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Hybrid Sliding Mode and Model Predictive Control for Unmanned Surface Vehicles
(Kawalan Mod Gelincir dan Kawalan Ramalan Model Hibrid untuk Kenderaan Permukaan Tanpa Pemandu)

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ABSTRACT

Unmanned Surface Vehicles (USVs) are widely deployed in marine exploration and operations and thus require heading controllers that remain robust to model mismatch and environmental disturbances while safeguarding rate-limited actuators from chatter-induced stress. Classical sliding-mode control (SMC) offers disturbance tolerance but suffers from high-frequency chattering; model predictive control (MPC) enforces constraints but, when naively integrated with SMC, often produces discontinuous mode transitions that degrade smoothness and feasibility. Prior SMC–MPC approaches typically (i) embed the sliding manifold in a single MPC objective or (ii) hard-switch between SMC and MPC, leaving residual chatter and command jumps unresolved. This study proposes a dual-MPC framework with soft-switching that architecturally decouples the reaching and sliding phases and continuously blends them by a weight scheduled with the distance to the sliding surface. Two homogeneous, Reaching-MPC and Sliding-MPC are combined under unified amplitude constraints on the final command with saturation re-injection to restore prediction–execution consistency. Evaluated on the Cybership-II 3-DOF model with environmental disturbances in Matlab, the proposed controller achieves guaranteeing chattering suppression without compromising robust performance. It eliminates the residual chatter characteristic of Single-MPC schemes and avoids the command discontinuities inherent in hard-switching approaches.

Keywords: Unmanned Surface Vehicles, Sliding Mode Control, Model Predictive Control

ABSTRAK

Kenderaan Permukaan Tanpa Pemandu (USV) digunakan secara meluas dalam penerokaan dan operasi marin, dan oleh itu memerlukan pengawal haluan yang kekal teguh terhadap ketidakpadanan model serta gangguan persekitaran, sambil melindungi penggerak yang terhad kadarannya daripada tegasan akibat "chatter". Kawalan mod gelincir (SMC) klasik memang toleran terhadap gangguan namun mengalami "chattering" frekuensi tinggi; kawalan ramalan berasaskan model (MPC) pula menguatkuasakan kekangan tetapi, apabila digabungkan secara langsung dengan SMC, sering menghasilkan peralihan mod yang tidak berterusan sehingga menjejaskan kehalusan dan kebolehlaksanaan. Pendekatan SMC–MPC terdahulu lazimnya (i) membenamkan permukaan gelincir ke dalam satu fungsi objektif MPC atau (ii) melakukan pertukaran keras (hard-switch) antara SMC dan MPC, yang meninggalkan chatter sisa dan loncatan arahan. Kajian ini mencadangkan rangka kerja dwi-MPC dengan pertukaran lembut (soft-switching) yang memisahkan secara seni bina fasa "reaching" dan "sliding", serta mengadunnya secara berterusan melalui pemberat yang dijadualkan mengikut jarak ke permukaan gelincir. Dua MPC homogen—Reaching-MPC dan Sliding-MPC—digabungkan di bawah kekangan amplitud bersepadu pada arahan akhir, dengan suntikan semula ketepuan (saturation re-injection) untuk memulihkan konsistensi ramalan-pelaksanaan. Dinilai pada model Cybership-II tiga darjah kebebasan (3-DOF) dengan gangguan persekitaran dalam Matlab, pengawal yang dicadangkan membuktikan penekanan chatter tanpa menjejaskan prestasi keteguhan, menghapuskan chatter sisa yang lazim dalam skim MPC tunggal, dan mengelakkan ketakselanjaran arahan yang wujud secara intrinsik dalam pertukaran keras.

Kata Kunci: Kenderaan Permukaan Tanpa Pemandu , Kawalan Mod Gelincir , Kawalan Ramalan Berasaskan Model

INTRODUCTION

Unmanned surface vehicles (USVs) are vessels that can be remotely operated via networked communication equipment, eliminating the need for onboard personnel during missions. With advances in modern technology, maritime scientific investigations increasingly require access to environments with severe wind and wave conditions, which can pose substantial risks to researchers' safety. Against this background, the development and study of USVs have emerged as a practical and timely response. In the early stages, USV technology was primarily applied to military reconnaissance. During World War II, for example, the military employed USVs for mine countermeasures to secure navigation routes for transport ships. Subsequently, with progress in surveillance and precision-guidance technologies, USVs owing to their operational efficiency and maneuverability have become attractive platforms for maritime strategic reconnaissance and monitoring, precision engagement, and intelligence collection. In the civilian domain, USVs can carry sonar systems to detect seabed fish distributions, thereby improving fishing efficiency. They can also be equipped with meteorological sensors to observe the formation of tropical storms at sea and provide reliable data support for land-based weather forecasting centers. In addition, USVs have demonstrated promising performance in applications such as oil exploration and maritime search-and-rescue logistics. Overall, USVs offer advantages including strong interoperability with other platforms and intelligent onboard systems, rapid response enabled by remote control, and reduced operational risk. These characteristics facilitate more effective exploration and utilization of marine resources. As a result, USVs are expected to see broad future development and hold significant research value in both civilian and military contexts.

This study uses the CyberShip II experimental USV developed at the Norwegian University of Science and Technology as research subject. Measuring approximately 1.25 meters in length and weighing 24 kilograms, and it features a vectored-thrust configuration. CyberShip II is commonly employed in low-speed dynamic positioning and path-following research.

FIGURE 1. Photograph of the CyberShip II

As USVs typically operate in dynamic and uncertain marine environments characterized by persistent wave, wind, and current disturbances, achieving reliable trajectory tracking and robust control performance is paramount. Among various control strategies, sliding mode control (SMC) is a nonlinear control method distinguished by its strong robustness against model uncertainties and external disturbances.

The fundamental principle of SMC involves designing a stable sliding surface, $s(x) = 0$, in the state space. A discontinuous control law is then synthesized to force the system state trajectories to reach this surface in finite time and subsequently remain constrained to it. Once the sliding motion is established, the system dynamics are governed by the reduced-order dynamics of the sliding surface itself, exhibiting invariance to so-called matched perturbations. This invariance forms the theoretical cornerstone of SMC's robustness. Consequently, its design process is structurally straight forward, primarily comprising the design of the sliding surface and the reaching control law. However, the practical application of SMC is significantly impeded by the chattering phenomenon. In theoretical analysis, the control law assumes ideal, infinitely fast switching. In physical implementations, this discontinuity interacts with inevitable non-idealities such as actuator saturation, sensor noise, and computational delays. This conflict manifests as high-frequency, finite-amplitude oscillations of the state trajectory around the sliding surface, rather than ideal sliding. Chattering is highly undesirable as it not only degrades control precision and tracking performance but also induces excessive wear on mechanical actuators, increases energy loss, and may potentially excite unmodeled high-frequency dynamics, thereby threatening system stability. Therefore, chattering suppression remains a critical research focus in advancing SMC for real-world applications.

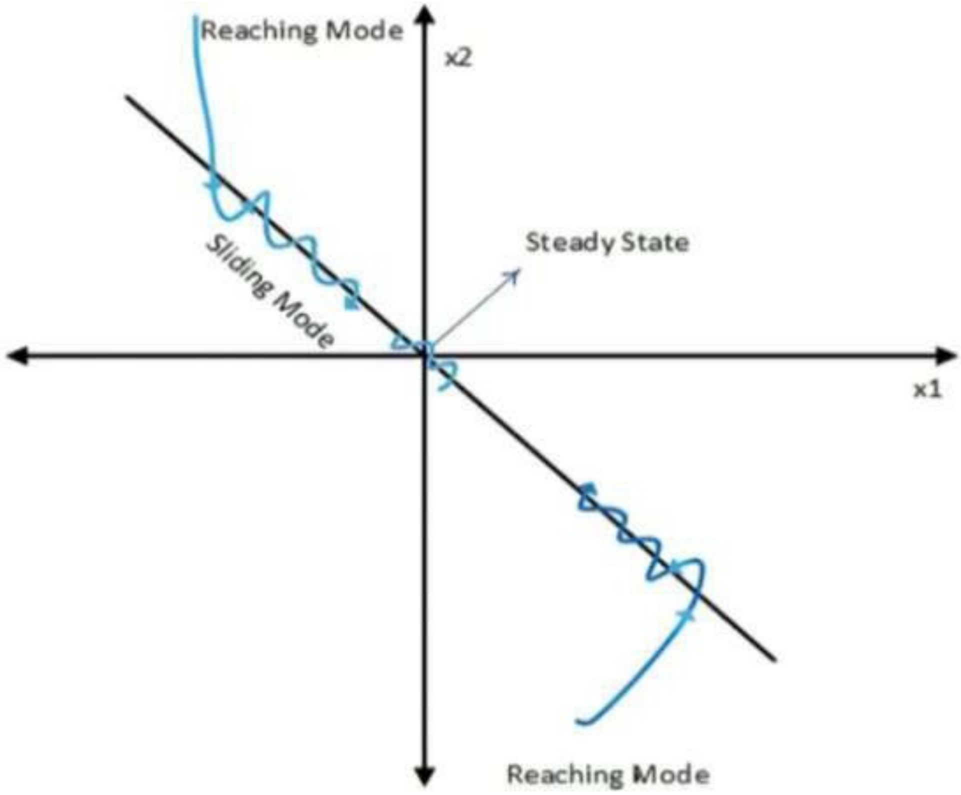

FIGURE 2. Schematic Diagram of Sliding Mode Control

Model predictive control (MPC) represents a widely adopted advanced control strategy, particularly suited for systems with constraints and complex dynamics. In contrast to traditional controllers, MPC employs a receding-horizon optimization framework. At each sampling instant, it solves a finite-horizon optimal control problem based on a dynamic model of the system, explicitly incorporating state and input constraints. Only the first control action of the optimized sequence is applied to the plant, and the process is repeated at the next time step with updated state information. This mechanism enables MPC to handle multi-variable coupling, actuator limits, and explicit path constraints in a unified and systematic manner, making it highly effective for precise trajectory tracking and collision avoidance in dynamically changing marine environments. However, its practical implementation is often constrained by the substantial computational burden associated with solving online optimization problems in real time, which poses a key challenge for platforms with limited processing capabilities.

The integration of SMC with MPC presents a promising approach for the motion control of USVs. This hybrid control strategy effectively combines the strengths of both methodologies, with MPC providing an explicit framework for constraint handling and multi-variable optimization while SMC delivers robust tracking performance in the presence of model uncertainties and external disturbances. The primary advantage of this integration lies in its ability to address the complementary limitations of each method when applied individually. Specifically, the MPC framework's rolling-horizon optimization can be utilized to compute an optimal reference trajectory or a nominal control input that respects system constraints, such as actuator limits and environmental boundaries. This optimized signal then serves as the command for a subsequent SMC layer, which ensures precise tracking despite environmental disturbances and unmodeled hydrodynamic effects. This architecture not only preserves the inherent robustness of SMC but also enhances overall system performance through the predictive and constraint-aware capabilities of MPC. Furthermore, this hybrid approach offers potential benefits in chattering mitigation. By integrating the predictive model and smoothing capabilities of MPC into the control law design, the discontinuous switching inherent in conventional SMC can be softened or strategically regulated, thereby reducing high-frequency oscillations while maintaining disturbance rejection characteristics. This results in improved control precision and reduced mechanical wear on actuators, which is particularly advantageous for USVs operating in complex and variable marine conditions.

Previous studies on SMC-MPC integration have primarily adopted two strategies. The first category embeds the sliding variable $s(x)$ into the MPC formulation, e.g., by penalizing $s(x)$ in the cost function or enforcing $(s(x) = 0)$ as a soft/hard constraint, aiming to steer the state toward and keep it near the sliding manifold while satisfying input/state constraints. A key limitation is that the reaching/maintenance mechanism of sliding mode is realized only indirectly through continuous optimization, so the closed-loop behavior is often practical convergence and can be sensitive to weight tuning. Moreover, discretization and non-ideal actuator effects such as saturation/rate limits introduce prediction–execution mismatch, which may lead to residual oscillations near the manifold and complicate stability and recursive-feasibility analysis. The second category relies on hard switching between SMC and MPC, e.g., using SMC when the error is large or the state is far from the manifold to enhance disturbance rejection, and switching to MPC near the manifold to explicitly enforce constraints and improve smoothness. This strategy is straightforward to implement, but it commonly exhibits two issues: (i) switching can introduce input discontinuities and, under noise/disturbances, lead to frequent mode switching; (ii) since the feasible sets and closed-loop dynamics of the two controllers are not necessarily compatible, switching may cause transient performance degradation and increase the risk of operating near constraint boundaries, making unified guarantees of continuity, feasibility, and stability harder to establish.

To address these limitations, we propose an improved SMC-MPC integration framework for USVs that explicitly separates the reaching and sliding objectives using two-phase MPC and connects them via distance-scheduled continuous blending. Specifically, we formulate two homogeneous MPC problems Reaching-MPC and Sliding-MPC each optimized under the same input constraints, and generate the final control command through a continuous weighting function parameterized by the distance to the sliding manifold, rather than threshold-based hard switching. This design aims to reduce command jumps and mode chattering while retaining constraint awareness throughout the maneuver. The proposed framework is intended to improve command continuity and practical implement ability while preserving robustness against disturbances and model uncertainties.

This study makes two main contributions: (i) We introduce a two-phase (reaching/sliding) dual-MPC architecture that structurally decouples the objectives while enforcing a unified set of input constraints in both phases. (ii) We propose a distance-to-manifold–scheduled continuous blending mechanism that replaces hard switching and improves command continuity near the manifold.

METHODOLOGY

In this study, the proposed controller is implemented and evaluated through modeling and simulation in MATLAB/Simulink. The overall workflow consists of (i) establishing the USV kinematic and dynamic models, (ii) formulating the constrained control problem, and (iii) designing and testing the proposed SMC–MPC framework under representative disturbances and actuator limitations.

When describing the motion of a USV on the horizontal plane, two reference frames are commonly employed. One is the earth-fixed frame, also referred to as the global or inertial frame, denoted as $\{n\}$ $= (x_n, y_n, z_n)$. This frame is fixed to the Earth and is typically employed to define the USV's position and

TABLE 1. Comparison between prior SMC-MPC methods and the proposed hybrid method

Method	Controller structure	Phase handling	Switching	Constraint handling	Main limitation
(A) SMC embedded in single MPC	Single MPC with sliding constraint	Reaching & Sliding coupled in one objective	No explicit switching	Constraints enforced in MPC, but may suffer prediction–execution mismatch	Practical convergence; weight-sensitive; residual oscillations near manifold
(B) Hard switching SMC-MPC	Two controllers (SMC + MPC)	Phase separation via thresholds	Hard switching (threshold-based)	MPC handles constraints only in its mode; switching may violate continuity	Command jumps; mode chattering under noise/disturbances; feasible-set mismatch across modes
Proposed hybrid method	Two-phase dual MPC (Reaching-MPC + Sliding-MPC)	Explicit reaching vs sliding objectives	Distance-scheduled continuous blending	Unified input constraints in both MPCs + prediction–execution consistency handling	Higher computational cost (two optimizations)

orientation. The other is the body-fixed frame, attached to the USV itself and denoted as $\{b\} = (x_b, y_b, z_b)$. This frame is generally utilized for analyzing physical quantities such as acceleration and rudder angle. The kinematic relationship between these two frames can be established through coordinate rotation transformations.

When a USV operates on the water surface, it has six degrees of freedom(6-DOF): translations along x_b, y_b, z_b— surge, sway, heave; and rotations about x_b, y_b, z_b— roll, pitch, yaw. In low-speed dynamic positioning and path following problems, the motion of USV is commonly simplified to the horizontal plane (xoy), considering only 3-DOF: surge, sway, and yaw.

For the 3-DOF kinematic model of the USV, the vector $\nu = [u \;\; v \;\; r]^T$ represents the surge, sway, and yaw velocities in body fixed frame. Vector $\eta = [x \;\; y \;\; \psi]^T$ represents the surge and sway positions and the yaw angle in the earth-fixed frame. The relationship between these two vectors is given by Equation (1):

$$\dot{\eta} = R(\psi)\nu \qquad (1)$$

where $R(\psi)$ is the transformation matrix from the body-fixed frame to the earth-fixed frame (satisfying $R^{-1} = R^T$), expressed as Equation (2):

$$R(\psi) = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

As for the dynamic model, defining $\nu = [u \;\; v \;\; r]^T$ as body-fixed velocities and $\tau = [X \;\; Y \;\; N]^T$ as generalized forces. The motion obeys Equation (3):

$$M\dot{\nu} + D\nu = \tau + d \qquad (3)$$

where M includes added mass, D is linear damping, and d denotes environmental disturbances. A

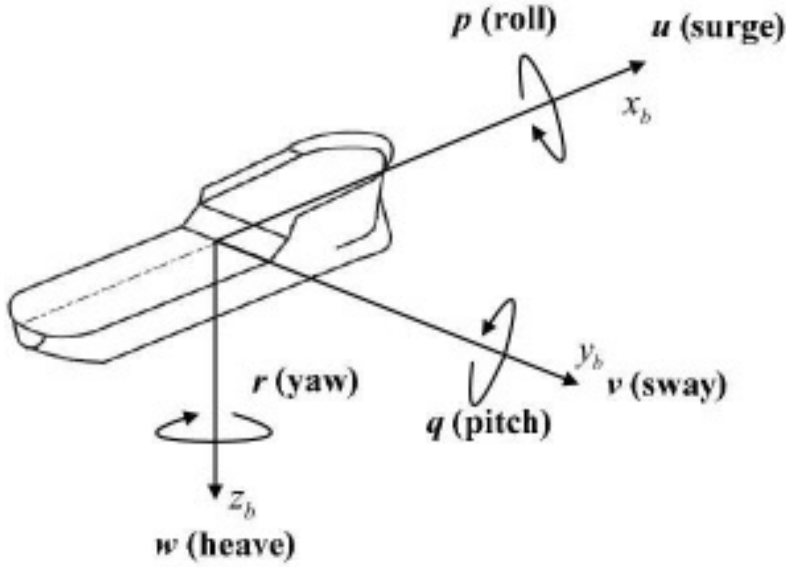

FIGURE 3. Six Degrees of Freedom of USV

conventional sliding-mode controller is implemented as a baseline heading controller.

The control input is the yaw moment N, and the generalized force vector is set to $\tau = [0,0,N]^{\mathrm{T}}$. Let ψ and r denote the heading angle and yaw rate, respectively, and ψ_{d} be the heading reference with its first and second derivatives $\dot{\psi}_{\mathrm{d}}$ and $\ddot{\psi}_{\mathrm{d}}$. The heading error is defined using an angle-wrapping form to avoid discontinuities:

$$e_{\psi} = atan2(\sin(\psi - \psi_{\mathrm{d}}), \cos(\psi - \psi_{\mathrm{d}})) \quad (4)$$

The sliding variable is chosen as

$$s = (r - \dot{\psi}_{\mathrm{d}}) + \lambda e_{\psi} \quad (5)$$

where $\lambda > 0$ is a design parameter. The yaw moment command is composed of an equivalent term and a switching term:

$$N_{\mathrm{cmd}} = N_{\mathrm{eq}} + N_{\mathrm{sw}} \quad (6)$$

The equivalent term N_{eq} compensates the nominal yaw dynamics and incorporates the reference acceleration, while the switching term enforces the reaching law

$$N_{\mathrm{sw}} = -I_z k \; sign(s) \quad (7)$$

with $k > 0$ In simulations, $\lambda = 0.4$ and $k = 0.5$ are used. This baseline uses a pure sign function and is therefore expected to exhibit residual chattering, which motivates the proposed SMC–MPC integration introduced later.

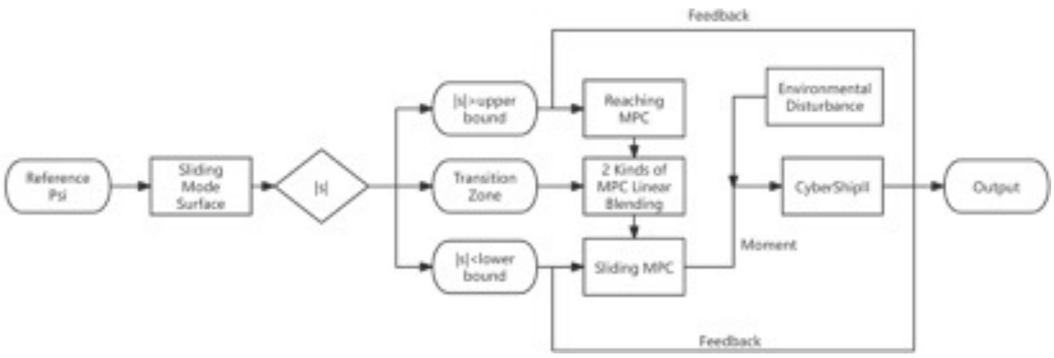

FIGURE 4. Workflows of SMC-MPC Simulation System

This study replaces a single-MPC or hard-switching scheme with a dual-MPC architecture specialized for the two SMC phases and blended by a distance-scheduled soft transition. This study designs distinct cost functions for the two MPC controllers. The Reaching-MPC is responsible for the reaching phase when the system state is far from the sliding surface. Its cost function is designed to prioritize convergence speed:

$$J_{Reach} = \sum_{k=0}^{N}(Q_{Reach}|s(k)|^2 + R_{Reach}\Delta N_{Reach}(k)^2) \quad (8)$$

where a high weighting Q_{Reach} on the sliding surface distance $|s|$ encourages aggressive control action to drive the state towards the manifold rapidly.

Conversely, The Sliding-MPC takes over in the vicinity of the sliding surface. Its cost function is tailored for stability and smoothness:

$$J_{Slide} = \sum_{k=0}^{N}(Q_{Slide}|s(k)|^2 + R_{Slide}N_{Slide}(k)^2 + S_{Slide}\Delta N_{Slide}(k)^2) \quad (9)$$

Large penalties on both the control input N_{Slide} and its rate ΔN_{Slide} are key to suppressing chattering at the source by explicitly penalizing high-frequency control activity.

The blending weight $w \in [0,1]$ is linearly interpolated over a distance band $[s_{\text{lower}} \ s_{\text{upper}}]$ to avoid hard switching. Equation is as follows:

$$w = (|s| - s_{\text{lower}})/(s_{\text{upper}} - s_{\text{lower}}) \quad (10)$$

$$N_{\text{Final}} = wN_{\text{Reaching}} + (1-w)N_{\text{Sliding}} \quad (11)$$

To define the blending weight (w), two parameters, s_{lower} and s_{upper}, are selected. The parameter s_{lower} represents a lower bound on the distance from the sliding surface, which corresponds to a region where the system is close to the sliding surface but has not yet achieved full stability. It is typically chosen based on the desired convergence speed and system dynamics. s_{upper}, on the other hand, represents an upper bound that defines the region far from the sliding surface where the system is not yet close to the desired state. The values of s_{lower} and s_{upper} are typically set experimentally, ensuring that the weight (w) transitions smoothly from 0 to 1 as the system approaches the sliding surface. For example, s_{lower} might be set to a small threshold based on empirical tests of the system's behavior, while s_{upper} could be set based on the distance at which the system is far enough from the sliding surface that aggressive control actions are no longer needed. These values are tuned in the simulation to achieve smooth blending and avoid discontinuities inherent in hard-switching schemes both MPCs and the blended command share the same torque and rate limits. The executed command N_{Final} after limiting is re-injected to initialize the next prediction. Then:

$$|\Delta N_{\text{Final}}| \le |\Delta w| \cdot |N_{\text{Reaching}} - N_{\text{Sliding}}| + max(w, 1-w)\Delta_{\text{max}} \quad (12)$$

Linear interpolation makes $|\Delta w|$ small. The soft-switching law (11) ensures that N_{Final} is a continuous function of the sliding surface distance $|s|$. This eliminates the discontinuous jumps inherent in hard-switching schemes. The bounded change in the weight w directly leads to a bounded change in the final command $|\Delta N_{\text{Final}}|$, theoretically guaranteeing smoothness and mitigating chatter.

RESULTS & DISCUSSION

In MATLAB/ Simulink, two control architectures were implemented for comparison: a baseline SMC and the proposed MPC–SMC hybrid controller. The simulation was conducted for 60 s, and the resulting responses are shown in Figures 5 – 7.

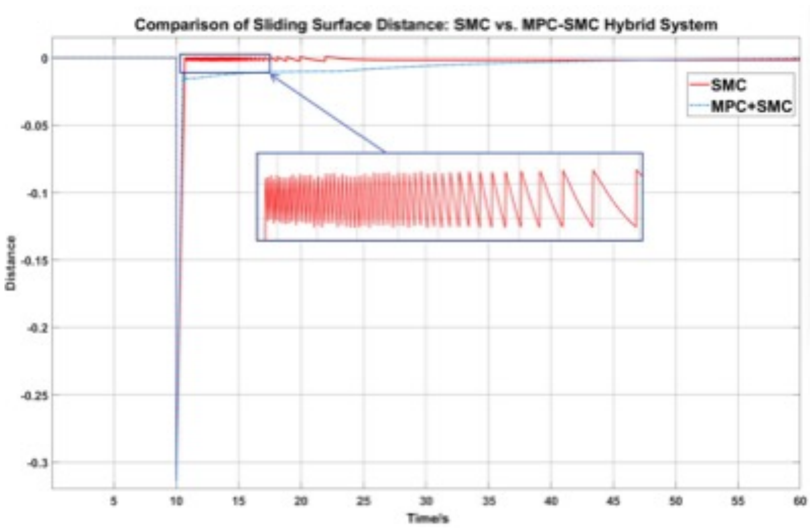

FIGURE 5. Comparison of Sliding Surface Distance between Conventional SMC and Proposed Hybrid SMC-MPC System

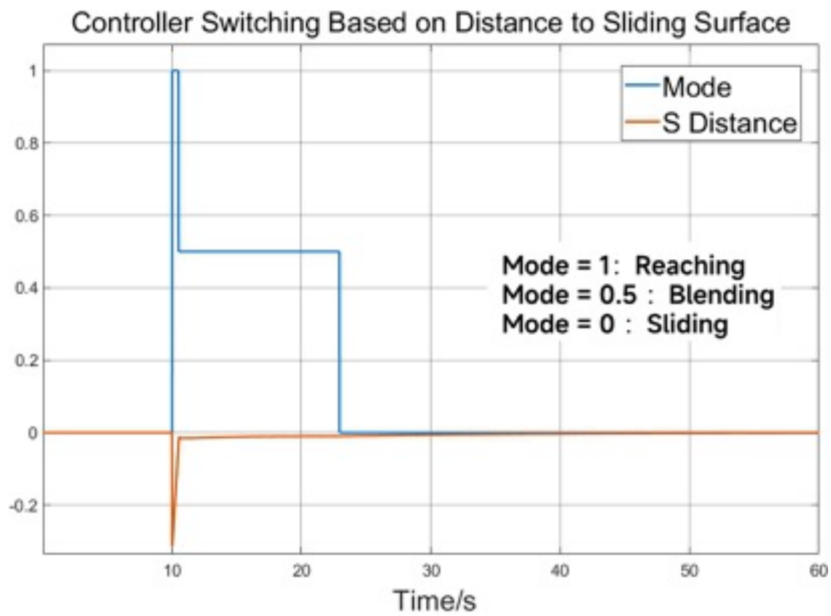

FIGURE 6. Controller Switching Based on Distance

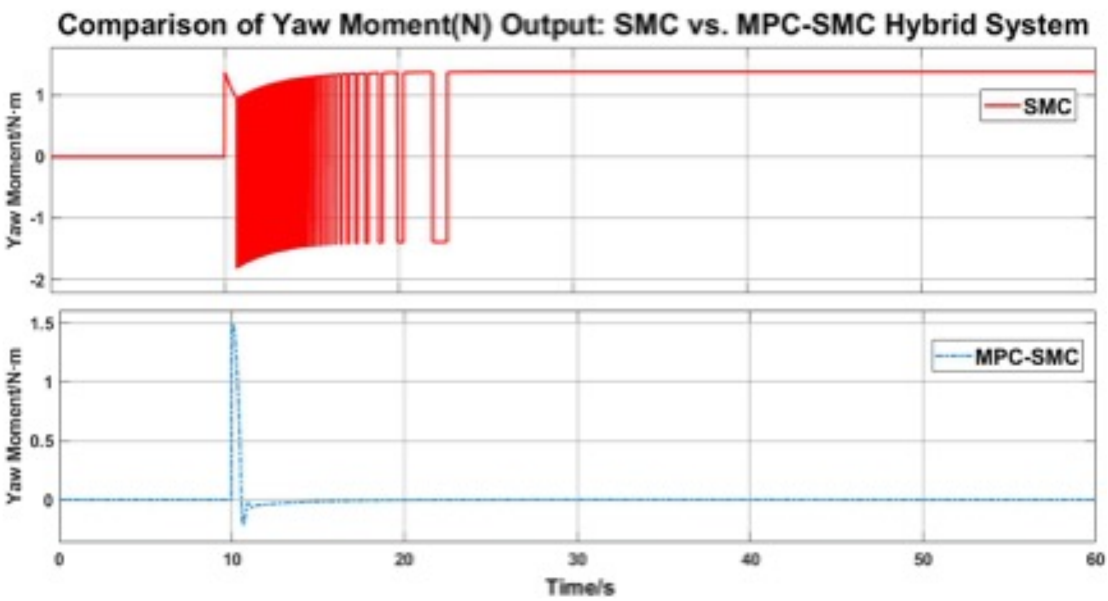

FIGURE 7. Comparison of Moment Output

Figure 5 compares the time histories of the sliding-surface distance for the conventional SMC and the proposed MPC-SMC hybrid controller. Both approaches drive the system state toward the sliding manifold after the heading command is applied; however, their behaviors near the manifold differ markedly. The hybrid controller converges to the vicinity of the sliding surface with a smooth and gradually decaying trajectory, indicating that the reaching process is regulated without inducing high-frequency activity. In contrast, once the conventional SMC reaches the manifold, the sliding variable exhibits pronounced oscillations with a clear high-frequency component, which is characteristic of chattering arising from the discontinuous switching term. The zoomed-in view further highlights that these oscillations persist for an extended period, implying sustained switching activity even after the nominal sliding condition is achieved.

Figure 6 provides additional insight into the internal operation of the proposed framework by showing the mode evolution of the two MPC components as a function of the distance to the sliding surface. Consistent with the distance-scheduled design, the controller initially assigns a higher weight to the reaching-oriented MPC when $|s|$ is large, thereby prioritizing rapid convergence toward the manifold. As $|s|$ decreases and enters the predefined transition band, the blending weight is smoothly adjusted, resulting in a progressive handover from the reaching MPC to the sliding MPC. Once the state approaches the manifold sufficiently closely, the sliding-oriented MPC dominates, emphasizing stability and command smoothness in the vicinity of $s \approx 0$. Importantly, the mode trajectory does not exhibit rapid toggling, suggesting that the proposed distance-based blending effectively avoids the frequent switching behavior that is often observed in threshold-based hybrid schemes under measurement noise and disturbances.

The corresponding control effort is summarized in Figure 7 through the yaw-moment responses. The conventional SMC generates a torque command with rapid sign changes and large high-frequency components, which directly reflect the discontinuous switching action used to enforce the reaching law. Such a control pattern can impose substantial actuator stress, increase energy consumption, and amplify wear, especially in practical systems with finite bandwidth, saturation, and rate limits. By contrast, the proposed MPC–SMC hybrid system produces a smooth and well-behaved yaw-moment command. The control action remains continuous and exhibits significantly reduced high-frequency content, demonstrating that the hybrid framework effectively suppresses chattering at its source rather than merely filtering it after generation.

Taken together, the results in Figures 5 – 7 indicate that the proposed method preserves the ability to drive the system toward the sliding manifold while substantially improving input smoothness and practical implement-ability, thereby enabling more stable and efficient actuation for USV heading control.

CONCLUSION

This study designed and validated a hybrid SMC-MPC control framework for USV heading regulation, aiming to reconcile fast transient performance with practical actuator-friendly command generation. By adopting a dual-MPC architecture with a distance-scheduled soft transition, the proposed method addresses key limitations of existing SMC-MPC integration strategies: it suppresses the residual oscillations that may remain when the sliding objective is embedded in a single MPC formulation, and it avoids the command discontinuities and mode chattering commonly induced by hard-switching schemes. Simulation results demonstrate that the hybrid controller drives the system toward the sliding manifold while producing a smooth yaw-moment command, thereby improving practical implement-ability and reducing actuator stress.

Despite these benefits, several limitations remain. First, the proposed framework may require a longer settling time in heading regulation under certain tuning choices, reflecting an inherent trade-off between aggressive convergence and smoothness in the vicinity of the sliding surface. Second, the dual-MPC structure increases computational demand relative to conventional SMC or single-MPC baselines, which may constrain real-time deployment on embedded hardware without further optimization. In addition, the current validation is limited to simulation with a specific plant model and disturbance setting; broader robustness assessments under more diverse sea conditions, sensor noise levels, and model uncertainty patterns are needed.

Future work will therefore focus on improving convergence speed through systematic weight/ horizon tuning and adaptive scheduling, as well as reducing computational complexity via efficient QP solvers, model reduction, and warm-start strategies. Moreover, we will extend the insights gained from this heading-control study to the more challenging problem of path tracking for USVs, where coupled dynamics, stronger environmental disturbances, and stricter constraints require tighter integration between guidance and control.

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