass increases toward infinity. Thus, by combining the above two equations, the total energy E of the particle is

$$E = mc^2 = \frac{E_0}{\sqrt{1-\frac{v^2}{c^2}}}. \quad (2.3)$$

Here $E_0 = m_0 c^2$ is called the rest mass energy of the particle. The relativistic kinetic energy of the particle is determined by subtracting the rest mass energy from the total energy.

Q2.2.1	Write the relativistic and nonrelativistic expressions for the kinetic energy of a particle of mass m_0. Show that when $v \ll c$, the relativistic expression reduces to the nonrelativistic expression of the kinetic energy of the particle.	0.5 pts

Classically, the linear momentum p of a particle of rest mass m_0, moving with velocity v is defined as $p = m_0 v$. Momentum, p and kinetic energy, KE of the classical particles are related by $p = \sqrt{2m_0(KE)}$. However, the relationship between relativistic energy and relativistic momentum is different from the classical expression.

Q2.2.2	Use the relativistic mass formula to show that the relationship between the relativistic momentum, kinetic energy, and the rest mass energy is given by $p = (1/c)\sqrt{(KE)^2 + 2(KE)E_0}$	1.0 pts

Until 1900, scientists assumed that light travelled in the form of waves. However, phenomena such as the photoelectric effect and Compton scattering showed that it also possesses a particle nature. Even other forms of electromagnetic energy, such as X-rays and gamma rays, comprise quanta of energy called photons. What we know now is that photons are energy-containing particles. In nuclear reactions, the product nuclei are formed and are usually in the excited state. These excited nuclei emit gamma rays in the form of high-energy photons. When the gamma rays are emitted some energy is given to the recoiling nuclei to conserve the momentum. In light water reactors, the thermal neutrons combine with the hydrogen $\left({}_1^1H\right)$ to form deuterons $\left({}_1^2H\right)$. The deuteron formed is in an excited state and it emits a gamma ray to come to the ground state. Assume that the initial kinetic energies of the hydrogen and deuteron are negligible. The atomic weights of hydrogen, neutron, and deutron are, respectively, 1.007825 amu, 1.008665 amu, and 2.014102 amu.

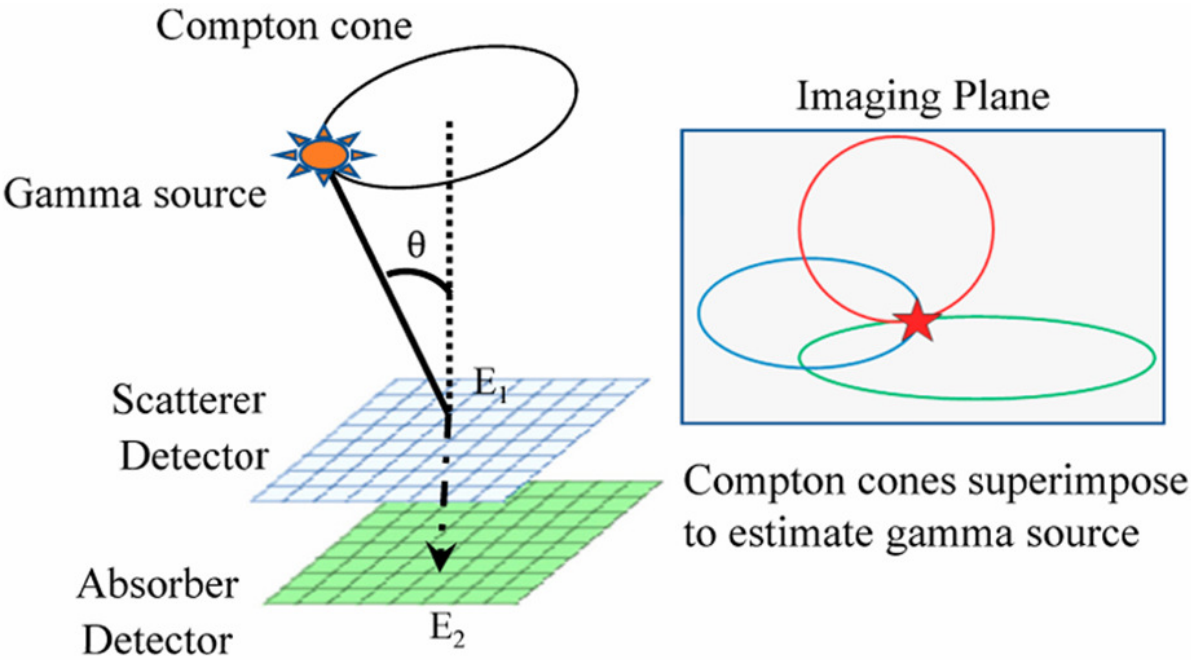
Q2.2.3	Find the energy of the gamma-ray emitted in this reaction. Also, calculate the kinetic energy and recoil velocity of the deuteron.	2.0 pts

Gamma rays can be detected with the help of Compton cameras. Compton cameras are based on the principle of Compton scattering and are used to determine the origin of observed gamma rays, allowing for imaging without the need for mechanical collimators. Compton cameras are used in fields like astrophysics, nuclear medicine, and nuclear threat detection.

One famous application of Compton camera is the Imaging Compton Telescope (COMPTEL) on the Compton Gamma Ray Observatory (CGRO). This instrument was used to detect gamma ray energies between 0.75 MeV to 30 MeV. The instrument has a field of view of one steradian. In cosmic ray event detections, two nearly simultaneous interactions are required in a set of front and rear scintillators. From the angle between incoming and scattered gamma rays, the energy of the gamma ray can be determined. By placing a sufficient delay between measurements, most background events can be eliminated. After a series of several cosmic ray events, a map of the positions of the gamma ray cosmic sources along with photon fluxes and spectra can be determined.

Fig. 3 Compton Gamma Ray Observatory. Source: NASA

In Compton scattering, a gamma-ray photon interacts with an electron, and there is a change in energy and direction of the gamma-ray photon. The Compton camera utilizes two detectors: one is referred to as a scatterer, and the other is referred to as an absorber. A proportion of any incident gamma rays will Compton scatter within the scatterer. Subsequently, a proportion of these scattered gamma rays will then undergo photoelectric absorption within the absorber, as shown in the Figure below.



The scatterer, which is typically a thin detector, measures the energy and position of the scattered electron (and thus the scattered gamma ray). In contrast, the absorber, which is usually a thicker detector, measures the energy and position of the absorbed gamma ray. In an ideal case where the gamma ray is fully absorbed, the energy deposited in the scatterer, E_1, plus the energy deposited in the absorber, E_2 gives the energy of the incident gamma ray, E_γ. The position of the source will be somewhere at an angle θ from the interaction point in the scatterer. As the direction of the incident gamma ray is unknown, the source could be anywhere on the surface of a cone with an opening angle θ .

Q2.2.4	Using quantum theory of radiation, A H Compton showed that, after scattering from an electron of mass, m_e, the change in wavelength Δλ of the photon is given by $\Delta\lambda = \frac{h}{m_e c}\left(1 - \cos\psi\right)$, where ψ is the angle between the incident and scattering directions of the photon. Using this relationship, find an expression for, $E_\gamma - E_{\gamma'}$ where E_γ and $E_{\gamma'}$ are energies of the photon before and after the scattering, respectively.	0.75 pts

Q2.2.5	Find an expression for the kinetic energy of the recoil electron in terms of the scattering angle, ψ. Show that the maximum kinetic energy of the recoiled electron is given by $$(KE)_{max} = \frac{2\,h^2\,\nu^2}{m_e c^2} / \left(1 + \frac{2\,h\nu}{m_e c^2}\right),$$ where ν is the frequency of the incident photon.	1.0 pts

Q2.2.6	Find an expression for the angle θ of the cone (in terms of energies E_1 and E_2), at the surface of which the gamma source can be found.	0.75 pts

Question 3.1 History of Nuclear Fission (2.0 pts)

After the discovery of neutrons in 1932, Enrico Fermi began bombarding various elements with neutrons to investigate the resulting reactions. He mainly used a radon-beryllium (Rn-Be) source, where alpha particles emitted from the decay of Rn-222 interact with beryllium nuclei, producing neutrons through the (α, n) reaction. Fermi's experiments led to the discovery of nuclear fission in 1938 by Otto Hahn, Lise Meitner, and Fritz Strassmann when they bombarded the heaviest naturally occurring element (uranium) with neutrons. In nuclear reactions, the mass excess (Δ), defined as the difference between the actual mass of a nucleus and its mass number, is used to calculate reaction Q-values and energy balances. The mass excess of nuclear particles involved in the reactions are provided in Table 1.

Table 1: *Mass excess of nuclear particles.*

Particles	Mass Excess, Δ [MeV]	Particles	Mass Excess, Δ [MeV]
$^{239}_{92}U$	50.574	$^{12}_{6}C$	0.00
$^{238}_{92}U$	47.309	$^{9}_{4}Be$	11.348
$^{236}_{92}U$	42.446	$^{8}_{4}Be$	4.942
$^{235}_{92}U$	40.920	$^{4}_{2}\alpha$	2.425
$^{222}_{86}Rn$	16.374	$^{1}_{0}n$	8.071
$^{218}_{84}Po$	8.358	$^{1}_{1}p$	7.289

Note: Express answers with **4 significant figures** and use the following constants for the questions below:

$$1u = 931.5\ \mathrm{MeV/c^2} \qquad c = 2.998 \times 10^8 \mathrm{m/s}$$

Q3.1.1	Calculate the Q-value in MeV of the Rn-222 alpha decay.	0.2 pts

Q3.1.2	Calculate the kinetic energy of the alpha particle emitted from the decay of Rn-222. Note that the daughter nuclide also has a recoil speed.	1.0 pt

Q3.1.3	What is the maximum possible kinetic energy of the neutrons produced from an $\mathrm{Be}(\alpha, n)\mathrm{C}$ reaction?	0.3 pts

When a neutron is absorbed by a fissionable nuclide, a compound nucleus is formed with excitation energy (E_x) that is roughly equal to the binding energy of the last neutron (B_n) plus its kinetic energy. For fission to occur, the E_x must exceed a threshold energy (E_{th}) to overcome the fission barrier.

Table 2: *Threshold energy (E_{th}) and binding energy of last neutron (B_n) of uranium isotopes*

Particles	E_{th} (MeV)	B_n (MeV)
${}^{238}_{92}U$	7.0	5.5
${}^{235}_{92}U$	6.5	6.8

Q3.1.4	Based on the data provided in Table 2, which uranium nuclide can be fissioned with thermal neutrons ($E = 0.025$ eV)?	0.2 pts

Q3.1.5	Based on the result of Q3.1.3, if the Rn-Be neutron source is used to bombard a natural uranium sample, which uranium nuclide will fission?	0.3 pts

Question 3.2 Control of Fission (4.0 pts)

Fission of a U-235 nucleus produces two lighter fragments, releases additional neutrons, and ~200 MeV of energy. On average, 2-3 neutrons are also emitted per neutron-induced fission. Neutrons produced from fission are classified into two types. Prompt neutrons are released within seconds and account for over 99% of the total. Delayed neutrons make up a small fraction and are emitted seconds to minutes later due to the decay of certain fission products.

The presence of both prompt and delayed neutrons plays an important role in determining how the neutron population changes over time. The change in neutron population is quantified by the multiplication factor k, which is defined as the ratio of the number of neutrons produced in the current generation (N_i) to those in the previous generation (N_{i-1}).

The reactor's criticality state or ability to sustain fission reaction is determined as follows: $k = 1$ means the reactor is critical and self-sustaining; $k < 1$ means it is subcritical, with the neutron population declining; and $k > 1$ means it is supercritical, with the neutron population and power increasing. Deviation from criticality is an important aspect of reactor safety and is measured in terms of reactivity (ρ):

$$\rho = \frac{k-1}{k}$$

Positive ρ means supercritical and negative ρ means subcritical. It is a dimensionless quantity that is often expressed in pcm (percent mille), where 1,000 pcm = 0.01 $\Delta k/k$.

Note: Provide numerical answers with **5 significant figures** for the questions below.

Q3.2.1	Measurements on an experimental thermal nuclear reactor show that for every 100 fast neutrons produced from fission, as the neutrons slow down, 8 neutrons are lost due to leakage, but no neutrons are absorbed. After becoming thermalized, another 25 neutrons leak out. Among the thermal neutrons that are absorbed within the reactor, 61% are absorbed in the fuel, while the rest are absorbed in non-fuel materials. The reproduction factor (η) or neutrons produced per absorption in fuel is 2.07. Calculate the multiplication factor (k) of the reactor based on these observations. What is the reactor's criticality state?	0.6 pts

Q3.2.2	A neutron reflector is added in the reactor system to reduce the thermal neutron leakage by half; what is the new value of k? Did the criticality state change?	0.6 pts

The reactor period (T) is the time it takes for the neutron population (and reactor power) to change by a factor of $e =\sim 2.718$. The relationship is expressed as follows:

$$N\left(t\right) = N_0 e^{\frac{t}{T}}$$

where N_0 is the initial neutron population, and $N(t)$ is the population at time t. If T is positive, neutrons increase over time; if it is negative, neutrons decrease. The reactor period can be calculated from the prompt neutron generation time ($l_p = 1 \times 10^{-4}$ sec) and the reactivity ρ as follows, if delayed neutrons are not considered:

$$T = \frac{l_p}{\rho}$$

Q3.2.3	For a reactor system with multiplication factor $k = 1.0020$, if delayed neutrons are not considered, by what factor will the neutron population increase if the time elapsed is 0.5 seconds?	1.0 pt

Q3.2.4	Since the neutron population in a nuclear reactor is proportional to its power output, what is the implication of the result from Q.3.2.3? Select the correct answer from the options. A. It demonstrates that the reactor can efficiently ramp up power output when needed, improving response time during high demand. B. It highlights the importance of delayed neutrons in reactor control, as a reactor without delayed neutrons would experience a dangerously fast increase in power that is physically impossible to control. C. It suggests that the reactor is operating in a subcritical state, where neutron population naturally decreases over time. D. It implies that neutron absorbers should be removed to allow the reactor to reach criticality faster for better energy production.	0.2 pts

Delayed neutrons are emitted from the decay of certain fission products called precursors. Since different precursors decay at different rates, they are grouped into six delayed neutron groups, each with a different decay constant (λ_i), mean lifetime (l_i) and yields (β_i) as shown in Table 3.

Table 3: *Six-group delayed neutron precursors.*

Group	$\lambda_i = \left[s^{-1}\right]$	l_i $[s]$	β_i
1	0.01244	80.4	0.000215
2	0.03040	32.9	0.001425
3	0.07576	13.2	0.001270
4	0.22831	4.38	0.002565
5	0.96154	1.04	0.000845
6	3.61011	0.277	0.000270

The Nordheim equation, commonly known as the inhour equation, is derived from the point kinetics equations, which describe the time-dependent behavior of neutrons in a nuclear reactor. The inhour equation relates the reactivity (ρ) to the reactor period (T).

Using the effective delayed neutron fraction $\beta_{eff} = \sum_i^6 \beta_i$, the average prompt neutron generation time (l_p), and defining $\omega = 1/T$, the inhour equation is expressed as:

$$\rho = l_p\omega + \sum_{i=1}^{6} \frac{\beta_i \omega}{\omega + \lambda_i}$$

Where $\lambda_{eff} = \frac{\beta}{\sum_{i=1}^{6} \beta_i/\lambda_i}$ is the weighted average of the decay constant of the delayed neutron precursors. The inhour equation is often simplified using a one-group approximation for delayed neutrons by using β_{eff} and λ_{eff} as representing all delayed neutrons. This approximation is appropriate when only a rough estimate is needed or when the reactor dynamics are dominated by a single effective decay rate.

For small reactivities, the terms in the inhour equation that contains ω in the denominator can be ignored, resulting in the following simplified expression:

$$\rho = \omega\left(l_p + \sum_{i=1}^{6} \frac{\beta_i}{\lambda_i}\right)$$

Q3.2.5	Calculate the effective decay constant (λ_{eff}) and effective delayed neutron fraction (β_{eff}) using the data from Table 3.	0.6 pts

Q3.2.6	Write the expression for the period T for a reactor system with 1-group delayed neutrons and small reactivities.	0.4 pts

Q3.2.7	For a reactor with reactivity $\rho = 1.0015$, (a) calculate the reactor period if there are no delayed neutrons, and (b) using the expression obtained from Q3.2.6.	0.6 pts

Question 3.3 Innovative Nuclear Fuel (4.0 pts)

Fission-based technology keeps advancing, and one of the most promising innovations is the Tri-structural Isotropic (TRISO) fuel, a coated-particle nuclear fuel designed for High-Temperature Gas-Cooled Reactors (HTGRs) and other next-generation reactors. TRISO fuel particles consist of a fuel kernel, which can be in the form of uranium dioxide (UO2), uranium oxycarbide (UCO), or uranium carbide (UC). These approximately spherical kernels typically have a diameter of 500 μm. The kernels are encapsulated in multiple concentric layers of pyrolytic carbon and silicon carbide to provide structural integrity, act as a containment barrier, and retain fission products even under extreme conditions. HTGRs, using helium as a coolant, can achieve high thermal efficiencies while maintaining inherent safety features. Sample TRISO fuel parameters are provided in Table 4.

Table 4. TRISO fuel parameters.

Material	Density (g/cm^3)	Typical Thickness (μm)
Buffer layer (porous carbon)	1.00	100
Inner PyC	1.90	40
SiC	3.21	35
Outer PyC	1.90	40

Q3.3.1	Given a 30 MW HTGR that operates with a core power density of 2.5 W/cm^3, estimate the total number of TRISO particles required to fuel the reactor. Note that 99.2% of the core consists of graphite moderators, coolant channels, and voids. Assume that each TRISO particle contributes equally to the power density.	0.5 pts

Q3.3.2	Calculate the total mass of 8% enriched uranium in a single TRISO particle if the kernel consists of UO_2 with a density of 10.5 g/cm^3 and a diameter of 500 μm. Express your answer in milligram. Note: $M_{U235} = 235.044$ g/mol, $M_{U238} = 238.051$ g/mol, and $M_O = 16.000$ g/mol.	0.8 pts

Q3.3.3	Calculate the total initial uranium inventory in a $12\mathrm{m}^3$ HTGR core. Express your answer in kg.	0.3 pts

Q3.3.4	Determine the burnup in % FIMA (Fissions per Initial Metal Atom) for a 30 MW HTGR assuming constant power operation for 2 years. Note: Avogadro's Number, $N_A = 6.022 \times 10^{23}\ \mathrm{atoms/mol}$, energy released per fission 3.204×10^{-11} J.	0.7 pts

One of the key safety advantages of TRISO fuel is its ability to retain fission products. The SiC layer is particularly effective in preventing the release of volatile fission products such as cesium-137 (Cs-137) and strontium-90 (Sr-90). However, noble gases like xenon (Xe) and krypton (Kr) may diffuse out over time.

Q3.3.5	In an experimental HTGR setup, TRISO fuel particles that have been irradiated to a burnup of ~10% FIMA were heated at 1600°C for 300 hrs to simulate an accident scenario. The fuel initially generated 1200×10^{-6} mol of Cs-137 per gram of uranium. At the end of the experiment, only 2400×10^{-9} mol of Cs-137 was measured to have escaped through the SiC layer from a sample containing 5.000 g of uranium. Determine the percentage of Cs-137 retention and explain its implication. Express your answer with **4 significant figures**.	0.4 pts

Fick's second law describes how the concentration of a diffusing substance changes over time due to diffusion. It can be used to estimate the diffusion of gases from a fuel containment. For a constant source concentration and diffusion coefficient, Fick's second law can be simplified as:

$$n_t = n_0 \mathrm{erf}\left(\frac{x}{\left(2\sqrt{Dt}\right)}\right)$$

where:

n_t is the concentration of the diffusing particle at after passing through material thickness x after time t

n_0 is the concentration of the diffusing particle at $x, t = 0$

D is the diffusion coefficient

t is the time it takes for the particles to diffuse through x

erf is the error function that can be expressed in elementary terms with the Maclaurin series:

$$\mathrm{erf(z)} = \frac{2}{\sqrt{\pi}}\left(z - \frac{z^3}{3} + \frac{z^5}{10} + \ldots\right)$$

For $z << 1$, the error function can be truncated to $\mathrm{erf(z)} = \frac{2}{\sqrt{\pi}} z$

Q3.3.6	Write the expression for time t from the above equation. Assume that the argument for the error function is $<< 1$.	0.8 pts

Q3.3.7	Given that the diffusion coefficient of Krypton in SiC is $6.5 \times 10^{-20}\ \mathrm{m^2 s^{-1}}$, estimate the number of years required for 1% of Krypton to escape through a 35 μm thick SiC layer.	0.5 pts

Q4 Radioactivity in the Environment

Radionuclides occur both naturally and as a result of human activities, contributing to background radiation in the environment. Some, like Uranium-238 and Thorium-232, are found in trace amounts in the Earth's crust and decay over billions of years, releasing radiation continuously. Others, such as Curium-244, are man-made and serve critical roles in technological applications - most notably in deep-space exploration, where their decay heat can be used to power spacecraft systems. Understanding how radioactive materials behave over time is crucial for assessing their utility, environmental impact, and associated radiological risks.

4.1 Curium in spacecraft (1.0 pts)

Curium-244 is used in spacecraft to provide heat to power generators. One of these generators contains 500 g of Curium-244 when it is manufactured. One gram of Curium-244 has a power density of 2.8 W/g. Curium-244 has a half-life of 18.1 years.

Q4.1.1	Calculate the average energy released per second by the generator after 72.4 years.	1.0 pts

4.2 Deposit of uranium (2.5 pts)

A uranium ore deposit contains 0.2% Uranium-238 by mass. Assuming the total mass of the deposit is 5,000 metric tons, calculate:

Q4.2.1	The total mass of Uranium-238 present in the deposit.	0.5 pts

Q4.2.2	The number of moles of Uranium-238 in the deposit. (Given atomic mass of U-238 = 238 g/mol)	1.0 pts

Q4.2.3	The total activity (in becquerels) of Uranium-238 in the deposit. (Half-life of Uranium-238 = 4.468×10^9 years)	1.0 pts

4.3 Deposit of thorium (2.0 pts)

A thorium ore deposit contains 1.5% Thorium-232 by mass. The total mass of the deposit is 8,000 metric tons. Given that:

• Atomic mass of Thorium-232 = 232 g/mol

• Half-life of Thorium-232 = 1.405×10^{10} years

• 1 year = 3.154×10^{7} seconds

Q4.3.1	Calculate the total mass of Thorium-232 present in the deposit.	0.5 pts

Q4.3.2	Determine the number of moles of Thorium-232 in the deposit.	0.5 pts

Q4.3.3	Calculate the total number of Thorium-232 atoms in the deposit.	0.5 pts

Q4.3.4	Using the decay law, calculate the total activity (in becquerels) of Thorium-232 in the deposit.	0.5 pts

4.4 Uranium decay (2.5 pts)

A sample of uranium ore originally contained 500 g of Uranium-238. Today, only 473 g remains. The half-life of Uranium-238 is 4.47×10^{9} years.

Q4.4.1	How many half-lives have passed?	0.5 pts

Q4.4.2	Estimate the age of the uranium ore sample.	0.5 pts

Q4.4.3	If the sample originally had 100 g of Pb-206 (U-238 decay product), how much Pb-206 is present today?	0.75 pts

Q4.4.4	What is the ratio of Pb-206 to U-238 in the sample now?	0.75 pts

4.5 Activity concentration of radionuclides (2.0 pts)

A 10 g sample of uranium ore contains 70% Uranium-238, 28% Thorium-232, and 2% Uranium-235 by weight.

The decay constants are:

Uranium-238: $4.92 \times 10^{-18}\ s^{-1}$

Thorium-232: $1.57 \times 10^{-18}\ s^{-1}$

Uranium-235: $9.85 \times 10^{-18}\ s^{-1}$

Given atomic masses:

Uranium-238 = 238 g/mol,

Thorium-232 = 232 g/mol,

Uranium-235 = 235 g/mol

Q4.5.1	Calculate the number of atoms of each isotope in the sample at t=0.	1.0 pts

Q4.5.2	Determine the total activity (in Bq) of the samples.	1.0 pts

Q5 Fuel Burnup and Utilization of Natural Uranium Resources (10.0 Points)

In comparing the merits of different types of reactors an important consideration is the efficiency with which each is capable of utilizing natural uranium resources. Since most countries lack abundant indigenous uranium resources, uranium utilization is often a key factor in a national decision to adopt a particular type of reactor and fuel cycle. Some reactors such as Canada Deuterium Uranium (CANDU) reactors use natural uranium. However, in the case of Light Water Reactors (LWRs), enriched fuel is used. The energy in a nuclear power plant (NPP) is produced by the fission of fissile nuclei such as $^{235}_{92}U$. Fissioning of a ^{235}U nucleus results in the release of energy carried by various particles. On average, about 168 MeV energy is carried by fission fragments, 8 MeV by β-particles, 7 MeV by γ-rays, 12 MeV by neutrinos, and 5 MeV by fission neutrons. The neutrons produced during fission are eventually absorbed somewhere inside the core of the nuclear power plant, emitting about 12 MeV capture gamma rays. The atomic weights of ^{16}O, ^{235}U, and ^{238}U, respectively, are 15.9994 amu, 235.0439 amu, and 238.0508 amu.

Q5.1	How much recoverable energy in MeV is produced per fission?	0.4 pts

Consider a reactor fuelled by $^{235}_{92}U$ and operating at a thermal power level of P Megawatts.

Q5.2	Find the mass of $^{235}_{92}U$ in terms of *P*, fissioned daily to operate this reactor.	0.6 pts

The fissile nuclei are consumed both in fission and radiative capture. Radiative capture is a process in which instead of fissioning $^{235}_{92}U$ it is converted into $^{236}_{92}U$ and a gamma ray after absorbing a neutron. About 15% of the neutrons absorbed in $^{235}_{92}U$ results in radiative capture.

Q5.3	Find the mass of $^{235}_{92}U$ in terms of *P*, consumed daily, to operate this reactor.	0.5 pts

The total energy released in fission by a given amount of nuclear fuel is called the fuel burnup and is measured in megawatt days (MWd). One MWd is the thermal energy generated by a one-megawatt reactor during one day of its operation. Similarly, one gigawatt year (GW-yr) is the energy generated by on gigawatt reactor during one year (365 days). The fission energy released per unit mass of the fuel is called the specific burnup of the fuel and is usually expressed in megawatt days per metric ton or per kilogram (i.e., MWd/t or MWd/kg) of the heavy metal (Uranium and/or Plutonium) originally contained in the fuel at the time of fuel loading.

Q5.4	How many kilowatt hours are there in one gigawatt year?	0.5 pts

Consider a light water reactor (LWR) that operates at an overall efficiency of 34%. The overall efficiency of a nuclear power plant is defined as the ratio of the rate of electrical energy output to the rate of thermal energy output from the reactor. The reactor is loaded with 30 metric tons of uranium dioxide (UO_2) fuel in which the uranium is enriched to 3 $^{\omega}/_{o}$ of ^{235}U. The reactor operates at a power level of 1000 MWe for 275 days before it is shut down for refueling.

Q5.5	What is the specific burnup of the fuel at shutdown of this reactor?	1.5 pts

Now consider a CANDU reactor that uses natural uranium and operates at an overall efficiency of 30%. Consider a CANDU reactor producing an electrical energy of 0.75 GWe-yr. Its fuel cycle starts with about 145 tons of U_3O_8. About 1.2% uranium is lost during fuel fabrication.

Q5.6	Calculate the specific burnup of this CANDU reactor.	1.5 pts

The natural Uranium resource utillization depends on the enrichment process. Specifically, suppose it is required to produce M_P kg of Uranium enriched to x_P weight fraction in ^{235}U from M_F kg of feed with enrichment x_F, leaving M_T kg of tails of enrichment x_T. Since there is little or no loss of Uranium in the plant, it follows that

$$M_F = M_P + M_T$$

The total amount of ^{235}U is also the same before and after enrichment, so that

$$x_F M_F = x_P M_P + x_T M_T$$

For natural Uranium as feed $x_F = 0.00711$, the value of x_T is set by the operator, however, in most of the plants its value is 0.2 $^{\omega}/_{o}$.

Q5.7	a) Show that the natural uranium feed required to produce enriched fuel containing a given mass of ^{235}U is almost independent of the fuel`s enrichment level. b) How much natural uranium feed is needed to produce enriched fuel containing 1.0 kg of ^{235}U?	1.2 pts 0.3 pts

The nuclear resource utilization, U, is quantitatively defined as the ratio of the amount of fuel that fissions in a given nuclear system to the amount of natural uranium input required to provide those fissions.

$$U = \frac{\text{fuel fissioned}}{\text{resource input}}$$

In reactors using fuel based on natural Uranium, the resource input is equal to the fuel load inside the reactor. However, for reactors using enriched fuel, the resource input should be more than the fuel load inside the reactor.

Q5.8	Find the resource utilization for the CANDU reactor.	1.5 pts

Q5.9	Find the resource utilization of the light water reactor (LWR).	1.5 pts

Q5.10	Which reactor type has better fuel utilization?	0.5 pts