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Incorrect Thinking Process Prediction for Negative Numbers Subtraction Operation Involving Positive with Negative Integers
(Meramal Proses Kesilapan Pemikiran untuk Operasi Penolakan Nombor Negatif Melibatkan Integer Positif dengan Negatif)

ELANGO PERIASAMY

ABSTRACT

This study was divided into two parts. The first part was to identify the incorrect answer produced by the respondents for each item and its frequency. Then, the second part was to predict the incorrect thinking process with respect to its frequencies that respondent might have adapted in solving such sentence questions incorrectly. The respondents of this study were five mathematics teachers and 124 students aged 14 years old from Malaysian secondary school. The finding shows types of mistakes made by the students for each type of items tested and the prediction of incorrect thinking process respectively. This paper focused on the third category: subtraction operation involving positive with negative integers. The findings revealed that in depth analysis into the students thinking process is an essential knowledge for teachers to re-assess their teaching and correct students their misconceptions.

Keywords: Negative numbers, subtraction, predict, teaching and learning mathematics

ABSTRAK

Kajian ini dibahagikan kepada dua bahagian. Bahagian pertama mengenal pasti kesilapan-kesilapan jawapan yang diberikan oleh responden untuk setiap item dan kekerapannya. Kemudian, bahagian kedua meramal kesilapan proses pemikiran responden yang memungkinkan responden mencapai penyelesaian yang salah dengan berdasarkan kekerapannya. Responden kajian ini adalah lima orang guru Matematik dan 124 murid berusia 14 tahun daripada sekolah menengah di Malaysia. Dapatan kajian menunjukkan jenis-jenis kesilapan yang dilakukan oleh murid untuk setiap item yang dikaji serta ramalan proses kesilapan pemikiran murid. Kertas ini berfokus kepada kategori ketiga: operasi penolakan integer positif dengan negatif. Dapatan kajian menunjukan bahawa analisis yang terperinci ke dalam proses pemikiran murid adalah ilmu yang penting untuk guru dalam mentafsir semula pengajaran mereka dan memperbetulkan miskonsepsi murid.

Kata kunci: Nombor negative, penolakan, meramal, pengajaran dan pembelajaran matematik

INTRODUCTION

According to Lembaga Peperiksaan Malaysia (1993), the Lower Secondary Examination (PMR) report from the Malaysian examination Board showed that students were unable to master the skills and understanding the abstract concepts that involves negative number operation in fraction, transformation and algebra. Moreover, in the 2002 PMR examination, 47% showed clear weaknesses in operation involving negative number such as (-17+14), (-17+22+8), (-17-14) and (-17+30) (Lembaga Peperiksaan Malaysia 2002). Therefore, a study with 124 students aged 14 years old from two secondary schools in Malaysia was carried out by Elango Periasamy and Halimah Badioze Zaman (2009) which revealed the existence of difficulties in solving negative numbers subtraction operation involving two integers. This phenomenon is explained by Naylor (2006) as situations whereby negative numbers extend our number line and greatly simplify our calculations, but sometimes students struggle with the concepts.

A review of literature showed that teachers were very creative and innovative in teaching the concept of subtraction and addition operation involving negative numbers by integrating various communication tools such as line graph, coloured stones, coloured chips, gain-owe techniques and computer courseware in their effort to help students acquire the knowledge of solving negative numbers subtraction and addition operation. These efforts shows the commitment and creativeness of teachers that should be acknowledged as an ongoing process that are continuously evolving in searching ways and mean to help students acquire knowledge related to subtraction and addition operation in negative numbers. This was to help students avoid arising at incorrect assumption, conclusions, thought process and generalizations which is also important for them to determine what things are as well as what they are not (Brumbaugh & Rock, 2006). The first part of this study was to identify the incorrect answer produced by the respondents for each item and its frequency respectively. The second part of this study was to predict the ITP with respect to its frequencies that respondent might have adapted in solving such sentence questions incorrectly. The focus of this paper was on the subtraction operation involving two single and double positive with negative integers only.

RELATED WORKS

According to Chen and Hung (2003), a central function of the mind is to process the information, sort them in a meaningful way is determined by the rules and principles employed, thus learning is then perceived as appropriating these rules and principles and being able to apply (or process information) according to these rules. Therefore, the knowledge of how children construct their early knowledge can be effectively gained from observing and interviewing during explicit teacher set tasks, that is if a student compute that 8 – 5 = 6, and from examination of work samples the teacher would immediately conclude that the child was experiencing difficulty with the subtraction process but further observation as the child works through examples: 7-3 = 2; 10 – 7 = 3; 2 – 1 = 4 the teacher quickly realises the source of the errors that is the child is confusing the digits 2 and 5 (Carnellor 2004). Moreover, a study to refine students' skills of addition and subtraction including negative numbers with a seventh grade student, turned out that errors were due to bug rules and the lack of a critical production when executing a purely algebraic solution were identified based on a cognitive task analysis using several possible ways of calculation (Terao et al., 2005).

Furthermore, findings suggest that adults' representations of operation with negative numbers are not as well established as their representations of operations with positive numbers (Prather & Alibali, 2004) because in operation involving negative numbers, some students assume many mathematical things to be universally true and because of this they are at times, amazed to realize their assumptions have been false (Brumbaugh & Rock 2006). Such phenomenon was found existed among two secondary school students in Malaysia in solving subtraction operation involving two integers (Elango Periasamy & Halimah Badioze Zaman 2009). For example,

> …some students are not aware that the commutative property for addition operates in sets other than the counting number. A series of questions or problems like -3 + +7 = and +7 + -3 = could help lead to the appropriate conclusions and can be amplified with problems involving subtraction where commutativity does not generally hold, sometimes that same students assume to be true (-5 - +8 = and +8 - -5 =).
>
> (Brumbaugh & Rock 2006, Pg 295)

> ... 3 + 3 = 6. Counting it out on your fingers can prove the accuracy of the equation. We can see apples and oranges in clusters of 3 or 6. It is reasonably easy to visualize the concept of addition of positive numbers. But, despite what all our algebra teachers have instructed about negative numbers, when we try to add 3 apples to

a pile consisting of a (-3) apples, things do not work out so simply. I get a queasy feeling in my stomach every time I try to work with negative numbers. It makes me quite uneasy to think that my bowl containing 3 apples will be swept off into a vortex and lost forever if I were to add them to a pile containing a minus 3 apples, yet the pile of 3 apples would remain intact if I were to place them into an empty container. The mystery of where the 3 apples would travel absolutely baffles me. And, yet, it would be a rare mathematician who would concede that negative numbers are an illusion. The mathematicians don't care if the rules and concepts they employ are idiotic as long as they can arrive at precise answers time after time. In other words, they know full well that negative numbers are fraudulent, but, since they are useful tools, they are happy to continue with the illusion. To my way of thinking, the smallest number of anything would have to be zero. When there are no apples on the plate, it is empty. It would take a strange metaphysical phenomenon indeed to allow me to place 3 apples on the plate and watch them vanish. Since when did the sceptical people of science allow such portals that consume apples to be considered "normal" behaviour? This is not to say that such portals cannot exist, but it is to say that such portals could not be called upon to operate in a totally predictable manner each and every time someone placed a hyphen before a number converting it from a positive number, or something, into a negative number, or a weird thing that is less than nothing.

(Stanford 2003, Pg 3)

Although different strategies were used by various researchers in helping students gain the knowledge of solving negative numbers subtraction operation, nevertheless real objects manipulation for subtraction operation of negative numbers is an illusion. Therefore, Stanford claimed that students have been given absurd rules to apply to this weird concept, such as a negative number when multiplied by another negative number becomes a positive number which is an unadulterated nonsense Stanford (2003). Therefore, the misconceptions among students need to be addressed via predicting their ITP. It would give a guideline on how to hinder such misconceptions on negative numbers subtraction operation. Moreover, immediate practice of corrective thinking process can be instigated and further difficulties avoid.

METHOD

The demographic information of this research was 124 respondents aged 14 years old and among them were 53 boys and 71 girls. The number of respondent achieved a grade A is 26 (20.97%), grade B 58 (46.77%) and grade C 40(32.26%) for their Primary School Evaluation Examination (UPSR) in mathematics subject. The questionnaires were divided into two sections. The first section consists of demography data to understand the respondent profile. The second section consists of 24 negative number subtraction operation test items with only one correct answer for each item as in Table I. Face validity was done with five Mathematics teachers from three schools from a district in Malaysia. Those teachers had an experience of teaching Negative Number topic for at least five years. The questionnaire for this research was created by Elango Periasamy & Halimah Badioze Zaman (2009). A pilot test was carried out by Elango Periasamy and Halimah Badioze Zaman (2009) with a subject of 35 school students aged 14 years old from a secondary school in Malaysia. The calculation of reliability coefficient using Kuder-Richardson formula is use for dichotomy question with right wrong answer such as the objective questions (Alias Baba, 1999). The Kuder-Richardson (KR20) reliability estimation value of this instrument was 0.919544. The reliability was calculated using the KR20 formula (Mervis & Spagnolo, 1995) with Microsoft Office Excel 2007. According to Mervis and Spagnolo (1995), when the test format has only one correct answer then KR20 is algebraically equivalent to Cronbach alpha. Therefore, in this case the KR20 reliability estimation value of this pilot test is equivalent to Cronbach alpha coefficient.

The 24 items of this study (as in Table 1) were rearranged into four categories by Elango Periasamy and Halimah Badioze Zaman (2011) as follows:

i. First category – Subtraction Operation Involving Two Positive Integers
ii. Second category – Subtraction Operation Involving Negative with Positive Integers
iii. Third category – Subtraction Operation Involving Positive with Negative Integers
iv. Forth category –Subtraction Operation Involving Negative with Negative Integers

Therefore, each category consists of 6 items with respect to its theme. The focus of this paper was limited to the third category to share and predict the ITP of subtraction operation involving a positive with a negative integers ($a - b$, $a>0$, $b<0$) only.

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TABLE 1. Negative number subtraction operation test items

No	Item	No	Item
1	5 - 2 =	13	-8 - 13 =
2	-5 - 2 =	14	8 - 13 =
3	-5 - (-2) =	15	-8 - (-13) =
4	5 - (-2) =	16	8 - (-13) =
5	-2 - 5 =	17	16 - 23 =
6	2 - 5 =	18	-16 - 23 =
7	-2 - (-5) =	19	-16 - (-23) =
8	2 - (-5) =	20	16 - (-23) =
9	13 - 8 =	21	-23 - 16 =
10	-13 - 8 =	22	23 - 16 =
11	-13 - (-8) =	23	-23 - (-16) =
12	13 - (-8) =	24	23 - (-16) =

ITP of subtraction operation involving a positive with a negative integers (a – b, a>0, b<0) only.

In conjunction, this study was divided into two parts. The first part was to identify the incorrect answer produced by the respondent for each item and its frequency for the third category. Then, the second part was to predict the ITP with respect to its frequencies that that respondent might have adapted in solving such sentence questions incorrectly. Therefore, all possible ITP that student would have used in order to arrive at those incorrect answers need to be derived explicitly by analyzing students prior knowledge and teachers teaching approach for negative numbers subtraction operation that might have been responsible for such conflict in adapting the correct thinking or rules in solving subtraction operation. Then, re-confirming with five mathematics teachers.

FINDINGS

The first part was to identify the incorrect answer produced by the respondent for each item and its frequency. Table 2 shows the result of the first part of this study. The highest incorrect solution was for item 16 (46.77%) whereby 2, 30 and 26 students gave incorrect solution 5, -5 and -21 respectively; followed by item 12 (43.55%) whereby 33, 15 and 6 students gave incorrect solution 5, -21 and -5 respectively; then item 4 (41.13%) whereby 25, 6 and 20 students gave incorrect solution 3, -3 and -7 respectively; for item 20 meanwhile 15, 8, 24 and 2 students gave incorrect solution -39, 7, -7 and -8 respectively; continued by item 8 (36.29%) then 10, 17 and 18 students gave incorrect solution 3, -3 and 7 respectively; and finally for item 24 (33.8%) whereby 12, 18 and 12 students gave incorrect solution -39, -7 and 7 respectively.

TABLE 2. Subtraction of positive with negative integer

No	Item	Incorrect Solution	Frequency	Total (%)
4	5 – (-2) =	3	25	51 (41.13%)
		-3	6	
		-7	20	
8	2 – (-5) =	3	10	45 (36.29%)
		-3	17	
		-7	18	
12	13 – (-8) =	5	33	54 (43.55%)
		-21	15	
		-5	6	
16	8 – (-13) =	5	2	58 (46.77%)
		-5	30	
		-21	26	
20	16 – (-23) =	-39	15	49 (39.52%)
		7	8	
		-7	24	
		-8	2	
24	23 – (-16) =	-39	12	42 (33.8%)
		-7	18	
		7	12	

- The S3T1 type

The S3T1 type addressed the subtraction operation where the magnitude value of the first positive integer was greater than the magnitude value of the second negative integer. An example of the S3T1 type item is 15- (-7). Based on this item, a prediction of respondents' ITP for S3T2 type item was made and discussed in Table 3.

TABLE 3. Predict S3T1 Type ITP

ITP	Types of Solution	Predict ITP
1	15- (-7) = -8	Move 1: positive sign in front of number 15 multiply negative sign in front of number 7 which gives the sign for final answer, in this case negative. Move 2: Now the question is rewritten as 15-7= which gives 8 as its solution. Then, with reference to move 1, the answer becomes -8, thus 15- (-7) = -8
2	15- (-7) = -22	Move 1: Positive sign in front of number 15 multiply negative sign in front of number 7 which gives the sign for final answer, in this case negative. Move 2: There exist two negative signs in between 15 and 7, thus negative multiply negative is positive. Then perform 15 + 7 which gives 22. Now, from move 1 the final answer will have negative sign. Thus, 15-(-7) = -22.
3	15- (-7) = 8	Move 1: Positive sign in front of number 15 multiply negative sign in front of number 7 which gives a negative sign. Then, multiply it with the negative sign in between 15 and -7 which gives the sign for final answer, in this case positive. Now the question is rewritten as 15 – 7. Move 2: Perform 15 – 7 which gives 8 as its solution. Now from move 1 the final answer will have negative sign. Thus, 15- (-7) = 8

- The S3T2 type

The S3T2 type addressed the subtraction operation feature where the magnitude value of the first positive integer was smaller than the magnitude value of the second negative integer. An example of the S3T2 type item is 7- (-15). Based on this item, a prediction of respondents' ITP for S3T2 type item was made and discussed in Table 4.

TABLE 4. Predict S3T2 type ITP

ITP	Types of Solution	Predict ITP
1	7- (-15) = 8	Move 1: Positive sign in front of number 7 multiply negative sign in front of number 15 which give negative sign. Then, multiply it with the negative sign in between 7 and -15. Now the question is rewritten as 7 – 15. Now the question becomes a first category type of sentence question. Move 2: Number 15 is bigger than 7. Then perform 15 – 7 which give 8, thus 7 – (-15) = 8.
2	7- (-15) = -22	Move 1: Positive sign in front of number 7 multiply negative sign in front of number 15 which gives the sign for final answer, in this case negative. Move 2: There exist two negative sign in between 7 and 15, thus negative multiply negative is positive. Then, rewrite sentence question as 7 + 15 and perform 7 +15 which gives 22. Now, from move 1 the final answer will have negative sign, thus 7- (-15) = -22
3	7- (-15) = -8	Move 1: positive sign in front of number 7 multiply negative sign in front of number 15 which gives the sign for final answer, in this case negative. Move 2: Now the question is rewritten as 7-15 (first category type sentence question) but performs 15 -7 which gives 8 as its solution. Then, with reference to move 1, the answer becomes -8, thus 7- (-15) = -8

DISCUSSION

Items 4, 8, 12, 16, 20 and 24 from the S3 group which were the subtraction operation feature of a positive number with a negative number in parenthetical. Items 4, 12 and 24 were the subtraction operation feature of a positive integer number which had greater value with a negative integer number which had smaller absolute value in parenthetical ($a - b =, a>0, b<0, a>|b|$). The research findings showed that the respondents' use of the three different thinking process techniques gave rise to wrong answers. Among the three thinking process techniques, the ITP 3 technique was found to be more dominant followed by the ITP 2 and the ITP 1, respectively. While, items 8, 16 and 20 from the S3T2 group were the subtraction operation feature of a positive integer number which had a smaller value with a negative integer number that had a larger absolute value in parenthetical ($a - b =, a>0, b<0, a<|b|$). The research findings showed that the respondents' use of the three different thinking process techniques gave rise to wrong answers. Among the three thinking process techniques, the ITP 3 technique was found to be more dominant followed by the ITP 2 and the ITP 1, respectively. Therefore, these incorrect thinking processes created opportunities for teachers to reflect on their teaching and learning process on the specific domain.

The process of predicting ITP was a very tedious and time consuming. The process needed special diagnostic sentence questions which could create conflict in the students' thinking process in solving them and with proper analysis and synthesis when predicting ITP and followed by reconfirming the prediction. Nevertheless, this study was an interesting experience towards exploring the ITP respondents acquired, moreover the findings can be important in helping mathematics educators to be aware of such ITP could exist and proper precaution should be taken into consideration during teaching and learning of negative numbers subtraction operation or remedial works. It is because such misconceptions firstly, interfere with learning when students use them to interpret new experiences and secondly, students are emotionally and intellectually attached to their misconceptions because they have actively constructed them and students give up their misconceptions, which can have such a harmful effect on learning, only with great reluctance (Rose et al., 2007). But to teach in a way that avoids creating and misconceptions is not possible and we have to accept that students will make some incorrect generalizations that will remain hidden unless the teacher makes

specific efforts to uncover them (Askew & William, 1995) in Rose et al., (2007).

Therefore, according to Carnellor (2004), there is no simple one answer to guide specific practice and teachers must provide a wide variety of methods through their diverse repertoire of class room practices in their lesson planning, the topic presented, the instructional experiences and activities incorporated in the learning session and their responses to children's questions. Moreover, Brumbaugh and Rock (2006) suggested that assistance is provided to the discovery process through a carefully developed set of problems that guide the student to appropriate responses. Nevertheless, Chen and Hung (2003) said that by analysing the way the experts think and by teaching students these expert ways of thinking, cognitivists hope to instruct students in order to emulate expert thinking and develop the students' expertise is a particular domain of knowledge.

CONCLUSION

In conjunctions, this study was to identify and predict the ITP of respondents in solving negative numbers subtraction operation involving two integer sentence questions limited to single double digit integers. Even though, all students in a class room are taught equally and simultaneously but the way they perceive and process the knowledge are in their own unique way should be acknowledged with great enthusiasm. Therefore, the most sadness of this study was that the teachers and students were unaware of the existence of the ITP until a study of this kind was conducted. Askell-Williams et al. (2007) perceives it as a situation where students develop robust mental models of teaching and learning during their school years, and as such, often teach as they were taught-possibly perpetuating practices that limit intellectual inquiry in classrooms.

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Elango Periasamy (Dr.)
IPG Kampus Pendidikan Teknik
Kompleks Pendidikan Nilai,
71760 Bandar Enstek,
Nilai, Negeri Sembilan.
E-mel: surensutha@yahoo.com