Jurnal Personalia Pelajar 29(1): 57-63
https://doi.org/10.17576/personalia.2901.2026.06
ISSN 0128-2735

Predicting Next Event Time Of Event Series Using Barycentric Coordinates
(Ramalan Masa Kejadian Seterusnya dalam Siri Peristiwa Menggunakan Koordinat Barisentrik)

YUNPENG ZHANG & YOSHITO HIRATA

ABSTRACT

Many complex systems generate sequences of discrete events rather than uniformly sampled time series. Forecasting the next event time from such irregular event series is important in applications ranging from neuroscience to fault monitoring and seismicity. Motivated by reconstruction theory for non-uni-form sampling, we study how to predict the time until the next event using only event times, without access to the underlying continuous state. We propose a local geometric predictor that (i) represents each event by an inter-event-interval (ISI) delay-coordinate state, and (ii) predicts the next interval or the next event time via barycentric coordinates computed by linear programming. The barycentric con-straints enforce an interpretable convex-combination structure while an $\boldsymbol{L_1}$ fitting objective improves robustness. Using a Rössler-system proof-of-concept, we evaluate multiple variants and baselines under a strict time split and an explicit guard against future-information leakage. The results demonstrate that barycentric geometry can extract additional predictive power beyond neighbor averaging in determin-istic event-generation settings, providing a principled and interpretable foundation for extending the framework to noisy real-world event series.

Keywords: Event time prediction, Event series, Inter-event interval embedding, Barycentric coordinates, Linear programming

ABSTRAK

Banyak sistem kompleks menghasilkan jujukan peristiwa diskret dan bukannya siri masa yang disampel secara seragam. Meramal masa peristiwa seterusnya daripada siri peristiwa tidak teratur sedemikian adalah penting dalam pelbagai aplikasi, daripada bidang neurosains hinggalah kepada pemantauan kerosakan dan seismisiti (kegempaan). Didorong oleh teori pembinaan semula bagi pensampelan tidak seragam, kami mengkaji cara untuk meramal masa sehingga peristiwa seterusnya dengan hanya menggunakan masa peristiwa, tanpa akses kepada keadaan selanjar yang mendasarinya. Kami mencadangkan satu peramal geometri tempatan yang: (i) mewakili setiap peristiwa melalui keadaan koordinat-lengah selang-antara-peristiwa (ISI), dan (ii) meramal selang masa seterusnya atau masa peristiwa seterusnya melalui koordinat barisentrik yang dihitung dengan pengaturcaraan linear. Kekangan barisentrik tersebut menguatkuasakan struktur gabungan cembung (convex-combination) yang boleh ditafsir, manakala objektif penyuaian $\boldsymbol{L_1}$ *meningkatkan keteguhan (robustness). Menggunakan bukti konsep sistem Rössler, kami menilai pelbagai varian dan garis dasar (baselines) di bawah pembahagian masa yang ketat serta perlindungan eksplisit terhadap kebocoran maklumat masa hadapan. Keputusan kajian menunjukkan bahawa geometri barisentrik mampu mengekstrak kuasa ramalan tambahan melangkaui purata jiran (neighbor averaging) dalam tetapan penjanaan peristiwa berketentuan (deterministic), sekali gus menyediakan asas yang berprinsip dan boleh ditafsir untuk memperluas rangka kerja ini kepada siri peristiwa dunia sebenar yang hingar (noisy).*

Kata Kunci: Ramalan masa peristiwa, Siri peristiwa, Benaman selang-antara-peristiwa (inter-event interval embedding), Koordinat barisentrik, Pengaturcaraan linear.

INTRODUCTION

Many critical systems in nature and engineering do not produce continuous signals but rather generate sequences of discrete events: earthquakes, neural spikes, failure occurrences, threshold exceedances, and other episodic phenomena. A central task for such data is to predict the next event time, typically expressed as the remaining time until the next event, $y(t) = T_{"\#!} - t$. At the current time t in ($T_{"}$, $_{"\#!}$). Unlike conventional time-series prediction, the observation is an irregular event sequence $T_{"}$ with no uniformly sampled state. This raises two challenges: (i) how to construct a meaningful state representation from event times alone, and (ii) how to perform local prediction in that state space in an interpretable and physically consistent manner. State-space reconstruction from non-uniform samples has a long history. Takens [1] established foundational embedding results for uniformly sampled observations, and subsequent work extended the theory to non-uniform sampling. Sauer [2] proposed reconstruction using interspike (inter-event) intervals, suggesting that the ISI sequence itself can serve as an observation function for reconstructing dynamics. Huke and Broomhead [3] further developed embedding theorems for non-uniformly sampled dynamical systems, providing theoretical support for reconstruction from irregular observations. On the prediction side, Hirata and Shiro [4] demonstrated accuracy improvements for predicting maxima of a flow by reconstructions using local cross sections, emphasizing that tailored reconstruction strategies can significantly affect prediction performance. Separately, Hirata *et al.* [5] proposed approximating high-dimensional dynamics by barycentric coordinates via linear programming, offering a geometric and interpretable local modelling tool. Building on these ideas, we focus on event-series prediction and evaluate barycentric-coordinate prediction in an ISI-based reconstructed state space.

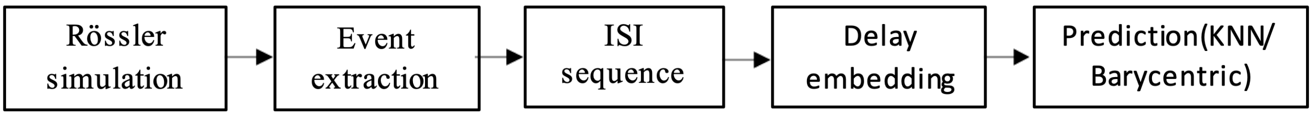

FIGURE 1. Overall workflow of the proposed event-time prediction pipeline

METHODOLOGY

Event Series and Prediction Target

Let $T_{"}$ denote an event time sequence extracted from an underlying continuous dynamical system. For any query time t with $T_{"} < t < T_{"\#!}$, the remaining time to the next event is $y(t) = {}_{"\#!} - t$. We evaluate predictors that use only the history of event times strictly prior to t.

$$y(t) = T_{"\#!} - t, \quad \text{for } T_{"} < t < T_{"\#!} \tag{1}$$

ISI Representation and Delay-coordinate Embedding

Define the inter-event interval (ISI) sequence $\Delta_{"} = T_{"\#!} - T_{"}$. Following the interspike-interval reconstruction viewpoint [2], we form a delay-coordinate state vector from consecutive ISIs. With embedding dimension m = 3 (used throughout this study), the state associated with event i is

$$\Delta_{"} = T_{"\#!} - T_{"} \tag{2}$$

$$x_{"} = [\Delta_{"\$!}, \Delta_{"\$\%}, \Delta_{"\$\&}]' \tag{3}$$

The choice m = 3 matches the low-dimensional nature of the Rössler attractor and keeps local modeling stable with finite data. In practice, the embedding is computed from the ISI sequence using a standard delay embedding routine.

Local Neighbor Selection

Given a current index i (defined as the last event before t), we locate K nearest neighbors of $x_{"}$ in the reconstructed space. Distances are computed using the Euclidean metric in the m-dimensional embedding space. The neighbor index set consists of the K smallest distances among a library of admissible past states.

ISI k-nearest-neighbor averaging (pt0)

The baseline predictor estimates the next inter-event interval using neighbor averaging. Let the neighbors be $\{j_{!}, \dots, j_{(}\}$. We predict the next interval by the mean of the neighbors' subsequent ISIs and then convert it to remaining time by subtracting the elapsed time since the last event:

$$\Delta 3_i = {}^{!}\sum\nolimits^{)}_{)} \; \Delta_{(+!} \quad {}_{*!} \tag{4}$$

$$y5(t) = \max 9\bar{\bar{\Delta}} - (t - T_{"}), 0= \tag{5}$$

Proposed Method: Barycentric Coordinates via Linear

Programming

Our main method replaces neighbor averaging with a geometric interpolation based on barycentric coordinates. We seek weights $w = [w_!, \ldots, w_(]'$ that represent the current state $x_"$ as an affine combination of neighbor states, subject to convex-combination constraints. To improve robustness to outliers and local nonlinearity, we fit the representation by minimizing an $L_!$ reconstruction error using slack variables $e_, \geq 0$. We also include a scalar offset a_- that captures systematic local bias.

Specifically, we select the K nearest neighbors. Then, we solve the following linear program:

2
min D$e_¸$
.",0,1
,+!
subject to
$-e_, \leq x_" - \mathrm{F}a_- \cdot 1 + \Sigma^)(+!\, w_(x_(, \mathrm{J} \leq e_,$
$\Sigma) (+!\, w_(= 1, \qquad w_(\geq 0, e_¸ \geq 0.$ (6)

Once $(a_-$, w) are obtained, the next interval is predicted by applying the same weights to the neighbors' subsequent ISIs:

$\Delta^3 i = a_- + \Sigma^)(+!\, w_(\Delta_{*(},$ (7)
$y5(t) = \max(\Delta^3 i - (t - T_"), 0)$ (8)

This formulation inherits the interpretability of barycentric weights (each $w_($ quantifies the contribution of a neighbor) and enforces a physically meaningful structure through convexity. It is also closely aligned with the barycentric-LP framework for approximating dynamics, specialized here to event-series prediction.

Variants

To isolate which design choices matter, we evaluate additional variants: (pt3) a no-offset barycentric model ($a_- = 0$) with the same convex constraints; (pt4, pt5) vector-offset heuristics that introduce a separate offset per embedding dimension and map it to interval prediction by either summation (pt4) or averaging (pt5). For completeness, we also report a $(T - t)$-based local barycentric predictor (pt) that operates directly on time-to-past-events features instead of ISIs. We formalize pt as a local barycentric predictor in the $(T{-}t)$feature space. For a query time $t \in [T_"$, $T_{"\#!})$, define $Z_((t) = T_(- t$. We then select K nearest neighbors in the space $\mathrm{F}Z_*(t)$, $Z_{*\$!}(t)$, $Z_{*\$\%}(t)\mathrm{J}$ and fit a convex local linear map by solving the following linear program:

$0b^{\mathrm{Q}}_- , \{v5\}\S = \arg \min_{3",\{5\#\}} \Sigma" h"$
subject to
$-h_" \leq Z_{"\#!} - \mathrm{F}b_- + \Sigma^\% ,+\text{-}\, v_, Z_{"\$,}\mathrm{J} \leq h"$,
%
D $\quad v_, = 1, \;\; v_, \geq 0.$
,+-

After estimating b^{Q}_- and $\{v5_¸\}$, we have the following prediction formula:

$\hat{z}_{*\#!} = b^{\mathrm{Q}}_- + \Sigma^\% v5_¸ Z_{*\$_¸}.$

The rationale for ISI embeddings can be understood from two complementary perspectives. First, Takens-type embedding results establish that delay-coordinate maps generically provide diffeomorphic reconstructions of the underlying attractor when observations are sufficiently rich. For event series, the sampling is intrinsically non-uniform; nevertheless, embedding theorems for non-uniform sampling and interspike intervals provide conditions under which the ISI sequence retains enough information for reconstruction. Sauer [2] explicitly proposed reconstruction from interspike intervals, and Huke and Broomhead [3] analyzed embedding conditions for non-uniformly sampled dynamical systems, supporting the use of ISI delay coordinates as a state proxy in deterministic settings.

Second, from a geometric viewpoint, event extraction such as maxima detection can be interpreted as constructing a local cross section of the flow. Hirata and Shiro [4] showed that appropriate reconstructions based on local cross sections can improve prediction of extrema. In our setting, the ISI sequence encodes successive return times to such a section. This motivates using ISI coordinates as the state representation and then performing local interpolation in that space.

These theoretical considerations do not imply that perfect prediction is achievable in noisy or nonstationary data; rather, they justify the proof-of-concept design and clarify why the proposed method is expected to perform well in a deterministic simulation.

Experiments

The model will be validated using data generated from the Rössler system [6], a system of three non-linear ordinary differential equations.

The Rössler equations are defined as

$\dot{x} = -y - z \,\mathrm{Y} \quad \dot{y} = x + ay$
$\dot{z} = b + z(x - c)$

Typical parameters used for generating chaotic behavior are a = 0.36, b = 0.4, and c = 4.5.Figures 2-4 illustrate how the event series is constructed from the

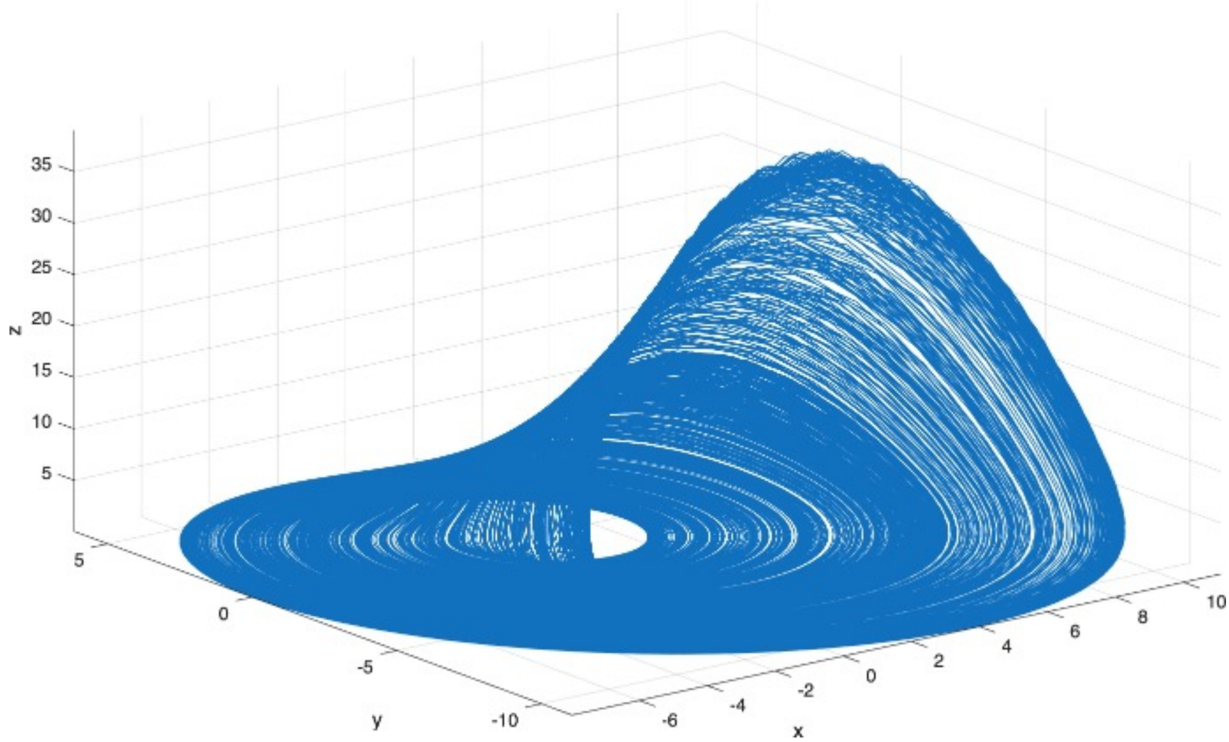

FIGURE 2. Rössler attractor

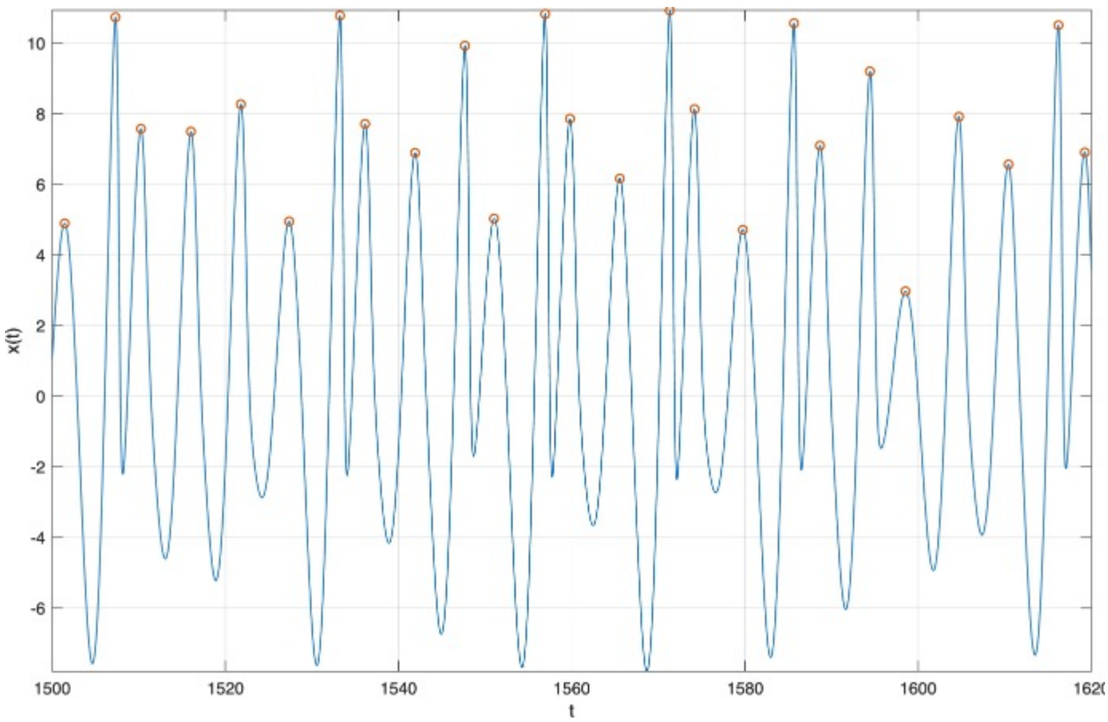

FIGURE 3. Time series $x(t)$ with their detected maxima marked by circles

underlying dynamical system. Figure 2 shows the synthetic Rössler attractor generated by numerical integration, providing a representative chaotic trajectory in three-dimensional state space. Based on this trajectory, we define events as the times of local maxima of the observable ($x(t)$). Figure 3 presents a zoomed segment of ($x(t)$), where the detected maxima times (T_n) are marked to visually confirm that events correspond to peak locations. Finally, Figure 4 summarizes the resulting inter-event intervals (ISI),

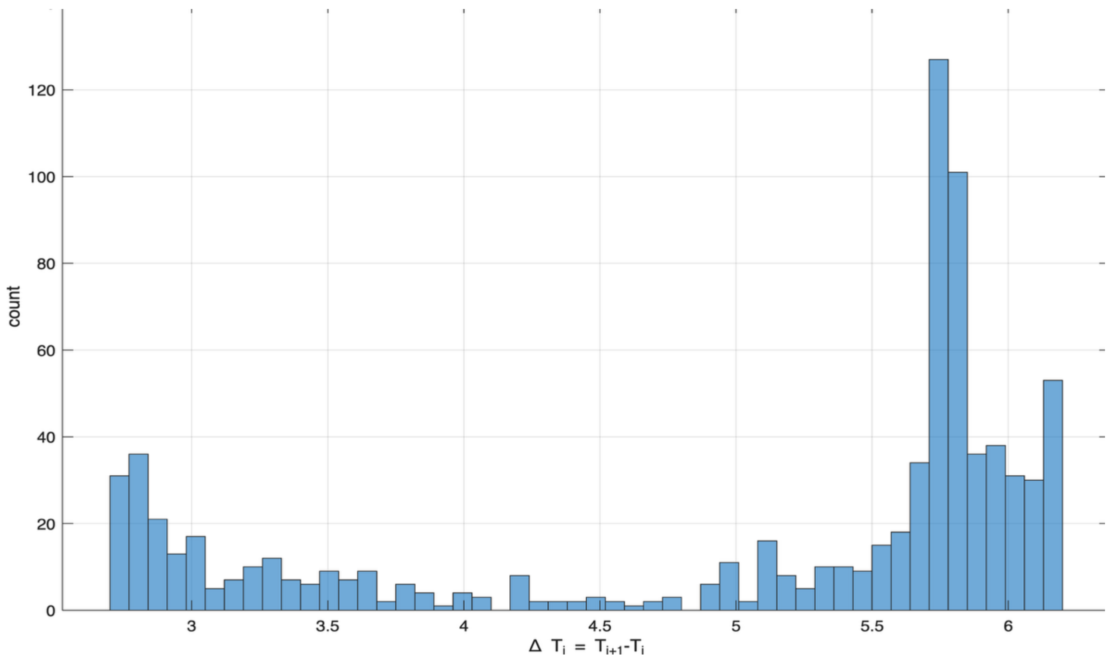

FIGURE 4. histogram of inter-event intervals ΔT

($\Delta_{"} = T_{"\#!} - T_{"}$), as a histogram, highlighting the variability of waiting times that motivates modeling and forecasting in the ISI domain.

We validate the proposed event-time predictors using a synthetic event series generated from the Rössler system, and define events as the times of local maxima of the observable $x(t)$, yielding an event-time sequence $\{T_{"}\}$ and inter-event intervals $\Delta_{"} = T_{"\#!} - T_{"}$. For any query time $t$satisfying $T_{"} < t < T_{"\#!}$, the prediction target is the remaining time to the next event, $y(t) = T_{"\#!} - t$. For ISI-based methods, we use the same notation as the manuscript and reconstruct state by an ISI delay vector with embedding dimension $m = 3$.

To evaluate under a strict causal protocol, we apply a time split at $t =$ initt . In the current MATLAB implementation used to produce Table 1, we set initt0 $= 30$, initt $= 200$, and tend $= 400$. In this setting, the number of events is mean 33.7 and standard deviations 1.0, which are estimated from the following 20 samples. We sample $N_{\text{train}} = 50{,}000$ training query times uniformly from [initt0, initt] to construct the library for the $(T{-}t)$-based baseline (pt), and sample $N_{\text{test}} = 50{,}000$ test query times uniformly from [initt, tend $-$ initt0] for evaluation. We repeat this procedure for 20 random seeds to reduce sampling variability. All local methods use $K = 5$ nearest neighbors, and performance is as-sessed by Pearson correlation (Corr) and root-mean-square error (RMSE) between predicted and true remaining times.

All experiments were conducted in MATLAB R2024a.

Evaluation Metrics

We report two complementary accuracy measures: (i) Pearson correlation between predicted and true remaining times, which reflects trend consistency, and (ii) root-mean-square error (RMSE), which directly measures absolute accuracy. Correlation alone can be misleading in event-time problems because it does not penalize systematic scale or bias; RMSE is therefore treated as the primary metric for comparing methods in this study.

RESULTS &DISCUSSION

Result

Table 1 reports the performance comparison with $K = 5$ for all methods. The $(T{-}t)$ local bary-centric-LP baseline (pt) achieves the best overall accuracy, with Corr = 0.963520 and RMSE = 0.442006. Among ISI-based methods, the no-offset barycentric-LP variant (pt3) performs best (Corr = 0.915412, RMSE = 0.706078), outperforming the kNN-mean baseline (pt0) and the offset-based var-iants (pt1/pt4/pt5).In particular, introducing offsets does not improve the results under this configura-tion: pt1 (scalar offset) and the vector-offset heuristics pt4/pt5 yield lower correlation and larger RMSE than pt3.

TABLE 1. Performance comparison (K = 5).

Method	Description	Corr	RMSE
pt	(T−t) local barycentric-LP	0.963520	0.442006
pt0	ISI embedding + kNN mean	0.886750	0.785900
pt1	ISI barycentric-LP + scalar offset	0.887654	0.801935
pt3	ISI barycentric-LP	0.915412	0.706078
pt4	ISI barycentric-LP + vector-offset sum	0.882760	0.826842
pt5	ISI barycentric-LP + vector-offset mean	0.898473	0.748115

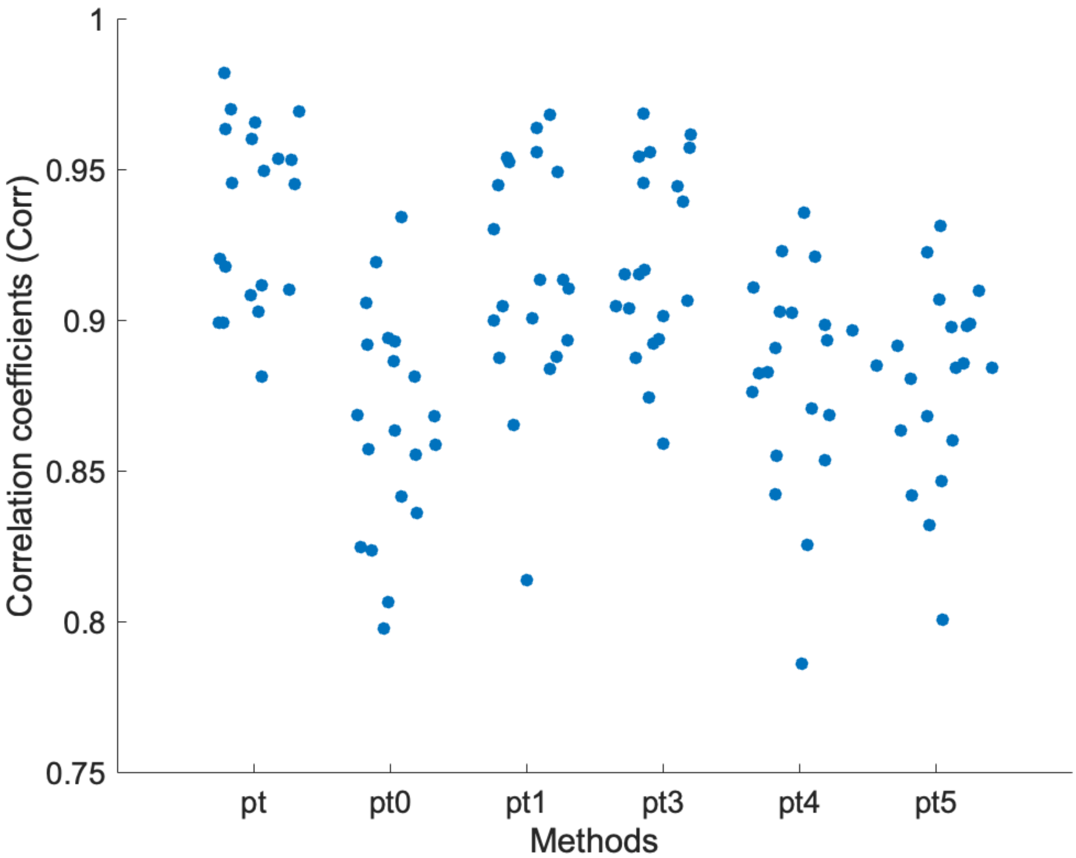

FIGURE 5. Corr distribution over 20 trials (K=5)

DISCUSSION

A key observation from Table 1 is that, under the current strict time-split setting with a relatively short historical window available for training (events prior to t = initt), the direct $(T-t)$-based representation (pt) provides the most accurate remaining-time predictions. This is notable because the man-uscript's motivation emphasizes that $(T-t)$ feature vectors change with the query time t and may distort neighborhood geometry, whereas ISI vectors are translation-invariant in time. The present results suggest that, in the limited-history regime used here, the dense sampling of training query times for pt (uniformly over [initt0,initt] in the implementation) can compensate for the time dependence of $(T-t)$ features and yield a more effective local interpolation set for remaining-time prediction.

For the ISI family, pt3 outperforming pt1/pt4/pt5 indicates that removing offsets can improve robustness when K is small and the admissible neighbor library is restricted to early data. Offsets increase degrees of freedom in the local fit; with limited representative neighbors, they can amplify vari-ance without reducing bias, leading to degraded RMSE. In contrast, pt3 enforces a simpler convex interpolation structure, which appears better aligned with the available information content of the ISI states in this experiment.

Finally, Corr and RMSE capture complementary aspects of predictive quality: Corr is scale-in-variant and reflects trend consistency, whereas RMSE measures absolute error in the remaining-time units and is more sensitive to bias and dispersion. In Table 1, the ranking by RMSE is consistent with the practical accuracy requirement for forecasting remaining time, and therefore RMSE should be treated as the primary criterion when selecting among variants. Future experiments should further probe sensitivity to the time-split location and the amount of historical data (e.g., extending the training interval) to clarify when ISI embeddings become advantageous, and whether the relative benefit of offset terms changes with larger Kor richer libraries.

CONCLUSION

In this study, we investigated remaining-time prediction for irregular event series by comparing local interpolation/regression approaches based on (i) the $(T-t)$ representation and (ii) ISI delay-coordinate embeddings. Using the strict time-split evaluation protocol described in the Experiment section and setting $K = 5$, the $(T-t)$ local barycentric-LP baseline (pt) achieved the best overall performance (Table 1), attaining the highest correlation and the lowest RMSE among all compared methods. Within the ISI-based family, the no-offset barycentric-LP variant (pt3) consistently outperformed the kNN-mean baseline (pt0) and the offset-based variants (pt1/pt4/pt5), indicating that a simpler convex interpolation structure can be more robust than introducing additional offset degrees of freedom under a limited-history regime.

These results suggest that, under the current configuration, directly modeling the remaining-time target in the $(T-t)$feature space provides a stronger local predictive signal than a low-dimensional ISI embedding with offsets. Future work will systematically study the sensitivity to hyperparameters (e.g., K and embedding dimension m) and to the amount of historical data available before the split, as well as extend the evaluation to additional dynamical settings and real-world event sequences. Improving reproducibility and statistical reporting-especially by storing per-trial error statistics consistently for all metrics-will further clarify the regimes in which ISI embeddings become advantageous.

REFERENCES

Gaspard, P. (2005). Rössler systems. Encyclopedia of Nonlinear Science 231, 808-811.

Hirata, Y., & Shiro, M. (2022). Improving time series pre*diction accuracy for the max*ima of **a f**low by reconstructions using local cross sections. Chaos: An Interdisciplinary Journal of Nonlinear Science 32(6) 063103. htt*ps://doi.org/***1**0.1063/5.0092433

Hirata, Y., Shiro, M., Takahashi, N., Aihara, K., Suzuki, H., & Mas, P. (2015). Approximating high-dimensional dynamics by barycentr*ic coordinate***s** with linear programming. Chaos: An Interdisciplinary Journal of Nonlinear Science 25(1), 013114. https://doi.org/10.1063/1.4906746

Huke, J. P., & Broomhead, D. S. (2007). Embedding theorems for non-uniformly *sampled dynamical systems. Nonlinearity 20(9), 2205-2244***.** http://dx.doi.org/10.1088/0951-7715/20/9/011

Sauer, T., Yorke, J.A. & Casdagli, M. (1991). Embedology. J Stat Phys 65, 579–616 https://doi.org/10.1007/BF01053745

Takens, F. (1981). Detecting strange attractors in turbu*lence. In: Rand, D., Young, LS. (eds) Dynamical Systems a***nd** Turbulence, Warwick 1980. Lecture Notes in Mathematics, vol. 898, Springer, Berlin, *Heidelberg. https://doi.org/10.10****07/***BFb0091924

Yunpeng Zhang* & Yoshito Hirata
Institute of Systems and Information Engineering, Graduate School of Science and Technology,
University of Tsukuba, 1-1-1 Tennodai, Tsukuba, Ibaraki 305-8573, Japan

* Corresponding author: yunpeng.zhang829@gmail.com